

# Convective atmospheric boundary layer using LES

U. Vigny<sup>a,b</sup>, L. Voivenel<sup>b</sup>, S. Zeoli<sup>a</sup>, and P. Benard<sup>b</sup>

<sup>a</sup>Université de Mons (UMONS), Polytechnic Faculty, Belgium.

<sup>b</sup>CORIA, CNRS UMR6614, Normandie Université, INSA and University of Rouen, 76801 Saint-Etienne-du-Rouvray, France.

E-mail: [ulysse.vigny@umons.ac.be](mailto:ulysse.vigny@umons.ac.be)

*Keywords:* Convective boundary layer, Thermal stratification, Monin-Obukhov similarity theory, LES

## 1 Introduction

According to the current energetic and environmental challenges, maximizing the electric power generated in wind-farms and minimizing the wind turbine fatigue is a societal concern. To increase generated power, rotors size have significantly increased over time, leading to hundred of meters diameter wind turbine. Such wind turbines are no longer in micro-scale windflow but at the interface between micro-scale and meso-scale [1]. Therefore, a better understanding of the atmospheric flow physics around wind turbines at these scales is necessary. The aim is to accurately analyze the interaction between wind turbines wake and the atmospheric boundary layer. Various physical phenomena are implied in atmospheric flows, such as thermal stratification, Coriolis force, humidity effect, inflow turbulence, surface terrain, etc. This work focuses on one physical phenomenon: the thermal stratification. It has a significant impact on wind turbines [2], in terms of wake recovery, velocity deficit, induced turbulence, power production, loads and fatigue. Because of the large range of scale motions, direct numerical simulation (DNS) of turbulent flows is unrealistic on such applications. To allow an accurate numerical prediction of such flows, the large-eddy simulation (LES) technique appears promising. However, a LES resolving the whole atmospheric boundary layer is computationally unaffordable. Therefore, a model is needed to correctly predict the wall velocity and temperature profiles. For this purpose, the Monin-Obukhov Similarity Theory [3, 4] is used.

The goal of this work is to implement into the YALES2 solver [5] a wall model taking into account a non-neutral atmospheric boundary layer and thus to obtain accurate wind turbines and wind farm simulation in realistic atmospheric conditions. To do so, the Monin-Obukhov Similarity Theory, detailed in the Section 2, is implemented. The validation of this methodology is performed through a test case developed by Willis and Deardorff [6], detailed in Section 3. Results are compared with experimental data obtained later on by the same authors [6] and from Deardorff and Willis extensive results [7] as well as from Schmidt and Schumann numerical results [8].

## 2 Methodology

Taking thermal stratification effects into account in a wall-modeled LES approach is non-trivial due to the various possible atmospheric configurations: neutral, stable, unstable boundary layer. These three cases can be split as a function of the thermal flux, named  $Q_w$ . The neutral boundary layer is the configuration with no thermal effect, i.e.  $Q_w = 0$ . The stable boundary layer is the configuration with a negative flux,  $Q_w < 0$ , going from the top to the bottom of the domain. It means that cold air is at the bottom and hot air at the top. The unstable or convective boundary layer is the configuration with a positive flux,  $Q_w > 0$ , which goes from the bottom to the top, leading to cold air at the top and hot air at the bottom. The Monin-Obukhov similarity theory proposes velocity and temperature profiles adequate to each configuration where the approach is identical, but correction terms will differ [9]. Velocity profile can be expressed as a logarithmic law, with a correction term  $\Psi_m$ :

$$\bar{u}(z) = \frac{u_*}{k} \left[ \ln \left( \frac{z}{z_0} \right) - \psi_m \left( \frac{z}{L} \right) + \psi_m \left( \frac{z_0}{L} \right) \right] \quad (1)$$

where  $u_* = \sqrt{\tau_w/\rho}$  is the friction velocity.  $\tau_w$  refers to the local shear stress at the wall.  $\rho$  is the fluid density.  $\kappa$  the von Karman constant.  $z_0$  the roughness length.  $\Psi_m$  the correction function.  $L$  the Obukhov length which represents the height above the surface from where buoyancy first dominates shear computed as  $L = -\frac{u_*^3 \theta_0}{\kappa g q_w}$ .  $g$  is the Earth's gravity.  $q_w$  the kinematic surface heat flux.  $\theta_0 = 299.8$  K is the mean flow temperature.

For neutral cases, the correction term are zero, leading to a simple logarithmic velocity profile. For the stable configuration, the correction term becomes:

$$\Psi_m(\xi) = 1 - \phi_m(\xi) \quad (2)$$

$$\text{where } \phi_m(\xi) = 1 + 5\xi \quad (3)$$

For the convective configuration, the same quantity writes:

$$\Psi_m(\xi) = 2 \ln \left( \frac{1 + \phi_m^{-1}(\xi)}{2} \right) + \ln \left( \frac{1 + \phi_m^{-2}(\xi)}{2} \right) - 2 \arctan(\phi_m^{-1}(\xi)) + \frac{\pi}{2} \quad (4)$$

$$\text{where } \phi_m(\xi) = (1 - 16\xi)^{-1/4} \quad (5)$$

From Equation 1 it appears that the velocity is based on Monin-Obukhov length  $L$ , which depends on the friction velocity  $u_*$ , itself related to the velocity. Thus, an analytical solution cannot be found, and a numerical approach is needed. Therefore, a convergence algorithm based on  $u_*$  is implemented to find the correct velocity. This algorithm needs input data to start. These input data vary in the literature [10]. Apart from classical input data such as roughness, mean flow temperature, density, the use of sensible heat flux or temperature as a surface boundary condition is questionable. Even if it seems that for stable boundary layer it changes the result [10], it is not the case for convective boundary layer. That is why in this work, where we study a convective boundary layer, we used surface sensible heat flux.

### 3 Simulation framework

#### 3.1 Flow solver

This methodology has been implemented in the massively-parallel finite-volume YALES2 flow solver [5], specifically tailored for Large-Eddy Simulation, which relies on a 4th-order central numerical scheme for spatial discretization associated to a 4th-order Runge-Kutta-like method for the time integration.

#### 3.2 Numerical setup

In order to validate our methodology and the Monin-Obukhov similarity theory implementation, the test case developed by Willis and Deardorff [6] is reproduced. Our numerical results will be compared to Willis and Deardorff first experimental results [6], Deardorff and Willis extensive results [7] and Schmidt and Schumann numerical results [8].

The case investigated corresponds to a Convective Boundary Layer uniformly heated from below and topped by a layer of uniformly stratified fluid (*ie* the inversion layer). It corresponds to a periodic box of size  $N_x \times N_y \times N_z = 256 \times 256 \times 128$  for a physical domain of size  $L_x \times L_y \times L_z = 8000 \times 8000 \times 2400$  m<sup>3</sup>. The spacial resolution is thus  $\Delta_x = \Delta_y = 31.25$  and  $\Delta_z = 18.75$  m for all three directions. The initial height of the inversion layer is also used to specify the initial condition. The initial temperature profile corresponds to the one of a mixed layer initially at  $\theta_0 = 299.8$  K topped by an inversion layer of uniform stability  $d(\theta)/dz = 0.0027$  K.m<sup>-1</sup>. Both temperature and velocity profiles are disturbed by a variable perturbation  $r$  randomly selected in  $[-0.5; 0.5]$ . The initial temperature profiles thus reads:

$$\theta(z) = \begin{cases} \theta_0 + 0.1r \left(1 - \frac{z}{z_m}\right) \theta_C^0 & \text{if } 0 < z \leq z_m, \\ \theta_0 + (z - z_m) \frac{d(\theta)}{dz} & \text{if } z > z_m. \end{cases} \quad (6)$$

Similarly, the initial velocity profile is given by:

$$w(z) = \begin{cases} 0.1r \left(1 - \frac{z}{z_m}\right) w_C^0 & \text{if } 0 < z \leq z_m, \\ 0, & \text{if } z > z_m, \end{cases} \quad (7)$$

where  $z_m = 1400$  m is the initial height of the mixed layer.

The fluid properties are typical of the ones encountered on a sunny day in southern Germany *ie* dry air of dynamic viscosity  $\nu = 2.15 \times 10^{-5} \text{ m}^2 \cdot \text{s}^{-1}$  and thermal diffusivity  $\alpha = 21.4 \times 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}$ .

As far as boundary conditions are concerned, the ground is heated by an uniform kinematic heat flux,  $q_w = 0.06 \text{ K} \cdot \text{m} \cdot \text{s}^{-1}$ , while the roughness height is  $z_0 = 0.16$  m. Schmidt & Schumann introduced a radiation boundary condition at the top of the domain to avoid spurious reflections of gravity waves. In our case, a sponge layer where source terms smoothly bring the velocity and scalar profiles up to their theoretical value is implemented for this purpose. This sponge layer has a height of 750 m and is discretized by 40 points. Our final domain dimension are then  $L_x \times L_y \times L_z = 8000 \times 8000 \times 3150 \text{ m}^3$  and the resolution remains the same as the one mentioned earlier. Finally, the lateral boundary conditions are considered periodic.

## 4 Results

Figures 1 and 2 represent the dimensionless horizontal and vertical velocity variance, respectively. The former is in overall lower than the latter, except at the edges of the mixed layer. Indeed, vertical velocity fluctuations are mostly due to buoyancy whereas horizontal velocity fluctuations mainly come from pressure fluctuations which are lower overall but predominant at the edges of the mixed layer. Both Figures 1 and 2 show that velocity profiles are matching those of Schmidt and Schumann. However, Deardorff and Willis's experimental data predicts higher horizontal velocity variances. Schmidt and Schumann [8] have suggested that horizontal variation of the surface heat flux, and thus the experimental setup itself, was the cause of those differences. Moreover, the Willis and Deardorff older measurements are much closer to all numerical results.

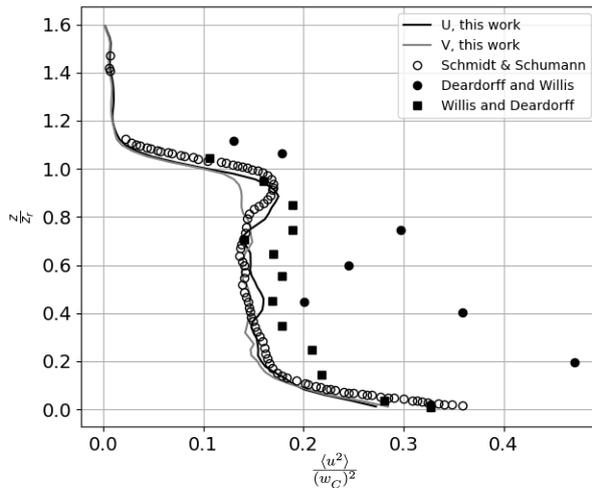


Figure 1: Dimensionless horizontal velocity variance function of dimensionless height.

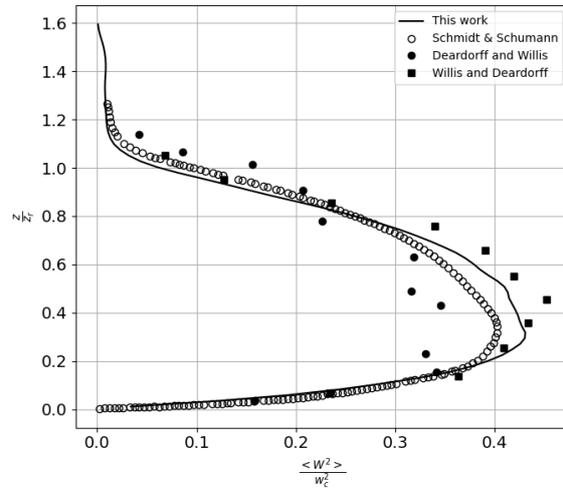


Figure 2: Dimensionless vertical velocity variance function of dimensionless height.

The dimensionless temperature and vertical heat flux variance are show on Figures 3 and 4, respectively. The temperature variance is low in overall, except at the edges of the mixed layer. Indeed, temperature variances are produced by the product of heat flux and temperature gradient, which is large near the surface and at the inversion height. On the other hand, vertical turbulent heat flux decrease linearly with height, up to the inversion layer. It implies a constant heating rate and thus the expected results. The obtained results match both Schmidt and Schumann's and the experimental data.

## 5 Conclusions

The implementation of the Monin-Obukhov similarity theory into YALES2 was performed and validated by reproducing a well known literature test case. Our results shown good agreement with experimental and numerical data. Therefore, convective boundary layer can now be used for real cases, such as wind turbine and wind farm

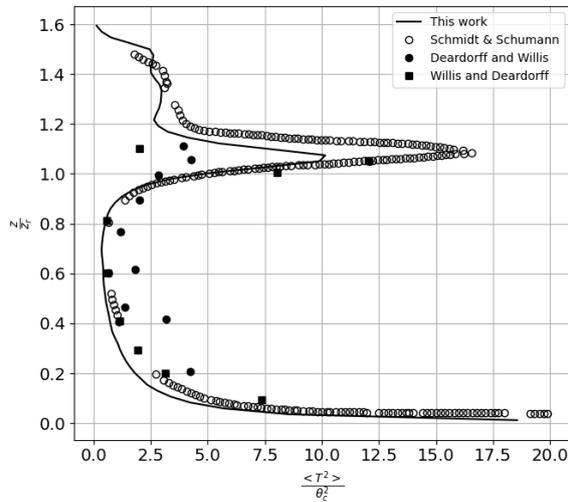


Figure 3: Dimensionless temperature variance function of dimensionless height.

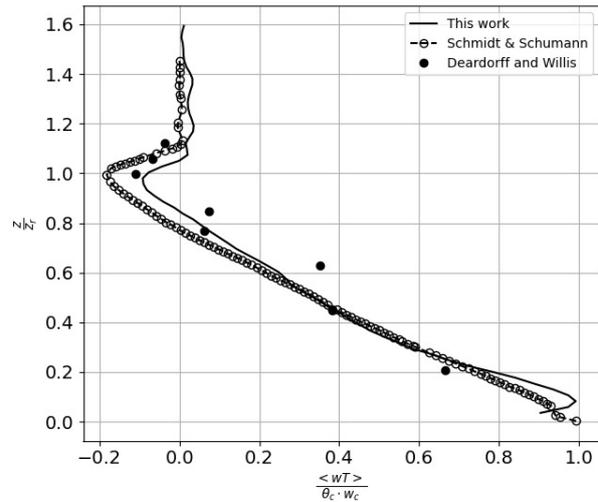


Figure 4: Dimensionless heat flux variance function of dimensionless height.

simulations. Yet, some investigation still needs to be done. Indeed it has been shown in [10] that this methodology, based on surface sensible heat flux, is suitable for convective boundary layer but not accurate for stable boundary layer. In this case, surface temperature based methodology leads to more reliable results. Therefore, it must be developed for this configuration.

## References

- [1] Paul Veers, Katherine Dykes, Eric Lantz, Stephan Barth, Carlo L Bottasso, Ola Carlson, Andrew Clifton, Johney Green, Peter Green, Hannele Holttinen, et al. Grand challenges in the science of wind energy. *Science*, 366(6464):eaau2027, 2019.
- [2] Majid Bastankhah and Fernando Porté-Agel. A new analytical model for wind-turbine wakes. *Renewable energy*, 70:116–123, 2014.
- [3] Andrei Sergeevich Monin and Aleksandr Mikhailovich Obukhov. Basic laws of turbulent mixing in the surface layer of the atmosphere. *Contrib. Geophys. Inst. Acad. Sci. USSR*, 151(163):e187, 1954.
- [4] LD Landau and EM Lifshitz. Fluid mechanics. pergamon press, oxford. *Section 92, problem, 2*, 1959.
- [5] Vincent Moureau, Pascale Domingo, and Luc Vervisch. Design of a massively parallel cfd code for complex geometries. *Comptes Rendus Mécanique*, 339(2-3):141–148, 2011.
- [6] GE Willis and JW Deardorff. A laboratory model of the unstable planetary boundary layer. *Journal of Atmospheric Sciences*, 31(5):1297–1307, 1974.
- [7] JW Deardorff and GE Willis. Further results from a laboratory model of the convective planetary boundary layer. *Boundary-Layer Meteorology*, 32(3):205–236, 1985.
- [8] Helmut Schmidt and Ulrich Schumann. Coherent structure of the convective boundary layer derived from large-eddy simulations. *Journal of Fluid Mechanics*, 200:511–562, 1989.
- [9] Jagadish Chandran Kaimal and John J Finnigan. *Atmospheric boundary layer flows: their structure and measurement*. Oxford university press, 1994.
- [10] Sukanta Basu, Albert AM Holtslag, Bas JH Van De Wiel, Arnold F Moene, and Gert-Jan Steeneveld. An inconvenient “truth” about using sensible heat flux as a surface boundary condition in models under stably stratified regimes. *Acta Geophysica*, 56(1):88–99, 2008.