






# Risk-Aware Battery Bidding With a Novel Benchmark Selection Under Second-Order Stochastic Dominance

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**Abstract**—This paper studies the risk management of a battery bidding in both day-ahead and intraday markets arising from the uncertain nature of electricity prices. To this end, a coherent risk measure, Second-order Stochastic Dominance (SSD), which is capable of expressing battery preferences in the form of a preset fixed benchmark (profit), is incorporated into the bidding model. The SSD serves the decision-maker as a risk-averse optimizer exploring for profit distribution members greater than a preset fixed benchmark. The most challenging facet of SSD-constrained methodologies is how to effectually define the preset fixed benchmark. In this regard, first, a generic approach is offered to find the feasible region for benchmark selection in SSD-constrained optimization problems. Then, a novel benchmark selection technique considering both the decision-maker's regret and out-of-sample profit, leverages the VIKOR method to get the ranking of different solutions and find the compromise benchmark in the risk-aware environment. Consequently, two decisive criteria from both *ex-ante* and *ex-post* tests are involved in the benchmark selection procedure, making the bidding problem regret- and consequence-aware. The numerical results of the developed methodology against risk-neutral and deterministic approaches show the efficiency of the proposed model.

**Index Terms**—Battery, bidding strategy, out-of-sample analysis, regret, risk management, second-order stochastic dominance (SSD).

## I. INTRODUCTION

**D**UE to technological maturity, Battery Storage Systems (BSSs) have shown excellent potential for providing grid services such as peak shaving [1], operating reserves and ancillary services [2], and decreasing renewable energy curtailment [3] in recent years. Despite their advances and economies of scale, the vast exploitation of BSSs in current electricity markets is still hindered mainly by high investment costs [4]. A major contributing factor to high investment costs is the degradation of BSSs due to arbitrary charging and discharging of these devices, which limits their useful lifetime over time. However, appropriate modeling of the BSS degradation cost within their operational strategies can effectively restrict their abrupt charging and discharging patterns [5]. Another viable solution for increasing BSSs deployment is to enhance their optimized revenues regarding the provided services. Energy arbitrage through market bidding is one of the primary practices with promising economic incomes for the BSS operators. Energy arbitrage is the practice of BSS charging and discharging at different times of the day to take advantage of price variability in the electricity market [6]. Therefore, BSSs utilization under proper consideration of degradation costs and optimum strategies for electricity arbitrage can affect their market revenues and thus the economic feasibility of large-scale exploitation.

A considerable part of the recent literature on BSSs energy arbitrage is dedicated to the optimal bidding strategies of these devices in the electricity markets [7], [8], [9], [10], [11], [12], [13], [14], [15], [16]. Authors in [7] proposed an energy arbitrage model for the BSS in the wholesale market by addressing its cycle aging cost. For joint energy and ancillary service markets, Ref. [8] proposed an optimal bidding model with multi-scenario settings to consider price uncertainty. As the abrupt charging and discharging cycles of BSSs significantly degrade their service life, the authors embedded the BSS cycle life model into the profit maximization problem to calculate the cycle life under various operational strategies. Similarly, a bidding structure based on the BSS cost functions was proposed in [9] for participating in energy and spinning reserve markets. In [10], an optimal bidding strategy for the BSS operation was implemented through a bi-level optimization model. The day-ahead arbitrage benefit of the BSS was maximized in the upper-level problem

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while the market was cleared in the lower-level. Furthermore, the cycling degradation was also taken into account to enhance the accuracy of BSS profit assessment. Ref. [11] investigated the synergies between energy arbitrage and fast frequency response in the wholesale electricity market with BSSs. Considering charging/discharging losses and the lifetime of the BSS, a reinforcement learning-based approach was suggested in [12] for learning the optimized bidding strategy and thus to enhance the profit of the BSS in power and regulation markets. The authors in [13] developed a bi-level distribution level bidding for distributed BSSs in day-ahead energy and reserve markets along with the balancing market and a new flexibility market. Similarly, in [14], a bi-level BSS bidding model was provided at the transmission level, with a decomposition approach employed to expedite the long-running problem. In [15], a co-optimized sizing and bidding model was proposed for a BSS in energy and frequency regulation markets under deterministic conditions. Furthermore, to facilitate the integration of BSSs, a joint bidding and market-clearing model was suggested in [16] while the BSS was in charge of submitting its cycling mileages.

A particular challenge for BSS operators is the attributed risks due to various sources of uncertainty in optimal bidding strategies. Nevertheless, the reviewed papers [8], [9], [10], [11], [12], [13], [14], [15], [16] ignored the risk associated with uncertainties. In addition to the reviewed studies, scholars addressed risk management in bidding strategies via a variety of concepts and approaches [17], [18], [19], [20], [21], [22], [23]. In [17], optimal offering and bidding models for the combined wind farm and energy storage systems were implemented while the risk arising from the stochastic parameters was handled by the conditional value-at-risk (CVaR) metric. The authors of [18] formulated a risk measure-based robust optimization bidding model for dispatching a wind farm in combination with the BSS. According to the method, the selection of the uncertainty set for robust optimization relied on the coherent risk measure of CVaR. A new scheme for handling the risk of stochastic and interval parameters based on CVaR and minimizing the deviation of interval objective function was developed in [19]. While the risk of stochastic parameters was ignored in [20], a hybrid stochastic-robust uncertainty modeling approach was developed for the microgrid bidding problem. A Second-order Stochastic Dominance (SSD) model was presented in [21] for risk management of a wind farm bidding in electricity markets. In [22], a distributionally robust optimal bidding model was derived for the wind-BSS aggregator participating in the day-ahead market. Distributionally robust optimization, as opposed to the conventional robust model, takes into account a group of potential distributions of the uncertain variables and seeks to optimize against the distribution's worst-case scenario. Information gap decision theory was utilized in [23] to handle the risk-averse and risk-seeker strategies for the optimal offering strategies of BSS and wind farms in the day-ahead energy market. The following aspects have been disregarded by current research [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23]:

First, apart from risk-based bidding structures introduced above [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17],

[18], [19], [20], [22], [23], there is little research that concentrates on expressing the decision-maker's preference in a risk-aware setting via SSD. The SSD criterion is a coherent risk metric enabling exploiters to hedge against the risk of uncertainties by imposing preferences in terms of acceptable earnings over a preset fixed benchmark. The existing literature on the SSD criterion fails to properly propose a generic approach to derive the feasible region for benchmark designation in SSD-constrained problems. Any benchmark selection outside this feasible region yields infeasible or unpractical outcomes. In [21], as one of the few works on SSD-constrained optimization, the authors did not adequately generalize the ground to establish the feasible region with suitable reasoning. For future implementation and development, it is necessary to articulate the explanation and justification behind the various steps to be taken to derive the feasible region.

Second, the final benchmark chosen by the decision-maker in SSD-constrained problems has drawn limited attention, which needs to be sensibly addressed since a poor selection may lead to useless outcomes in terms of risk management. There was no effort in the prior literature [21], [24], [25] to provide a practical method for selecting benchmarks within the SSD framework. The existing works relied on examining the SSD performance without devising a sensible architecture for picking the compromise benchmark.

Inspired by preceding studies, this paper extends the authors' previous work [26] on optimal battery bidding in day-ahead and intraday markets utilizing a detailed risk management tool, i.e., SSD, with its novel benchmark selection procedure. Risk is inevitable in almost all power system-related studies due to the number of uncertain sources that need to be tackled in advance (e.g., day-ahead). Accordingly, devising an appropriate risk management tool is one of the most important sides of decision-making under uncertainty. This paper thus utilizes the SSD criterion as a coherent risk measure in a stochastic environment to capture the risk of bidding and enforce decision-maker's preferences in terms of acceptable profits over a preset fixed benchmark. Compared to the frequently used CVaR criterion, which hunts the supreme elements of the profit distribution, the SSD explores profit distribution members greater than a preset fixed benchmark (profit). Coherent risk measures are those that meet the criteria laid forth by Artzner et al. [27], which fulfill the four conditions concurrently: *monotonicity*, *sub-additivity*, *positive homogeneity*, and *translation invariance*. A risk measure lacks coherence if any of these conditions are violated. The SSD meets all of these characteristics, resulting in substantial merits over incoherent risk measures. The SSD merits over incoherent risk metrics specifically depend on the missing criteria seen in incoherent risk measures. For example, the SSD is superior to value-at-risk, which fails to meet *sub-additivity*. *Sub-additivity* expresses the notion that diversification may lower risk (an amalgamation or a coalition does not raise the risk). The SSD appears to be more advantageous than value-at-risk, which cannot guarantee *sub-additivity* (diversification).

To effectively set the benchmarks for the SSD-constrained problem, the authors leverage *regret* theory and out-of-sample analysis to enter both *ex-ante* and *ex-post* tests into the

benchmark selection procedure. To do so, first, a generic approach is proposed to find the feasible region for inputting benchmarks into the SSD-constrained problems. Next, given the decision-maker's perspective, a number of equally-spaced benchmarks are extracted from the feasible region. Each input benchmark is then assessed based on its performance over a large number of samples (out-of-sample analysis) and the decision-maker's *regret*. Note that *regret* is characterized as the difference between the realized benefit and the gain we could realize if we were aware of the situation that would undoubtedly occur in advance [28]. Finally, the benchmarks' ranking and the compromise solution are obtained via VIKOR<sup>1</sup> method in accordance with two conflicting criteria (*regret* and out-of-sample performance). Summarizing, the contributions of the paper are listed as follows:

- Incorporating the *regret* theory into the SSD-constrained battery bidding problem to identify robust benchmarks entering the SSD constraints. This is the first time the *regret* theory is leveraged to detect robust benchmarks in an SSD-constrained problem. The superiority of the proposed approach is accordingly demonstrated over conventional approaches.
- Developing a generic approach to obtain the feasible region for benchmark selection in any problem subjected to SSD constraints.
- Proposing, for the first time, a novel benchmark selection method in SSD-constrained problems founded on decision-maker's *regret* and the out-of-sample performance, assuring the *regret*- and consequence-aware of the proposed methodology.

The organization of the paper is as follows. The mathematical model for the risk-neutral battery bidding model is introduced in Section II. Section III presents the mathematical model for the SSD-constrained bidding model along with the procedure to derive the benchmark feasible region. The proposed benchmark selection method is presented in Section IV. The numerical results and concluding remarks are given in Sections V and VI, respectively.

## II. RISK-NEUTRAL BATTERY BIDDING MODEL

This study aims to propose a bidding model for a price-taker lithium-ion BSS in both day-ahead and intraday electricity markets, while prices are exogenously characterized through scenarios. Various applications from grid-scale to residential-scale are attributed to lithium-ion BSSs due to their substantial merits over other battery technologies. Amongst visible advantages of lithium-ion BSS are low self-discharge rate and high energy density, whereas aging difficulties are the major disadvantages. Therefore, as stated in the introduction, a wide range of scientific studies have been committed to tackling the aging problem of lithium-ion BSS. Within all the approaches provided in the literature, the model presented in [7] is used in this paper to compensate for the aging cost of the BSS, which is defined as the function of depth of discharge (DoD). Regarding the aging

cost, the proposed energy arbitrage model is formulated as a two-stage stochastic problem where the stochastic parameters are the volatile electricity prices in day-ahead and intraday markets. While several scenario generation methods have been suggested in the literature [29], this article utilizes the method presented in [30]. Accordingly, the objective function of the proposed optimization problem can be described as (1a):

$$\begin{aligned} \text{Max} \quad & \sum_{\omega=1}^{\Omega} \pi_{\omega} \sum_{t=1}^T \sigma_{t,\omega}^{\text{DA}} \left[ \rho_t^{\text{DA},\text{dis}} - \rho_t^{\text{DA},\text{ch}} \right] + \\ & \sigma_{t,\omega}^{\text{IN}} \left[ \rho_{t,\omega}^{\text{IN},\text{dis}} - \rho_{t,\omega}^{\text{IN},\text{ch}} \right] - \sum_{s=1}^S \psi_s \left[ \varrho_{s,t}^{\text{DA},\text{dis}} + \varrho_{s,t,\omega}^{\text{IN},\text{dis}} \right] \end{aligned} \quad (1a)$$

where  $\pi_{\omega}$  is the probability of scenario  $\omega$ ,  $\rho_t^{\text{DA},\text{ch}}$  and  $\rho_t^{\text{DA},\text{dis}}$  represent the BSS charge and discharge bids into the day-ahead market, and  $\rho_{t,\omega}^{\text{IN},\text{ch}}$  and  $\rho_{t,\omega}^{\text{IN},\text{dis}}$  denote the BSS charge and discharge bids into the intraday market.  $\sigma_{t,\omega}^{\text{DA}}$  and  $\sigma_{t,\omega}^{\text{IN}}$  reflect day-ahead and intraday prices.  $\psi_s$  accounts for the slope of block  $s$  in the piecewise linear function of the BSS aging cost [7].

Equation (1a) consists of three terms. The first and second terms indicate profits from the BSS energy arbitrage in day-ahead and intraday markets, respectively. The third term, however, displays the aging cost of the BSS. Constraints (1b)–(1e) show the charge and discharge powers of the BSS in the day-ahead and intraday markets, which are equal to the sum of the charging and discharging powers in each segment  $s \in [1, S]$  of DoD (i.e.,  $\varrho_{s,t}^{\text{DA},\text{dis}}$ ,  $\varrho_{s,t}^{\text{DA},\text{ch}}$ ,  $\varrho_{s,t,\omega}^{\text{IN},\text{dis}}$ ,  $\varrho_{s,t,\omega}^{\text{IN},\text{ch}}$ ).

$$\rho_t^{\text{DA},\text{dis}} = \sum_{s=1}^S \varrho_{s,t}^{\text{DA},\text{dis}} \quad \forall t \quad (1b)$$

$$\rho_t^{\text{DA},\text{ch}} = \sum_{s=1}^S \varrho_{s,t}^{\text{DA},\text{ch}} \quad \forall t \quad (1c)$$

$$\rho_{t,\omega}^{\text{IN},\text{dis}} = \sum_{s=1}^S \varrho_{s,t,\omega}^{\text{IN},\text{dis}} \quad \forall t, \forall \omega \quad (1d)$$

$$\rho_{t,\omega}^{\text{IN},\text{ch}} = \sum_{s=1}^S \varrho_{s,t,\omega}^{\text{IN},\text{ch}} \quad \forall t, \forall \omega \quad (1e)$$

Furthermore, the BSS's charging and discharge powers must be scheduled without exceeding the operational limits in day-ahead and intraday markets, as described in (1f)–(1i).

$$0 \leq \rho_t^{\text{DA},\text{dis}} \leq P^{\text{dis}} \epsilon_t \quad \forall t \quad (1f)$$

$$0 \leq \rho_t^{\text{DA},\text{dis}} + \rho_{t,\omega}^{\text{IN},\text{dis}} \leq P^{\text{dis}} \epsilon_t \quad \forall t, \forall \omega \quad (1g)$$

$$0 \leq \rho_t^{\text{DA},\text{ch}} \leq P^{\text{ch}} \times (1 - \epsilon_t) \quad \forall t \quad (1h)$$

$$0 \leq \rho_t^{\text{DA},\text{ch}} + \rho_{t,\omega}^{\text{IN},\text{ch}} \leq P^{\text{ch}} \times (1 - \epsilon_t) \quad \forall t, \forall \omega \quad (1i)$$

In the above constraints,  $P^{\text{ch}}$  and  $P^{\text{dis}}$  express maximum charging and discharging powers of the BSS, and  $\epsilon_t$  is a binary decision variable for modeling the BSS discharging mode. It is of paramount importance that the charging and discharging powers of the BSS in the intraday market should be limited to a

<sup>1</sup>Vlekeritrijumsko KOMPromisno Rangiranje (in Serbian language).

certain extent [30], as shown in (1j) and (1k).

$$0 \leq \rho_{t,\omega}^{\text{IN},\text{dis}} \leq \alpha \times \rho_t^{\text{DA},\text{dis}} \quad \forall t, \forall \omega \quad (1j)$$

$$0 \leq \rho_{t,\omega}^{\text{IN},\text{ch}} \leq \alpha \times \rho_t^{\text{DA},\text{ch}} \quad \forall t, \forall \omega \quad (1k)$$

where  $\alpha$  is the coefficient for limiting intraday charging and discharging powers of the BSS. According to (1l), each segment of BSS charging and discharging powers must receive positive values in day-ahead and intraday markets:

$$\rho_{s,t}^{\text{DA},\text{dis}}, \rho_{s,t}^{\text{DA},\text{ch}}, \rho_{s,t,\omega}^{\text{IN},\text{dis}}, \rho_{s,t,\omega}^{\text{IN},\text{ch}} \geq 0 \quad \forall s, \forall t, \forall \omega \quad (1l)$$

Moreover, the hourly BSS State of Charge (SoC) in every block  $s$  and the resulting hourly BSS SoC are represented in (1m) and (1n), respectively.

$$\begin{aligned} \delta_{s,t,\omega}^{\text{SoC}} &= \delta_{s,t-1,\omega}^{\text{SoC}} - \left( \frac{\rho_{s,t}^{\text{DA},\text{dis}} + \rho_{s,t,\omega}^{\text{IN},\text{dis}}}{\Lambda^{\text{dis}}} \right) \\ &\quad + \left( \Lambda^{\text{ch}} \times [\rho_{s,t}^{\text{DA},\text{ch}} + \rho_{s,t,\omega}^{\text{IN},\text{ch}}] \right) \quad \forall s, \forall t, \forall \omega \quad (1m) \end{aligned}$$

$$\Delta_{t,\omega}^{\text{SoC}} = \sum_{s=1}^S \delta_{s,t,\omega}^{\text{SoC}} \quad \forall t, \forall \omega \quad (1n)$$

In (1m),  $\delta_{s,t,\omega}^{\text{SoC}}$  denotes the BSS SoC in block  $s$  of DoD,  $\Lambda^{\text{ch}}$  and  $\Lambda^{\text{dis}}$  show charging and discharging efficiencies of the BSS, and in (1n)  $\Delta_{t,\omega}^{\text{SoC}}$  expresses the hourly BSS SoC. In addition, the SoC of the BSS needs to be restricted within appropriate boundaries [31], as formulated in (1o) and (1p).

$$0 \leq \delta_{s,t,\omega}^{\text{SoC}} \leq \varkappa_s^{\text{SoC}} \quad \forall s, \forall t, \forall \omega \quad (1o)$$

$$0 \leq \Delta_{t,\omega}^{\text{SoC}} \leq E^{\text{SoC}} \quad \forall t, \forall \omega \quad (1p)$$

where  $\varkappa_s^{\text{SoC}}$  is the maximum allowable BSS SoC in block  $s$  of DoD, and  $E^{\text{SoC}}$  indicates the maximum BSS SoC.

### III. SSD-CONSTRAINED BATTERY BIDDING MODEL

In this section, first, the mathematical formulation is given for incorporating the SSD criterion into the risk-neutral formulation as presented in the previous section. Then, a generic approach is presented to derive the feasible region for benchmark selection in SSD-constrained problems.

#### A. Mathematical Formulation

Uncertain nature of electricity prices (day-ahead and intraday) modeled by stochastic samples turns the problem into a stochastic programming model, emphasizing the importance of risk management for assisting decision-makers in pursuing actions that are less likely to result in extremely unpleasant outcomes. The most frequent approach to implement risk management in stochastic optimization is to incorporate a risk criterion in the mathematical formulation, and accordingly take advantage of broadly employed risk criteria, such as expected shortage, shortfall probability, variance, value-at-risk, and CVaR [21]. The CVaR is the only criterion among the aforementioned risk measures that satisfies the coherence features of risk criteria [32]. In contrast to the commonly used CVaR criterion which seeks the supreme elements of the profit distribution, the SSD seeks profit

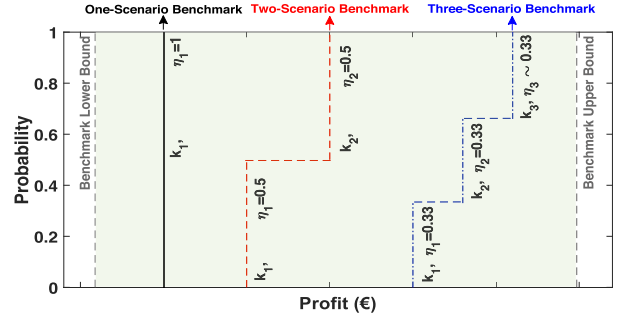


Fig. 1. Examples of benchmarks with different numbers of scenarios in an SSD-constrained problem.

distribution members that are greater than a preset benchmark (profit) [32]. The decision-maker is then able to make choices that stochastically dominate the preset benchmark. First-order stochastic dominance and SSD are among the well-documented models covered by stochastic dominance, whereas the first-order gets lesser attention owing to its inherent non-convexity. In this work, the SSD is therefore used to handle the risk by choosing various preset benchmarks from the feasible region. The SSD-constrained version of the risk-neutral model (1) can be formed as the following:

$$\begin{aligned} \text{Max} \quad & \sum_{\omega=1}^{\Omega} \pi_{\omega} \sum_{t=1}^T \sigma_{t,\omega}^{\text{DA}} \left( \rho_t^{\text{DA},\text{dis}} - \rho_t^{\text{DA},\text{ch}} \right) + \\ & \sigma_{t,\omega}^{\text{IN}} \left( \rho_{t,\omega}^{\text{IN},\text{dis}} - \rho_{t,\omega}^{\text{IN},\text{ch}} \right) - \sum_{s=1}^S \psi_s \left( \rho_{s,t}^{\text{DA},\text{dis}} + \rho_{s,t,\omega}^{\text{IN},\text{dis}} \right) \quad (2a) \end{aligned}$$

Subject to:

$$\begin{aligned} & \sum_{t=1}^T \sigma_{t,\omega}^{\text{DA}} \left( \rho_t^{\text{DA},\text{dis}} - \rho_t^{\text{DA},\text{ch}} \right) + \sigma_{t,\omega}^{\text{IN}} \left( \rho_{t,\omega}^{\text{IN},\text{dis}} - \rho_{t,\omega}^{\text{IN},\text{ch}} \right) \\ & - \sum_{s=1}^S \psi_s \left( \rho_{s,t}^{\text{DA},\text{dis}} + \rho_{s,t,\omega}^{\text{IN},\text{dis}} \right) \geq k_b - \zeta_{\omega,b} \quad \forall \omega, \forall b \quad (2b) \end{aligned}$$

$$\sum_{\omega=1}^{\Omega} \pi_{\omega} \zeta_{\omega,b} \leq \sum_{b'=1}^B \eta_{b'} \times \max(k_b - k_{b'}, 0) \quad \forall b \quad (2c)$$

$$\zeta_{\omega,b} \geq 0 \quad \forall \omega, \forall b \quad (2d)$$

$$\text{Constraints (1b)–(1p)} \quad (2e)$$

where  $k_b$  is the preset fixed benchmark by the decision-maker in scenario  $b$  with a given probability  $\eta_b$ , and  $\zeta_{\omega,b}$  is a variable calculating the profit shortfall under a preset fixed benchmark [32]. Hereafter, a one-scenario benchmark strategy (i.e.,  $b = 1$  and  $\eta_1 = 1$ ) is considered for the developed SSD-constrained model. Note that the proposed model is generic and capable of handling any number of benchmark scenarios. Fig. 1 displays examples of benchmarks with three different numbers of scenarios. As seen, we can move toward a Cumulative Distribution Functions (CDF) as a benchmark to be dominated by the proposed risk-aware model if we increase the number of benchmark scenarios. In other words, by increasing the number of scenarios in the preset

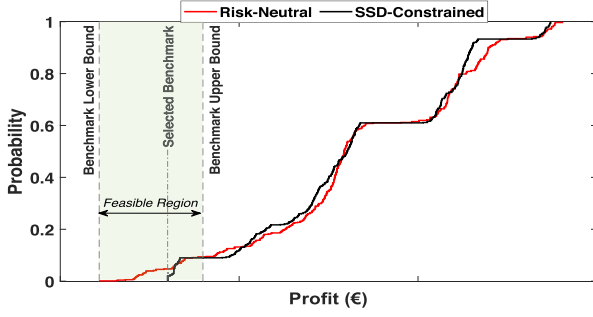


Fig. 2. Illustrative example of CDFs and benchmark feasible region in an SSD-constrained problem.

benchmark, the decision-maker can control the shape of the output CDF as a risk-controlling tool. This way, the range of output CDF and the CDF shape are controlled as scenario numbers get bigger. In contrast, the proposed approach gives higher flexibility to the shape of the output CDF despite controlling the range as the number of scenarios decreases. In this work, a single-scenario benchmark is chosen to keep the functional interpretation of the SSD as straightforward as possible.

As noticed, the objective function (2a) remains unchanged in the SSD-constrained formulation. The constraints associated with the SSD criterion are imposed through (2b)–(2d). To further exemplify how the SSD criterion works, the CDFs of the profits achieved by risk-neutral and SSD-constrained approaches for a typical problem are presented in Fig. 2. As observed in Fig. 2, the red CDF representing the risk-neutral case covers a broad range of profits. By applying the SSD criterion with a designated benchmark, the resulting CDF stochastically dominates the preset fixed benchmark. This way, profit members generated by the SSD-constrained formulation would dominate the preset benchmark. The primary challenge in SSD-constrained problems is that benchmarks should be chosen within a feasible region; otherwise, no solution would be found. The next subsection debates how to obtain this feasible region.

### B. Deriving Benchmark Feasible Region Under SSD Criterion

As discussed, the preset fixed benchmarks of the SSD-constrained model should be chosen accurately to prevent infeasible or unpractical outcomes. By focusing on the CDF of a given variable (e.g., profit), the benchmark feasible region is a rectangular area, as shown in Fig. 2 with green slime. The upper and lower bounds of this rectangle can be easily determined by solving two different optimization models before executing the main SSD-constrained problem, as detailed in the following:

- **Benchmark Lower Bound:** The benchmark lower bound is the lowest profit member (scenario  $\omega$ ) of the risk-neutral (1) CDF. According to Fig. 2, any benchmark smaller than this lower bound would turn the SSD-constrained problem into a risk-neutral model, implying that the given model already dominates the selected benchmark.
- **Benchmark Upper Bound:** The benchmark upper bound is the lowest profit member (scenario  $\omega$ ) of the following

CVaR maximization problem where confidence level approaches 1 ( $\beta \rightarrow 1$ ):

$$\text{Max } \xi - \frac{1}{1-\beta} \sum_{\omega=1}^{\Omega} \pi_{\omega} \varphi_{\omega} \quad (3a)$$

subject to:

$$\sum_{t=1}^T \sigma_{t,\omega}^{\text{DA}} \left( \rho_t^{\text{DA,dis}} \rho_t^{\text{DA,ch}} \right) + \sigma_{t,\omega}^{\text{IN}} \left( \rho_{t,\omega}^{\text{IN,dis}} - \rho_{t,\omega}^{\text{IN,ch}} \right) - \sum_{s=1}^S \psi_s \left( \varrho_{s,t}^{\text{DA,dis}} + \varrho_{s,t,\omega}^{\text{IN,dis}} \right) \geq \xi - \varphi_{\omega} \quad \forall \omega \quad (3b)$$

$$\varphi_{\omega} \geq 0 \quad \forall \omega \quad (3c)$$

$$\text{Constraints (1b) - (1p)} \quad (3d)$$

where  $\xi$  and  $\beta$  are value-at-risk and confidence level, respectively, and  $\varphi_{\omega}$  is a non-negative auxiliary variable required for CVaR evaluation. By setting  $\beta \rightarrow 1$  (here  $\beta = 0.99$ ), optimization problem (3) reflects the most conservative action that a risk-averse decision-maker could take [33]. In doing so, the decision-maker puts the most focus on extreme cases. The lowest profit member of this CVaR-maximization problem is the benchmark upper bound for the SSD-constrained model. According to Fig. 2, any benchmark greater than this upper bound would make the SSD-constrained problem infeasible, meaning that the SSD-constrained model can no way dominate the selected benchmark (which was obtained based on the most conservative action).

## IV. PROPOSED BENCHMARK SELECTION METHOD IN SSD-CONSTRAINED PROBLEMS

The predominant difficulty in SSD-Constrained problems is how decision-makers appropriately decide on a preset fixed benchmark. Despite the high level of attention paid to the features and performance of the SSD criterion in the relevant context, benchmark selection has received less engagement. While the vast majority of the prior art relied on empirical evidence for benchmark selection, in this work, a new architecture based on *ex-ante* and *ex-post* analyses is proposed for the same end. In this regard, the *regret* concept and the out-of-sample test are used as two metrics to assess the performance of benchmarks in the feasible region:

- 1) **Regret:** The *regret* theory is leveraged to judge the robustness of input benchmarks [28]. *Regret* might be characterized as the difference between the realized benefit and the gain we could realize if we were aware of the situation that would undoubtedly occur in advance [28]. In terms of stochastic programming problems, *regret* ( $R_{\omega}$ ) could be mathematically defined as the difference between the optimal (ideal) solution in each scenario ( $F_{\omega}^{\text{ideal}}$ ) in case of having full knowledge over uncertainties and the

solution of problem (2):

$$R_\omega = F_\omega^{ideal} - \left( \sum_{t=1}^T \sigma_{t,\omega}^{DA} \left( \rho_{t,\omega}^{DA,dis} - \rho_{t,\omega}^{DA,ch} \right) + \sigma_{t,\omega}^{IN} \left( \rho_{t,\omega}^{IN,dis} - \rho_{t,\omega}^{IN,ch} \right) - \sum_{s=1}^S \psi_s \left( \varrho_{s,t}^{DA,dis} + \varrho_{s,t,\omega}^{IN,dis} \right) \right) \quad \forall \omega \quad (4)$$

where  $F_\omega^{ideal}$  is a parameter calculated prior to handling the main problem.  $F_\omega^{ideal}$  is calculated using *ex-ante* scenarios while reformulating the two-stage stochastic problem in a manner that all decision variables are dependent on the realization of stochastic parameters (electricity prices). The resulting formulation is not a two-stage decision-making problem but rather a single-stage one with perfect insight into uncertainties. The problem formulation for calculating this parameter is reported in Appendix A. Decision-makers tend to minimize the *regret*, as a result, the regret-aware mathematical model for the intended problem is as minimizing the average *regret*:

$$\text{Min} \sum_{\omega=1}^{\Omega} \pi_\omega \times R_\omega \quad (5a)$$

Subject to:

$$\text{Constraints (2a)–(2e), (4)} \quad (5b)$$

In the above model, the average regret is minimized for each given benchmark from the feasible region in the SSD-constrained model. Each benchmark thus results in a distinct average profit and average *regret*. From the view point of this concept, the smaller the *regret*, the better the benchmark.

- 2) *Out-of-sample test*: Out-of-sample analysis is the second metric utilized to evaluate the quality of the results delivered by a stochastic programming setting [32]. Hence, an out-of-sample evaluation, similar to the one proposed in [34], is used to assess the efficacy of input benchmarks after solving a stochastic programming problem using information from a large-scale external model [32]. Each benchmark from the benchmark feasible region yields an average out-of-sample profit. The greater this value, the better the benchmark.

By having these two metrics as the judging criteria, the feasible region is divided into a number of equally spaced benchmarks ( $n$ ), and each benchmark is assessed in terms of these metrics. Ultimately, the benchmarks' ranking and the compromise solution are obtained via the VIKOR method in accordance with these two metrics. The VIKOR method has been extensively utilized to address numerous multi-criteria-based decision-making problems [35]. The VIKOR technique comes up with a ranking list based on how close a solution is to the ideal one [35], [36]. Algorithm 1 depicts the summarized procedure for implementing the VIKOR technique.

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**Algorithm 1: VIKOR Method Implementation.**


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- 1: Introduce rating function ( $f_{i,j}$ ) and calculate the worst and best values expressed by  $f_i^-$  and  $f_i^*$  for  $j = 1, 2, \dots, n$  according to (6) and (7). Note that  $n$  is the number of selected benchmarks,  $i$  is the index of metrics (*regret* and out-of-sample profit), and hereafter,  $(\cdot)^-$  and  $(\cdot)^*$  stand for the worst and best values of a given parameter within its set.

$$f_i^- = \text{Min}_j f_{i,j} \quad (6)$$

$$f_i^* = \text{Max}_j f_{i,j} \quad (7)$$

- 2: Calculate the values of the group utility measure ( $\Upsilon_i$ ) and individual regret measure ( $\Gamma_i$ ) designated for each solution with  $v_j$  as the weights of rating in (8) and (9). Note that  $\Upsilon_i$  differentiates from the previously defined *regret*.

$$\Upsilon_i = \sum_{j=1}^N v_j \times \frac{f_j^* - f_{i,j}}{f_j^* - f_{i,j}^-} \quad (8)$$

$$\Gamma_i = \text{Max}_j v_j \times \frac{f_j^* - f_{i,j}}{f_j^* - f_{i,j}^-} \quad (9)$$

- 3: Calculate the values of  $Q_i$  using (10).

$$Q_i = z \times \left[ \frac{\Upsilon_i - \Upsilon^*}{\Upsilon^- - \Upsilon^*} \right] + (1 - z) \times \left[ \frac{\Gamma_i - \Gamma^*}{\Gamma^- - \Gamma^*} \right] \quad (10)$$

where

$$\begin{aligned} \Upsilon^* &= \text{Min}_i \Upsilon_i, \quad \Upsilon^- = \text{Max}_i \Upsilon_i \\ \Gamma^* &= \text{Min}_i \Gamma_i, \quad \Gamma^- = \text{Max}_i \Gamma_i \end{aligned} \quad (11)$$

In which  $z$  is defined as the weight of the strategy of maximum group utility that is typically considered 0.5 [36].

- 4: Sort solutions in a ranking list following a decreasing order based on the values of  $Q_i$ .
  - 5: The most desirable solution is the one with the lowest value of  $Q$  [36].
- 

## V. NUMERICAL RESULTS

The performance of the proposed risk-averse SSD-constrained model for market bidding with a typical BSS is demonstrated in this section. The considered BSS is a 35 MW battery with a five-hour capacity. It is thus capable of storing a maximum of 175 MWh of received electricity. Both BSS charging and discharging efficiencies are considered  $\Lambda^{\text{dis}} = \Lambda^{\text{ch}} = 0.95$  [37]. Ref. [7] contains all data pertaining to the BSS aging cost, whereas this function is linearized with twenty blocks ( $S = 20$ ). The price scenarios (for both *ex-ante* and *ex-post* tests) are derived following a scenario generation process [30] for the 16<sup>th</sup> of May, 2022, Spanish market [38]. For the *ex-ante* analysis, first, one thousand scenarios for day-ahead and intraday prices are generated and then reduced to twenty for each [39]. These scenarios are depicted in Fig. 3. One thousand additional

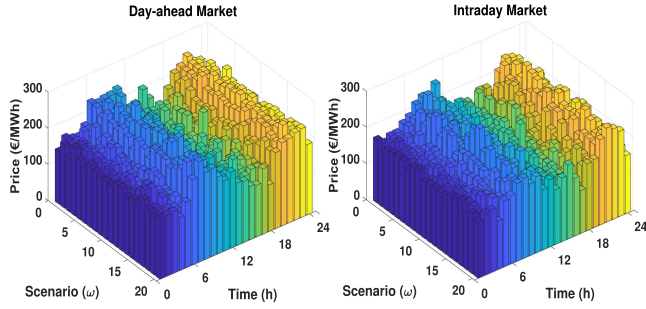


Fig. 3. Reduced day-ahead and intraday scenarios for the ex-ante analysis.

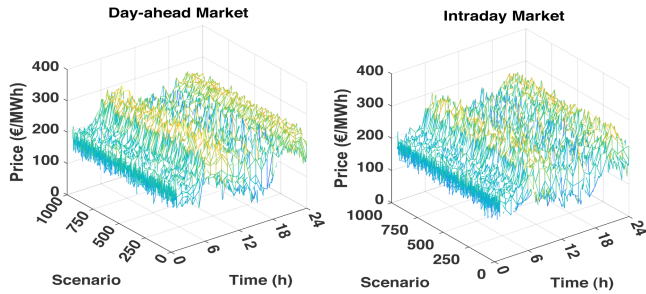


Fig. 4. One thousand day-ahead and intraday scenarios for the ex-post analysis.

scenarios are generated for the ex-post (out-of-sample) analysis, and the resulting scenarios are displayed in Fig. 4. The coefficient for limiting intraday charging and discharging powers is set to  $\alpha = 0.3$  [34], meaning that the BSS can only bid 30% of its day-ahead bid in the intraday market. It is worth noting that the 3<sup>rd</sup> intraday auction is the target trading floor in this study. All mathematical models developed in this work are mixed integer linear programming and solved with GAMS and solved using CPLEX. To clarify the effectiveness of the SSD-constrained bidding model, the results are broken down into two parts. First, a step-by-step explanation of how the suggested benchmark selection approach works in the SSD-constrained problem is provided. The constructed model is then evaluated in terms of *regret* and out-of-sample performance compared to other commonly used approaches.

#### A. Step-by-Step Implementation of the Proposed Benchmark Selection Method in the SSD-Constrained Problem

The first step in SSD-constrained problems is to derive the feasible region for benchmark designation. Following the approach described in Section III-B, the feasible region (rectangular area) in the CDF of BSS profit can be efficiently shaped by obtaining its upper and lower bounds:

- **Benchmark Lower Bound:** The benchmark lower bound is the lowest profit member of the risk-neutral (1) CDF. By solving model (1), the lowest profit member of the risk-neutral model is € 10,480. Both benchmark lower bound and the CDF of risk-neutral model are depicted in Fig. 5. The SSD-constrained problem would become a risk-neutral model for any benchmark less than € 10,480,

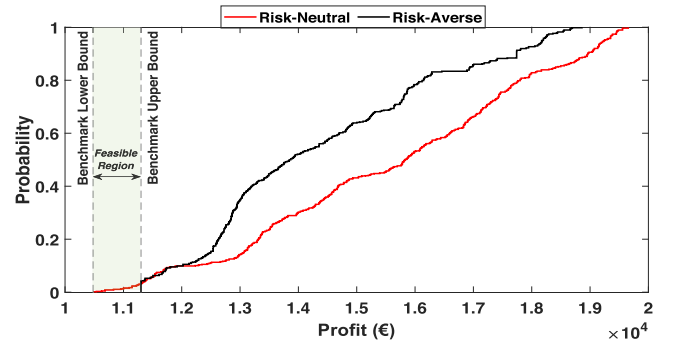


Fig. 5. Benchmark feasible region in the SSD-constrained bidding problem.

TABLE I  
RESULTS OF THE DEVELOPED ALGORITHM FOR ELEVEN DIFFERENT BENCHMARKS

$k_b$ (€)	Average Profit (€)	Average Regret (€)	Maximum Regret (€)	Average Out-of-Sample Profit (€)
10,480	15,594.69	2,401.64	5,365.82	15,463.46
10,562.1	15,591.96	2,404.37	5,453.79	15,524.28
10,644.2	15,588.53	2,407.79	5,457.35	15,537.21
10,726.3	15,585.01	2,411.31	5,460.94	15,500.14
10,808.4	15,580.14	2,416.18	5,458.81	15,547.38
10,890.5	15,560.87	2,435.45	5,588.63	15,502.29
10,972.6	15,533.90	2,462.42	5,805.69	15,465.32
11,054.7	15,499.56	2,496.76	6,101.99	15,486.25
11,136.8	15,450.54	2,545.78	6,296.46	15,429.22
11,218.9	15,220.55	2,775.77	6,472.73	15,177.73
11,301	14,901.59	3,094.74	6,695.22	14,699.97

indicating that the provided model already dominates the chosen benchmark.

- **Benchmark Upper Bound:** The lowest profit member of the CVaR-maximization problem (3) is the benchmark upper bound for the SSD-constrained model. By solving model (3), € 11,301 is obtained as the lowest profit member of model (3). Benchmark upper bound and the CDF of CVaR-maximization problem (labeled as “Risk-Averse”) are shown in Fig. 5.

The benchmark feasible region is now specified, and the decision-maker can divide the feasible region into certain evenly-spaced benchmarks ( $n$ ), and each benchmark is then assessed in terms of *regret* and out-of-sample performance. Here, the feasible region is divided into ten evenly-spaced areas; as a result, eleven benchmarks ( $n = 11$ ) with equally-spaced distances ( $\frac{11,301-10,480}{10} = € 82.1$ ) feed the proposed SSD-constrained model. It is worth to note that all these eleven benchmarks are listed in the first column of Table I.

The second step is to feed the *regret*-aware optimization problem (5) with the extracted benchmarks and then perform an out-of-sample test to check the quality of results in the presence of one thousand ex-post scenarios (Fig. 4). In this way, each input benchmark results in unique “average profit,” “average regret,” “maximum regret,” and “average out-of-sample profit.”

Table I reports the performance of each input benchmark in terms of the aforementioned items. In the context of *regret* theory, some decision-makers rely on average *regret*, which is the weighted regret over all scenarios, while some may lean on “maximum *regret*,” which represents the worst *regret* in all scenarios. This paper analyzes the effects of both strategies. Once [average *regret* + average out-of-sample profit] and another time [maximum *regret* + average out-of-sample profit] are the criteria to evaluate each benchmark and choose a compromise solution.

Table I concludes that:

- 1) The greater the benchmark, the smaller the average profit. Moreover, the closer the benchmarks to the upper bound, the greater the profit drop, meaning that the decision-maker needs to sacrifice more profit to dominate greater benchmarks.
- 2) Larger benchmarks yield higher average and maximum *regrets*. This implies that the *regret* indices increase as the algorithm attempts to dominate larger benchmarks. Similar to the previous point, the influence increases as the input benchmarks get closer to the benchmark upper bound.
- 3) The influence of input benchmarks on average out-of-sample profit does not follow a typical pattern, emphasizing the need of using a multi-criteria decision-making method such as VIKOR. However, when the decision-maker approaches the benchmark upper bound (becomes more and more risk-averse), the average out-of-sample profit decreases significantly. Therefore, as expected, the BSS's profitability will be negatively impacted by a highly risk-averse attitude.

The next step is to pick a benchmark that has least *regret* and highest out-of-sample profit. Here, the VIKOR method outlined in Algorithm 1 is implemented for such an end. The main focus of this study is on two distinct decision-making strategies with equal weighting ( $z = 0.5$ ) as noted above:

- 1) *Strategy 1*: Decision-making criteria are [average *regret*] and [average out-of-sample profit].
- 2) *Strategy 2*: Decision-making criteria are [maximum *regret*] and [average out-of-sample profit].

The parameter  $Q_i$  acquired in the third step of Algorithm 1 is used by the decision-maker to rank the benchmarks in ascending order from lowest to highest value. Table II displays the rankings of benchmarks adopting the two decision-making strategies. For the first strategy, the compromise solution is the benchmark with  $k_b = \text{€ } 10,644.2$ , while for the second strategy is the benchmark with  $k_b = \text{€ } 10,808.4$ . These compromise solutions along with their corresponding values of  $Q_i$  are discerned by shaded cells in Table II. Furthermore, Table III compares compromise solutions under decision-making strategies 1 and 2 in terms of principal variables. The first strategy triumphs over the second regarding average profit, average *regret*, and maximum *regret*, but the latter performs better in the out-of-sample test. Fig. 6 also shows the profit CDFs of these compromise solutions for the first and second strategies. As seen, the smaller benchmark of strategy 1 leads to a slightly broader range of profit distribution, resulting in a greater value of average profit. Clearly, the second strategy is more risk-averse than the first.

TABLE II  
RANKING OF INPUT BENCHMARKS UNDER TWO DISTINCT DECISION-MAKING STRATEGIES WITH  $z = 0.5$  (EQUAL WEIGHTING)

$k_b$ (€)	Strategy 1		Strategy 2	
	[Average <i>Regret</i> + Out-of-Sample Profit]		[Maximum <i>Regret</i> + Out-of-Sample Profit]	
	$Q_i$	Ranking	$Q_i$	Ranking
10,480	0.063785491	6	0.025124936	5
10,562.1	0.010322632	3	0.006080420	3
<b>10,644.2</b>	<b>0</b>	<b>1</b>	0.004257288	2
10,726.3	0.034473287	4	0.017734863	4
<b>10,808.4</b>	0.004578009	2	<b>0.002023471</b>	<b>1</b>
10,890.5	0.041348686	5	0.093387681	6
10,972.6	0.084279533	7	0.234411984	7
11,054.7	0.111004155	8	0.405093215	8
11,136.8	0.181665151	9	0.538744749	9
11,218.9	0.508405027	10	0.720974034	10
11,301	1	11	1	11

TABLE III  
COMPARISON BETWEEN THE OBTAINED RESULTS UNDER DECISION-MAKING STRATEGIES 1 AND 2

Strategy	Average Profit (€)	Average <i>Regret</i> (€)	Maximum <i>Regret</i> (€)	Average Out-of-Sample Profit (€)
Strategy 1	15,588.53	2,407.79	5,457.35	15,537.21
Strategy 2	15,580.14	2,416.18	5,458.81	15,547.38

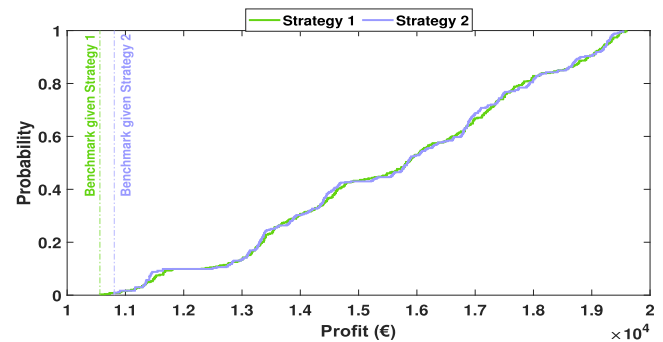


Fig. 6. CDF of BSS Profit for Decision-Making Strategies 1 and 2.

### B. Performance Analysis: A Comparative Study

The suggested SSD-constrained model is evaluated here in contrast to other frequently used approaches such as deterministic, pure stochastic, and robust programming. Details of robust programming formulation [40] are reported in Appendix B. The comparison is made in light of *regret* and out-of-sample performance metrics, and the results are given in Table IV. It is worth noting that for the deterministic analysis, the stochastic scenarios are substituted with the real market values of the Spanish Market on the 16<sup>th</sup> of May, 2022. Pure stochastic programming accounts for optimization model (5) without SSD constraints. For robust programming, various robustness parameters are explored and assessed in light of the aforementioned metrics, and eventually, Algorithm 1 is deployed to extract the compromise solution.

TABLE IV  
COMPARATIVE ANALYSIS OF DIFFERENT APPROACHES

Approach	Average Regret (€)	Maximum Regret (€)	Average Out-of-sample Profit (€)
Deterministic	2,664.66	6,881.12	15,457.66
Pure Stochastic Programming	2,401.64	5,365.82	15,458.49
Robust Programming	3,525.39	7,741.84	15,449.27
Proposed Model (Strategy 1)	2,407.79	5,457.35	15,537.21
Proposed Model (Strategy 2)	2,416.18	5,458.81	15,547.38

Based on the outcomes of Table IV, it can be observed that the robust programming has the worst performance in all metrics, especially in terms of *regret*. This lies in the fact that robust technique optimizes problems on the grounds of the worst-case scenario (extreme situation). The captured results thus lead to high levels of *regret* for decision-makers since they undergo worst-case scenarios that infrequently occur. We can draw the conclusion that robust programming may not be advisable when *regret* is a decision-making measure as the decision-maker would incur high losses by considering worst-cases over uncertainties. As deterministic analysis considers the point forecast of electricity prices, the *regret* metrics outdo robust programming (which deals with the worst-case scenario). On the other hand, pure stochastic programming outperforms the proposed model in light of *regret* metrics since it merely minimizes *regret* in the absence of SSD constraints. The preceding results show that imposing benchmarks negatively influences the *regret*. Nevertheless, the proposed models under both decision-making strategies offer more promising performance in terms of out-of-sample profit, indicating that the BSS could expect higher profits when relying on the given methodology.

## VI. CONCLUSION

The focus of this study was to propose a new benchmark selection method for the SSD-constrained market bidding problem with a BSS. Risk-averse decision makers can exploit the SSD to create profit distributions that dominate a preset fixed benchmark. Two main challenges in SSD-constrained problems, namely, benchmark feasible region extraction and final benchmark selection, were sensibly addressed. The following are some of the most important takeaways from the research that were carried out:

- 1) Benchmark feasible region allows decision-makers to more effectively seek desirable benchmarks and prevent infeasible or unpractical results.
- 2) The final benchmark in the SSD-constrained model is selected based on how close a benchmark is to the ideal values of judging metrics (i.e., *regret* and out-of-sample profit).
- 3) The *regret* deepens as the benchmarks become greater. As a result, imposing higher benchmarks leads to more regrets.

- 4) The BSS could expect larger revenues while relying on the suggested methodology as opposed to conventional procedures.
- 5) Robust optimization is not advocated when *regret* is a deciding factor, as there is a fundamental incompatibility between the *regret* measure and optimization under the worst-case scenario, which might result in severe losses.

## APPENDIX A

The ideal solution in each scenario  $F_{\omega}^{ideal}$  is found while assume having perfect insight into uncertainties by solving the following optimization problem:

$$\text{Max} \quad \sum_{\omega=1}^{\Omega} \pi_{\omega} \times F_{\omega}^{ideal} \quad (12a)$$

$$F_{\omega}^{ideal} = \sum_{t=1}^T \sigma_{t,\omega}^{DA} \left[ \rho_{t,\omega}^{DA,dis} - \rho_{t,\omega}^{DA,ch} \right] + \sigma_{t,\omega}^{IN} \left[ \rho_{t,\omega}^{IN,dis} - \rho_{t,\omega}^{IN,ch} \right] \\ - \sum_{s=1}^S \psi_s \left[ \varrho_{s,t,\omega}^{DA,dis} + \varrho_{s,t,\omega}^{IN,dis} \right] \quad (12b)$$

Subject to:

Constraints (1b)–(1p) while substituting :

$$\rho_t^{DA,dis} \rightarrow \rho_{t,\omega}^{DA,dis}; \quad \rho_t^{DA,ch} \rightarrow \rho_{t,\omega}^{DA,ch} \\ \varrho_{s,t}^{DA,dis} \rightarrow \varrho_{s,t,\omega}^{DA,dis}; \quad \varrho_{s,t}^{DA,ch} \rightarrow \varrho_{s,t,\omega}^{DA,ch} \quad (12c)$$

## APPENDIX B

The robust counterpart of the studied problem under bounded day-ahead and intraday prices holds a Max-Min structure that can be straightforwardly transformed to the following optimization problem using the duality theorem:

$$\text{Min} - [\det^{OF}] + (\chi^1 \varpi^{DA}) + (\chi^2 \varpi^{IN}) + \sum_{t=1}^T q_t^1 + q_t^2 \quad (13a)$$

$$\det^{OF} = \sum_{t=1}^T \sigma_t^{DA} \left[ \rho_t^{DA,dis} - \rho_t^{DA,ch} \right] + \sigma_t^{IN} \left[ \rho_t^{IN,dis} - \rho_t^{IN,ch} \right] \\ - \sum_{s=1}^S \psi_s \left[ \varrho_{s,t}^{DA,dis} + \varrho_{s,t}^{IN,dis} \right] \quad (13b)$$

where  $\det^{OF}$  is the deterministic objective function, the set of constraints  $\chi^1, \chi^2, q_t^1, q_t^2$  represent dual variables, and robustness parameters for day-ahead and intraday prices are  $\varpi^{DA}$  and  $\varpi^{IN}$  taking values in  $[0, T]$ . The constraints associated with (13a) are presented in the following.

$$q_t^1 + \chi^1 \geq \hat{\sigma}^{DA} y_t^1 \quad \forall t \quad (13c)$$

$$q_t^2 + \chi^2 \geq \hat{\sigma}^{IN} y_t^2 \quad \forall t \quad (13d)$$

$$q_t^1, q_t^2 \geq 0 \quad \forall t \quad (13e)$$

$$y_t^1, y_t^2 \geq 0 \quad \forall t \quad (13f)$$

$$\chi^1, \chi^2 \geq 0 \quad (13g)$$

$$\rho_t^{\text{DA},\text{dis}} - \rho_t^{\text{DA},\text{ch}} \leq y_t^1 \quad \forall t \quad (13h)$$

$$\rho_t^{\text{IN},\text{dis}} - \rho_t^{\text{IN},\text{ch}} \leq y_t^2 \quad \forall t \quad (13i)$$

$$\text{Deterministic form of constraints (1b)–(1p)} \quad (13j)$$

where  $y_t^1$  and  $y_t^2$  are auxiliary variables for transforming the Max-Min problem into a single-level optimization [40], and  $\hat{\sigma}^{\text{DA}}$  and  $\hat{\sigma}^{\text{IN}}$  reflect the deviation bounds day-ahead and intraday prices from their anticipated value (here,  $\pm 10\%$ ).

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