

# Renormalization of Maxwell asymptotic charges in $\text{AdS}_D$

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Based on [A. Campoleoni, AD, D. Francia, C. Heissenberg (2308.00476)]

## I. Plan and motivations

## II. Covariant phase space formalism

## III. Holographic renormalization

## IV. Symplectic renormalization

## V. Summary and future possibilities

**Let's start with some generic words...**

# I. Plan and motivations

- Probe the AdS/CFT correspondence

Study of the classical phase space of **asymptotically AdS gravity**

Asymptotic symmetries of AdS  $\leftrightarrow$  global symmetries of CFT

- Corner proposal [Donnelly-Freidel '16, Speranza '18, Ciambelli-Leigh '21, ...]

Gravitational theory  $\leftarrow$  **charges** and their algebra at **corners**

Organising principle for quantum observables

- Select the allowed metric fluctuations at infinity [Brown-Henneaux '86]

No requirement to fix any particular gauge but it is often convenient

For example: **Fefferman-Graham, Bondi gauge** [Starobinsky '83, Fefferman-Graham '85]

[Bondi-van der Burg-Metzner '62, Sachs '62]

# I. Plan and motivations

- **Charged diffeomorphisms** map inequivalent physical configurations

Symmetries used to gauge-fix can be charged (see, e.g., [Geiller-Goeller-Zwikel '21, Ciambelli-AD-Ruzziconi-Zwikel '23])

- **Charges** associated to asymptotic symmetries **can diverge**

Various techniques to **renormalize** the divergences [de Haro-Solodukhin-Skenderis '01, Barnich-Brandt '02, Papadimitriou-Skenderis '05, Mann-Marolf '06, Compère-Marolf '08, Freidel-Hopf Müller-Riello '19, Compère-Fiorucci-Ruzziconi '20, McNees-Zwikel '23, ...]

- In this talk: **modern review** of the renormalization techniques

Instructive and handy example of Maxwell fields in  $AdS_D$

**...now let's be more specific**

**In particular, what is an asymptotic symmetry?**

I. Plan and motivations

**II. Covariant phase space formalism**

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## II. Covariant phase space formalism: definitions

What is an **asymptotic symmetry**? For a gauge theory,

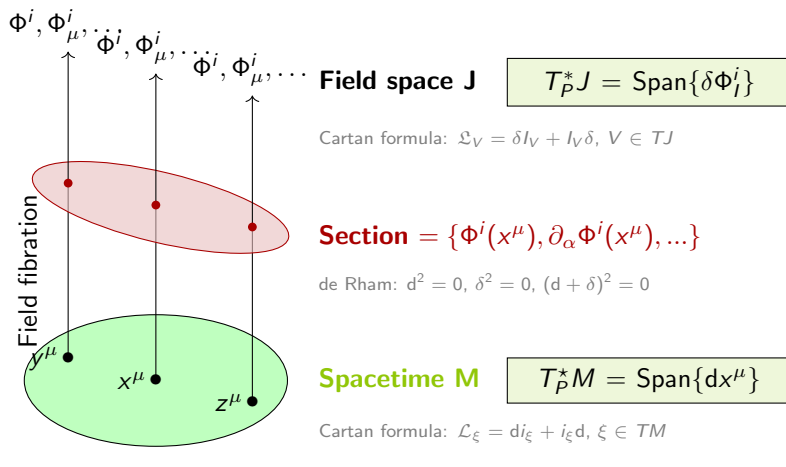
$$\text{Asymptotic symmetry group} = \frac{\text{Allowed diffeomorphisms}}{\text{Trivial gauge transformations}}$$

↪ Study the associated **charges**: how can we define Noether's charges?



↪ Extend Noether's first theorem [Lee-Wald '90, Wald-Zoupas '00, Barnich-Brandt '02]

## II. CPS formalism: variational bicomplex



$$\text{Jet bundle} = \{(x^\mu, \Phi^i_{(\mu)})\}$$

Horizontal derivative = exterior derivative  $d$   
 Vertical derivative = variational operator  $\delta$

Figure by [Compère-Fiorucci '18]

## II. CPS formalism: Lagrangian theory

- Action principle:

$$S = \int_M L, \quad L = \mathcal{L} d^D x$$

- Arbitrary field variation:  $\Phi \rightarrow \Phi + \delta\Phi \in J$

$$\delta L = (\text{eom})\delta\Phi + d\Theta \approx d\Theta, \quad \Theta = \Theta^\mu (d^{D-1}x)_\mu$$

- (Local) presymplectic two-form:

$$\omega = \delta\Theta, \quad \Omega = \int_{\partial M} \omega$$

- (Local) Noether current:  $V \in TJ$  isometry of  $\omega$ ,  $\mathcal{L}_V \omega = 0$

$$I_V \omega = -\delta J_V, \quad H_V = \int_{\partial M} J_V$$

## II. CPS formalism: Noether theorems and charges

- **Noether's first theorem:**  $V \in TJ$  global symmetry,  $\mathfrak{L}_V S = 0$

$$J_V = I_V \Theta, \quad dJ_V \approx 0, \quad H_V = \int_{\partial M} J_V$$

- **Noether's second theorem:**  $V = V_\xi \in TJ$  gauge symmetry,  $\mathfrak{L}_{V_\xi} S = \mathcal{L}_\xi S = 0$

$$J_{V_\xi} \approx I_{V_\xi} \Theta - i_\xi L \approx dQ_\xi \approx 0, \quad H_\xi \approx \int_{\partial^2 M} Q_\xi$$

- **Poisson bracket:**  $\xi, \zeta \in TM$ ,  $[\xi, \zeta] = \mathcal{L}_\xi \zeta$ ,  $\delta \kappa_{\xi, \zeta} = 0$

$$-\delta H_{[\xi, \zeta]} = I_{V_{[\xi, \zeta]}} \Omega = \delta \{H_\xi, H_\zeta\} \Rightarrow \{H_\xi, H_\zeta\} = \mathfrak{L}_{V_\xi} H_\zeta = -H_{[\xi, \zeta]} + \kappa_{\xi, \zeta}$$

- **Asymptotic symmetries:**
  - Dynamics on  $M$ , boundary conditions  $J|_{\partial M}$ , gauge fixings
  - Charges associated to residual symmetries

**Let's apply the CPS formalism to a  $U(1)$  gauge theory...**

## II. CPS formalism: classical electromagnetism

- Maxwell action:

$$S = \int_M L, \quad L = \frac{1}{4} F \wedge \star F$$

Faraday form and vector potential:

$$F = dA, \quad A = A_\mu dx^\mu, \quad dF = 0$$

- Arbitrary field variation:  $\delta L = (\text{eom})\delta A + d\Theta$

$$\text{eom} = d \star F \approx 0, \quad \Theta = \frac{1}{2} \star F \wedge \delta A$$

Gauge symmetry:

$$\delta_\lambda A = I_{V_\lambda} \delta A = d\lambda$$

- Surface charge:

$$\Omega = \int_{\partial M} \delta\Theta, \quad I_{V_\lambda} \Omega = -\delta H_\lambda, \quad H_\lambda \approx \int_{\partial^2 M} \lambda \star F$$

## II. CPS formalism: classical electromagnetism

- Maxwell action:

$$S = \int d^D x \mathcal{L}, \quad \mathcal{L} = -\frac{1}{4} \sqrt{-g} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}$$

Field strength:

$$\mathcal{F}_{\mu\nu} = \nabla_\mu \mathcal{A}_\nu - \nabla_\nu \mathcal{A}_\mu, \quad \partial_\mu \mathcal{F}_{\nu\rho} + \partial_\nu \mathcal{F}_{\rho\mu} + \partial_\rho \mathcal{F}_{\mu\nu} = 0$$

- Arbitrary field variation:  $\delta \mathcal{L} = (\text{eom})^\mu \delta \mathcal{A}_\mu + \partial_\mu \Theta^\mu$

$$(\text{eom})^\nu = \partial_\mu (\sqrt{-g} \mathcal{F}^{\mu\nu}) \approx 0, \quad \Theta^\mu = -\sqrt{-g} \mathcal{F}^{\mu\nu} \delta \mathcal{A}_\nu$$

Gauge symmetry:

$$\delta_\lambda \mathcal{A}_\mu = \nabla_\mu \lambda$$

- Surface charge:

$$\omega^\mu = \delta \Theta^\mu, \quad \omega_\lambda^\mu \approx -\partial_\nu (\sqrt{-g} \lambda \delta \mathcal{F}^{\mu\nu})$$

**...and consider an AdS  
background**

## II. Classical electromagnetism in AdS

- **Poincaré patch** of  $\text{AdS}_D$ : ( $a, b \in \{0, \dots, D-2\}$ )

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2} (\ell^2 dz^2 + \eta_{ab} dx^a dx^b)$$

Boundary of AdS is located at  $z \rightarrow 0$

- **Maxwell equations**: ( $\partial_\mu (\sqrt{-g} \mathcal{F}^{\mu\nu}) = 0$ )

$$\partial^a \mathcal{F}_{az} = 0, \quad \frac{1}{z\ell^2} (z\partial_z - D + 4) \mathcal{F}_{za} + \partial^b \mathcal{F}_{ba} = 0$$

Mixing with **Bianchi identities**: ( $\square = \partial^a \partial_a = \eta^{ab} \partial_a \partial_b$ )

$$\partial_z (z^{4-D} \partial_z \mathcal{F}_{ab}) = -z^{4-D} \ell^2 \square \mathcal{F}_{ab}$$

- **Radial gauge fixing**:

$$\mathcal{A}_z = 0, \quad \delta_\lambda \mathcal{A}_\mu = \nabla_\mu \lambda \quad \Rightarrow \quad \lambda = \lambda(x^a)$$

## II. Classical electromagnetism in AdS

- **Naive** computation of the **charge**: ( $\Omega_\lambda^z = -\delta H_\lambda$ ,  $\delta\lambda = 0$ )

$$\omega_\lambda^z \approx -\partial_a \left( \frac{\lambda}{\ell z^{D-4}} \partial_z \delta \mathcal{A}^a \right) \Rightarrow H_\lambda \approx \int d^{D-2}x \frac{\lambda}{\ell z^{D-4}} \partial_z \mathcal{A}^0$$

↪ Radial divergence(s) for  $D > 4$  when  $z \rightarrow 0$

- **Example ( $D = 6$ )**: asymptotic solution space ( $\partial \cdot A^{(3)} = 0$ )

$$\mathcal{A}_a = A_a^{(0)}(x^b) + \frac{z^2 \ell^2}{2} \left( \square A_a^{(0)} - \partial_a \partial \cdot A^{(0)} \right) + z^3 A_a^{(3)}(x^b) + \mathcal{O}(z^4)$$

and the surface charge

$$H_\lambda = \lim_{z \rightarrow 0} \int d^4x \frac{\lambda}{\ell} \left[ \underbrace{\frac{\ell^2}{z} \left( \square A_0^{(0)} - \partial_0 \partial \cdot A^{(0)} \right)}_{\text{divergent}} + \underbrace{\mathcal{O}(1)}_{\text{finite}} \right]$$

## II. Classical electromagnetism in AdS

- **Naive** computation of the **charge**: ( $\Omega_\lambda^z = -\delta H_\lambda$ ,  $\delta\lambda = 0$ )

$$\omega_\lambda^z \approx -\partial_a \left( \frac{\lambda}{\ell z^{D-4}} \partial_z \delta \mathcal{A}^a \right) \Rightarrow H_\lambda \approx \int d^{D-2}x \frac{\lambda}{\ell z^{D-4}} \partial_z \mathcal{A}^0$$

↪ Radial divergence(s) for  $D > 4$  when  $z \rightarrow 0$

- **Example ( $D = 5$ )**: asymptotic solution space ( $\partial \cdot A^{(2)} = 0$ )

$$\mathcal{A}_a = A_a^{(0)}(x^b) + z^2 \left[ A_a^{(2)}(x^b) - \log z \left( \square A_a^{(0)} - \partial_a \partial \cdot A^{(0)} \right) \right] + \mathcal{O}(z^4)$$

and the surface charge

$$H_\lambda = \lim_{z \rightarrow 0} \int d^3x \frac{\lambda}{\ell} \left[ \underbrace{2 \log z \left( \partial_0 \partial \cdot A^{(0)} - \square A_0^{(0)} \right)}_{\text{divergent}} + \underbrace{\mathcal{O}(1)}_{\text{finite}} \right]$$

**How can we renormalize the charges?**

**Via the quantities that define the charges**

**Via the action principle or the  
presymplectic potential**

- I. Plan and motivations
- II. Covariant phase space formalism
- III. Holographic renormalization**
- IV. Symplectic renormalization
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# III. Holographic renormalization

## Renormalize the action [de Haro-Solodukhin-Skenderis '01, Papadimitriou-Skenderis '05]

- **First step:** regularization – cut-off  $z \geq \epsilon$  ( $n_\mu = \delta_\mu^z$ )

$$S_{\text{reg}} = -\frac{1}{4} \int_{z \geq \epsilon} d^D x \sqrt{-g} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu} \approx \frac{1}{2\ell\epsilon^{D-4}} \int_{z=\epsilon} d^{D-1} x \mathcal{A}^a \mathcal{F}_{za}$$

↪ asymptotic expansion of fields and isolation of divergent terms ( $D = 6$ )

$$S_{\text{reg}} = -\frac{\ell}{2\epsilon} \int_{z=\epsilon} d^5 x A_{(0)}^a \partial \cdot F_a^{(0)} + \mathcal{O}(1), \quad F_{ab}^{(0)} = \partial_a A_b^{(0)} - \partial_b A_a^{(0)}$$

- **Second step:** counterterm and corner actions

$$S_{\text{ct}} = \frac{\ell}{2\epsilon} \int_{z=\epsilon} d^5 x A_{(0)}^a \partial \cdot F_a^{(0)} = -\frac{\ell}{4\epsilon} \int_{z=\epsilon} d^5 x F_{(0)}^{ab} F_{ab}^{(0)},$$

$$S_{\text{corner}} = -\frac{\ell}{2\epsilon} \int_{z=\epsilon} d^5 x \partial^b \left( A_{(0)}^a F_{ab}^{(0)} \right)$$

### III. Holographic renormalization

- **Third step:** inverted expansion and covariance ( $\gamma_{ab} = \eta_{ab}/\epsilon^2$ ,  $A_a^{(0)} = \mathcal{A}_a + \mathcal{O}(\epsilon^2)$ )

$$S_{\text{ct}} = -\frac{\ell}{4\epsilon} \int_{z=\epsilon} d^5x \mathcal{F}^{ab} \mathcal{F}_{ab} = -\frac{\ell}{4} \int_{z=\epsilon} d^5x \sqrt{-\gamma} \gamma^{ac} \gamma^{bd} \mathcal{F}_{ab} \mathcal{F}_{cd}$$

- **Fourth step:** renormalized action

$$S_{\text{ren}} = -\frac{1}{4} \lim_{\epsilon \rightarrow 0} \left[ \int_{z \geq \epsilon} d^6x \sqrt{-g} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu} + \ell \int_{z=\epsilon} d^5x \sqrt{-\gamma} \gamma^{ac} \gamma^{bd} \mathcal{F}_{ab} \mathcal{F}_{cd} \right]$$

- **Fifth step:** on-shell variation of renormalized action ( $F_{za}^{(2)} = 3A_a^{(3)}$ )

$$\delta S_{\text{ren}} \approx \lim_{\epsilon \rightarrow 0} \frac{1}{\ell \epsilon^2} \int_{z=\epsilon} d^5x \delta \mathcal{A}^a (\mathcal{F}_{za} - \ell^2 \epsilon \partial \cdot \mathcal{F}_a) = \frac{1}{\ell} \int d^5x \delta A_{(0)}^a F_{za}^{(2)}$$

- **Sixth and final step:** finite surface charge

$$\delta_\lambda S_{\text{ren}} \approx \frac{1}{\ell} \int d^5x \partial^a \left( \lambda^{(0)} F_{za}^{(2)} \right) \Rightarrow \boxed{H_\lambda = -\frac{1}{\ell} \int d^4x \lambda^{(0)} F_{z0}^{(2)}}$$

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# IV. Symplectic renormalization

## Renormalize the presymplectic potential [Freidel-Hopf Müller-Riello '19, McNees-Zwikel '23]

- **Main idea:** exploit the **ambiguities** of the CPS formalism ( $\delta L \approx d\Theta$ )
  - **boundary term:**  $L \rightarrow L + dB \Rightarrow \Theta \rightarrow \Theta + \delta B, \omega \rightarrow \omega$  ( $\delta^2 = 0$ )
  - **corner term:**  $d^2 = 0 \Rightarrow \Theta \rightarrow \Theta + dC, \omega \rightarrow \omega + d(\delta C)$
- Maxwell Lagrangian: ( $\mathcal{L} = -\frac{1}{4}\sqrt{-g}F^{\mu\nu}F_{\mu\nu}$ )

$$\mathcal{L} = z^{-(D-4)}\tilde{\mathcal{L}}, \quad \tilde{\mathcal{L}} = \frac{1}{2\ell}F^a{}_z F_{za} - \frac{\ell}{4}F^{ab}F_{ab}$$

Maxwell presymplectic potential: ( $\Theta^\mu = -\sqrt{-g}F^{\mu\nu}\delta\mathcal{A}_\nu$ )

$$\Theta^\mu = z^{-(D-4)}\tilde{\Theta}^\mu, \quad \tilde{\Theta}^z = \frac{1}{\ell}F_{az}\delta\mathcal{A}^a, \quad \tilde{\Theta}^a = \frac{1}{\ell}F_z{}^a\delta\mathcal{A}_z - \ell F^{ab}\delta\mathcal{A}_b$$

- **Asymptotic renormalization equation:** ( $\Theta^\mu \sim z^n\Theta_{(n)}^\mu, \mathcal{L} \sim z^n\mathcal{L}^{(n)}$ )

$$\delta\mathcal{L} \approx \partial_\mu\Theta^\mu \Rightarrow \boxed{(n-D+4)\tilde{\Theta}_{(n)}^z \approx \delta\tilde{\mathcal{L}}^{(n-1)} - \partial_a\tilde{\Theta}_{(n-1)}^a}$$

# IV. Symplectic renormalization

**Example:**  $D = 6$

- **Radial component** of the **presymplectic potential**:

$$\Theta^z = \underbrace{\frac{1}{z} \tilde{\Theta}_{(1)}^z}_{\text{divergent}} + \underbrace{\tilde{\Theta}_{(2)}^z}_{\text{finite}} + \mathcal{O}(z), \quad \tilde{\Theta}_{(1)}^z \approx \ell \partial \cdot F_a^{(0)} \delta A_{(0)}^a, \quad \tilde{\Theta}_{(2)}^z \approx -\frac{1}{\ell} F_{za}^{(2)} \delta A_{(0)}^a$$

- **Asymptotic renormalization equation**:

$$\tilde{\Theta}_{(1)}^z \approx \partial_a \tilde{\Theta}_{(0)}^a - \delta \tilde{\mathcal{L}}^{(0)}, \quad \tilde{\Theta}_{(0)}^a \approx -\ell F_b^{a(0)} \delta A_{(0)}^b, \quad \tilde{\mathcal{L}}^{(0)} \approx -\frac{\ell}{4} F_{(0)}^{ab} F_{ab}^{(0)}$$

- **Counterterm** presymplectic potential:

$$\Theta_{\text{ct}}^z = \frac{1}{z} \left[ \delta \tilde{\mathcal{L}}^{(0)} - \partial_a \tilde{\Theta}_{(0)}^a \right] \approx -\frac{\ell}{4z} \left[ \delta \left( F_{(0)}^{ab} F_{ab}^{(0)} \right) - 4 \partial_a \left( F_b^{a(0)} \delta A_{(0)}^b \right) \right]$$

- **Renormalized** presymplectic potential:

$$\Theta_{\text{ren}}^z = \lim_{z \rightarrow 0} (\Theta^z + \Theta_{\text{ct}}^z) = \tilde{\Theta}_{(2)}^z \approx -\frac{1}{\ell} F_{za}^{(2)} \delta A_{(0)}^a$$

**Poincaré patch of AdS does not allow a smooth flat limit,  $l \rightarrow \infty$**

**Bondi patch of AdS does**

# IV. Symplectic renormalization

- **Bondi patch** of  $\text{AdS}_D$ :  $(i, j \in \{1, \dots, D-2\})$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = - \left( 1 + \frac{r^2}{\ell^2} \right) du^2 - 2dudr + r^2 \gamma_{ij}(x^k) dx^i dx^j$$

Boundary of AdS is located at  $r \rightarrow \infty$

- **Asymptotic solution space**: field strength ( $D=6$ ) ( $\Delta = \gamma^{ij} \partial_i \partial_j$ )

$$\mathcal{F}_{ij} = F_{ij}^{(0)}(u, x^k) - \frac{\ell^2}{r} \partial_u F_{ij}^{(0)} + \frac{\ell^2}{2r^2} \Delta F_{ij}^{(0)} + \frac{1}{r^3} F_{ij}^{(3)}(u, x^k) + \mathcal{O}\left(\frac{1}{r^4}\right),$$

$$\mathcal{F}_{ir} = \frac{\ell^2}{r^2} F_{iu}^{(0)} - \frac{\ell^2}{r^3} \partial^j F_{ij}^{(0)} + \frac{1}{r^4} F_{ir}^{(4)} + \mathcal{O}\left(\frac{1}{r^5}\right),$$

$$\mathcal{F}_{ur} = \frac{\ell^2}{r^3} \partial^j F_{iu}^{(0)} + \frac{1}{r^4} F_{ur}^{(4)} + \mathcal{O}\left(\frac{1}{r^5}\right),$$

$$\mathcal{F}_{iu} = F_{iu}^{(0)}(u, x^j) - \frac{\ell^2}{r} \partial_u F_{iu}^{(0)} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

such that

$$0 = \frac{1}{\ell^2} \partial \cdot F_r^{(4)} + \ell^2 \partial \cdot F_u^{(0)} - \frac{\ell^2}{2} \Delta \partial \cdot F_u^{(0)} - \partial_u F_{ur}^{(4)}$$

# IV. Symplectic renormalization

- Radial component of the **presymplectic potential**:

$$\Theta^r = \underbrace{r \Theta_{(1)}^r}_{\text{divergent}} + \underbrace{\Theta_{(0)}^r}_{\text{finite}} + \mathcal{O}\left(\frac{1}{r}\right)$$

- **Asymptotic renormalization equation**: ( $\Theta^\mu \sim r^n \Theta_{(n)}^\mu$ ,  $\mathcal{L} \sim r^n \mathcal{L}^{(n)}$ )

$$\Theta_{(1)}^r \approx \delta \mathcal{L}_{(0)} - \partial_u \Theta_{(0)}^u - \partial_i \Theta_{(0)}^i$$

- **Renormalized presymplectic potential**:

$$\Theta_{\text{ren}}^r = \Theta_{(0)}^r \approx \frac{1}{\ell^2} F_{ir}^{(4)} \delta A_{(0)}^i + F_{ru}^{(4)} \delta A_u^{(0)} + \frac{\ell^2}{2} \left[ \partial^j \left( \partial_u F_{ij}^{(0)} + \partial_i F_{uj}^{(0)} \right) \delta A_{(0)}^i + 2F_{iu}^{(0)} \delta A_{(0)}^i \right]$$

- **Finite surface charge**:

$$\Theta_{\lambda, \text{ren}}^r \approx \boxed{\partial_u \left( \lambda F_{ur}^{(4)} \right)} - \frac{\partial^i}{2\ell^2} \left[ \lambda \left( 2F_{ir}^{(4)} - \ell^4 \left( \partial_u \partial \cdot F_i^{(0)} - 2F_{iu}^{(0)} + \partial_i \partial \cdot F_u^{(0)} \right) \right) \right]$$

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# V. Summary

## Main goal:

- Modern review of the asymptotic charge renormalization
- Instructive and handy example of Maxwell theory in  $AdS_D$

## Techniques:

Holographic		Symplectic	
✓	✗	✓	✗
Gauge inv.	Not systematic	Systematic	No gauge inv.
Diffeo inv.	Heavy computation	Soft computation	No diffeo inv.
Bdy action			No bdy action
Off-shell expr.			On-shell expr.

- Dual aspects: better to use the holographic renormalization
- Charge aspects: better to use the symplectic renormalization

# V. Summary

## Future possibilities:

- Linearized gravity and higher spins in  $\text{AdS}_D$  [WIP with A. Campoleoni, D. Francia, C. Heissenberg]
- Three-dimensional gravity coupled to higher spins [WIP with A. Campoleoni]
- Four-dimensional gravity [WIP with A. Campoleoni, S. Pekar, M. Petropoulos, D. Rivera-Betancour, M. Vilatte]
- Self-dual Yang-Mills and self-dual gravity [WIP with A. Campoleoni, S. Pekar, E. Skvortsov]
- ...

## IV. Summary



Emmy Noether and James Clerk Maxwell at Roma Tre University. Created with the assistance of DALL·E 2.

# Thank you for listening!