Lévy's phenomenon in the plane and on the disk

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Lévy's phenomenon concerns the rate of growth of entire functions $f(z) = \sum_{n=0}^{\infty} a_n z^n$. According to the Wiman-Valiron theory,

$$\max_{|z|\leq r} |f(z)| \leq \mu(r)(\log \mu(r))^{1/2}(\log\log \mu(r))^{1+\delta}$$

for any r outside a set of finite logarithmic measure, where $\delta > 0$ is a fixed number and μ denotes the maximum term of f. Erdős and Renyi showed in 1969 that if one attributes random signs to the coefficients a_n then, almost surely, one can replace the factor $(\log \mu(r))^{1/2}$ by $(\log \mu(r))^{1/4}$. Such a phenomenon was first observed, in a special case, by Paul Lévy in 1930. We show that the same result holds if one randomizes the series as $f(z) = \sum_{n=0}^{\infty} a_n X_n z^n$, where $(X_n)_n$ is an i.i.d. sequence of centred sub-gaussian random variables. This answers positively a question of O. B. Skaskiv and improves on a partial answer by A. Kuryliak (2017). We also consider the corresponding question for holomorphic functions on the unit disk. The results have an application to the dynamics of operators on the spaces $H(\mathbb{C})$ and $H(\mathbb{D})$.

This is joint work with Kevin Agneessens.