Super Null Infinity and the super good cuts

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Ongoing work with N. Boulanger and Y. Herfray

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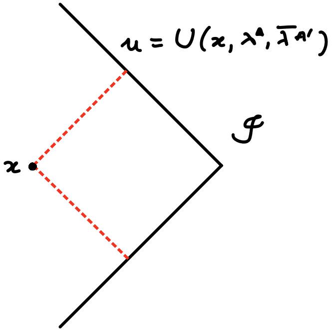
Context

- Newman 1976 : one of the first attempt to flat holography from null infinity (classical)
  - For an AF space define $\mathcal{H}$ as the space of cuts at null infinity
  - For Minkowski $\mathcal{H} \simeq M$



Context

- Newman 1976 : one of the first attempt to flat holography from null infinity (classical)
  - For an AF space define $\mathcal{H}$ as the space of cuts at null infinity
  - For Minkowski $\mathcal{H} \simeq M$
  - Non trivial : every AF self dual Einstein spacetime is one to one with $\mathcal{H}$
- Penrose : very powerful *non linear graviton theorem*, for asymptotic twistor spaces it gives the $\mathcal{H}$ spaces of Newman

Goal

In this talk :

- Using conformal compactification in a (super)twistor formulation
- And a decomposition in homogeneous spaces for the super Poincaré group
- We get a geometric description of super null Infinity
- And explicit form of the good cuts at super $\mathscr{I}$ for super Minkowski

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The non supersymmetric case

Conformally compactified Minkowski $\overline{M}^{1,3}$ is a homogeneous space for the conformal group

$$\overline{M}^{1,3} = \frac{\mathrm{SO}(2,4)}{\mathbb{R}^4 \rtimes (\mathbb{R} \times \mathrm{SO}(1,3))}$$

We can choose $\mathrm{ISO}(1,3) \subset \mathrm{SO}(2,4)$ and break the conformal invariance by imposing to stabilize the preferred degenerate direction called null infinity tractor $I^I = \begin{bmatrix} 1 \\ 0^{AA'} \\ 0 \end{bmatrix}$

$\rightarrow$ Split of $\overline{M}^{1,3}$ into orbits of Poincaré

$$\overline{M}^{1,3} = M^{1,3} \sqcup \mathscr{I} \sqcup \{I\}$$

Grassmannian definition of compactified Minkowski

Equivalent definition that generalizes to the susy case : (complexified)

$$\overline{M}^4 := \mathrm{Gr}(2, \mathbb{C}^4)$$
$$= \{\mathrm{span}(Z^{\alpha 1}, Z^{\alpha 2}) \mid Z^{\alpha b} = \begin{bmatrix} \omega^{Ab} \\ \pi_{A'}{}^{b} \end{bmatrix} \in \mathbb{C}^4 \text{ for } b = 1, 2\}$$

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$$\downarrow$$

$$\overline{M}_\ell^{4|2\mathcal{N}} := \mathrm{Gr}(2|0, \mathbb{C}^{4|\mathcal{N}})$$
$$= \{\mathrm{span}(Z^{\hat{\alpha} 1}, Z^{\hat{\alpha} 2}) \mid Z^{\hat{\alpha} b} = \begin{bmatrix} \omega^{Ab} \\ \pi_{A'}{}^{b} \\ \theta^{Ib} \end{bmatrix} \in \mathbb{C}^{4|\mathcal{N}} \text{ for } b = 1, 2\}$$

Parenthesis : chirality

Actually, it's more complicated... Complexify and :

$$\begin{tikzcd}[column sep=small]
 & \overline{M} = F(2|0, 2|\mathcal{N}, \mathbb{C}^{4|\mathcal{N}}) \arrow[dl, dashed, no head, "\pi_\ell"'] \arrow[dr, dashed, no head, "\pi_r"] & \\
\overline{M}_\ell = \mathrm{Gr}(2|0, \mathbb{C}^{4|\mathcal{N}}) & & \overline{M}_r
\end{tikzcd}$$

$$Z^{\hat{\alpha} b} = \begin{bmatrix} \omega_\ell^{Ab} \\ {\pi_{A'\ell}}^{b} \\ \theta_\ell^{Ib} \end{bmatrix}$$

Chiral right

Chiral left

Super Poincaré action

$$
\begin{array}{ccc}
SU(2,2) \twoheadrightarrow SO(2,4) & \rightarrow & \text{super conformal group } SU(2,2|\mathcal{N}) \\
SU(2,2) \circlearrowleft \overline{\mathbb{M}}^4 & \rightarrow & SU(2,2|\mathcal{N}) \circlearrowleft \overline{M}^{4|2\mathcal{N}} \\
I^{\alpha b} = \begin{bmatrix} 1^{Ab} \\ 0_{A'}{}^{b} \end{bmatrix} & \rightarrow & \text{preferred super null direction } I^{\hat{\alpha} b} = \begin{bmatrix} 1^{Ab} \\ 0_{A'}{}^{b} \\ 0^{Ib} \end{bmatrix} \\
ISO(1,3) & \rightarrow & ISO(1,3|\mathcal{N})_{\mathbb{C}} \subset SU(2,2|\mathcal{N})_{\mathbb{C}}
\end{array}
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\end{array}
$$

Result of the decomposition : more subspaces !

$$\overline{M}_{\ell}^{4|2\mathcal{N}} = M_{\ell}^{4|2\mathcal{N}} \sqcup \mathscr{I}_{\ell}^{(3|\mathcal{N})} \sqcup \iota_{\ell} \sqcup \mathcal{H} \sqcup \{I\}$$

Super Null Infinity

Coordinates on these orbits ? (reality conditions)

- On $M_\ell^{1,3|2\mathcal{N}} \simeq \frac{\mathrm{ISO}(1,3|\mathcal{N})}{\mathrm{SO}(1,3)\times\mathrm{SU}(\mathcal{N})}$ we find coordinates $(X_\ell^{AA'}, \theta^{A'}{}_I)$ such that one can write

$$X_\ell^{AA'} = X^{AA'} + \frac{i}{2}\theta^{A'}{}_I \bar{\theta}^A{}_{\bar{J}}\, \delta^{I\bar{J}},$$

for a real $X^{AA'}$

Chiral left coordinates appear naturally !

Super Null Infinity

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Chiral left coordinates appear naturally !

- On $\mathscr{I}_\ell^{(3|\mathcal{N})} \simeq \frac{\mathrm{ISO}(1,3|\mathcal{N})}{\mathbb{R}^3 \rtimes \left(\mathbb{R}^{0|\mathcal{N}} \rtimes (\mathrm{ISO}(2)\times\mathbb{R}\times\mathrm{SU}(\mathcal{N}))\right)}$ we find coordinates $(u_\ell, [\pi^A], \theta_I)$ such that one can write

$$u_\ell = u + \frac{i}{2}\theta_I \bar{\theta}_{\bar{J}}\delta^{I\bar{J}}$$

for a real u

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Ambitwistor space

The super ambitwistor space $\mathbb{A}$ is defined as the flag manifold $\mathbb{A} := F(1|0, 3|\mathcal{N}, \mathbb{C}^{4|\mathcal{N}})$.

$$\begin{array}{ccc} & F(1|0, 2|0, 2|\mathcal{N}, 3|\mathcal{N}, \mathbb{C}^4) & \\ \swarrow^{\pi_1} & & \searrow^{\pi_2} \\ \mathbb{A} = F(1|0, 3|\mathcal{N}, \mathbb{C}^{4|\mathcal{N}}) & & \overline{M} = F(2|0, 2|\mathcal{N}, \mathbb{C}^{4|\mathcal{N}}) \end{array}$$

$\rightarrow$ A point in $\mathbb{A}$ defines a super null ray in $\overline{M}$.

Ambitwistor space

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$$\begin{tikzcd}
 & F(1|0, 2|0, 2|\mathcal{N}, 3|\mathcal{N}, \mathbb{C}^4) \arrow[dl, dash, "\pi_1"'] \arrow[dr, dash, "\pi_2"] & \\
(Z^{\hat{\alpha}\, b}\lambda_b, \tilde{Z}_{\hat{\alpha}}{}^{b}\tilde{\lambda}_b) \in \mathbb{A} & & (Z^{\hat{\alpha} b}, \tilde{Z}_{\hat{\alpha} b}) \in M
\end{tikzcd}$$

Varying $(\lambda_b, \tilde{\lambda}_b)$ the super ambitwistor $(Z^{\hat{\alpha}\, b}\lambda_b, \tilde{Z}_{\hat{\alpha}}{}^{b}\tilde{\lambda}_b) \in \mathbb{A}$ corresponds to the super null cone passing through this point.

Super good cuts

The intersection of this super null cone with super $\mathscr{I}$ is the cut

$$(\lambda^a, \tilde{\lambda}^a) \quad \mapsto \quad \left(\begin{pmatrix} iX_+^{AB'}\lambda_{B'} & \tilde{\lambda}^A \\ \lambda_{A'} & 0 \\ \theta^{IB'}\lambda_{B'} & 0 \end{pmatrix}, \begin{pmatrix} \tilde{\lambda}_A & 0 \\ -i\tilde{X}_-^{A'B}\tilde{\lambda}_B & \lambda^{A'} \\ -\tilde{\theta}^{IB}\tilde{\lambda}_B & 0 \end{pmatrix} \right).$$

In the *chiral* coordinate system $(u_+, [\lambda_A], [\tilde{\lambda}_{A'}], \theta^I)$ for $\mathscr{I}_\ell$,

Solution to the super good cuts

$$(\lambda^a, \tilde{\lambda}^a) \longmapsto \left(X_+^{AA'}\lambda_A\tilde{\lambda}_{A'}, [\lambda_A], [\tilde{\lambda}_{A'}], \theta^{IB'}\lambda_{B'} \right).$$

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Conclusion

Short summary

- Super Minkowski case
- Supergeometry with a global approach, to describe the boundary
- Identify two remarkable subspaces inside the compactification : super Minkowski and super $\mathscr{I}$
- Expression of the super good cuts at super $\mathscr{I}$

Conclusion

Short summary

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Still in progress ...

- Generalisation to curved asymptotically flat superspaces
- Application to self dual supergravity

Thank you for your attention !