PROCEEDINGS OF SCIENCE

Carroll Fermions

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We consider different Carroll limits of relativistic Dirac fermions in any spacetime dimensions. One limit leads to Carroll fermions that are inert under internal Carroll boosts. We call these fermions electric Carroll fermions. Another limit makes use of projection operators and leads to a second type of Carroll fermion, called magnetic, that does transform non-trivially under Carroll boosts as a reducible but indecomposable representation of the Carroll group. We construct actions for both electric and magnetic Carroll fermions. In particular, in even dimensions we construct an action for a minimal magnetic Carroll fermion that has the same number of components as a Dirac spinor.

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1. Motivation

The Carroll group refers to a Inönü-Wigner contraction of the Poincaré group in which the speed of light c is taken to 0.[1] Originally studied by Lévy-Leblond [1] and Gupta [2] for a long time Carroll symmetries were considered as a theoretical curiosity with obscure features. In some respect, one can consider a Carroll limit as the opposite of a Galilei limit where one takes the limit in which the speed of light is taken to be infinity. These two limits have the effect that in the case of a Galilei limit the lightcone opens up and a particle can move everywhere in spacetime whereas when taking a Carroll limit the lightcone closes down and a Carroll particle cannot move [2], see Figure 1. In some sense, taking a Galilei limit of matter is as exotic as taking a Carroll limit. Souriau was the first to study such Galilei particles [6]. We note that in order to describe ordinary matter such as electrons and quarks one needs to extend the Galilei symmetries to centrally extended ones called Bargmann symmetries. This extension is required because in the non-relativistic case both the mass and the energy of ordinary matter is separately conserved while in the relativistic case, since mass is equivalent to energy, there is only one conservation law corresponding to a single Noether symmetry. We note that the Carroll symmetries do not allow such a central extension.

Galilei: lightcone opens up Carroll: lightcone closes down.

Figure 1: Under a Galilei limit (left) the lightcone opens up whereas under a Carrol limit (right) the lightcone closes down

Recently, Carroll symmetries have made a come-back due to several reasons. First of all, it turns out that a conformal extension of the Carroll group is the so-called BMS group that describes the asymptotic symmetries of flat spacetime at null infinity [7]. This relates the Carroll symmetries to the active field of flat space holography, see, e.g. [8–10]. Furthermore, any null hypersurface is described by a manifold whose structure group is the Carroll group. This applies for instance to black hole horizons [11]. In a rather different context, Carroll symmetries also naturally arise in the tensionless limit of string theory; see, e.g., [12] and references therein. Finally, Carroll symmetries also feature in a recent discussion of decoupling limits of M-theory [13].

Motivated by these applications of Carroll symmetry, various authors have recently considered the construction and study of Carroll invariant field theories, for an incomplete list, see [14–25]. Most of these papers concern the study of bosonic Carrollian field theories, see, however, [14, 18, 21, 22, 24, 26]. It is the purpose of this contribution to give a systematic treatment

[1]We use a slightly sloppy notation here. We actually mean to first replace c by λc, with λ a dimensionless contraction parameter, and next to take the limit $\lambda \to \infty$. Taking the limit in this way guarantees that c has disappeared from the action after taking the limit.

[2]This statement should be slightly weakened. First of all, when one considers a collection of Carroll particles only the center of mass cannot move [3] and secondly, taking the Carroll limit of a tachyon, one ends up with a new type of Carroll particle with zero energy that can move [4, 5].

of Carroll fermions thereby generalizing some of the results in the literature. As a warming-up exercise, we will first give a brief review of Carrollian scalar field theories.

2. Carroll Scalars

We are going to derive Carroll scalar field theories by taking a Carroll limit of a relativistic scalar field theory. We will take limits using the following two steps:

1. We first redefine the relativistic fields and symmetry parameters in terms of the would-be Carroll fields and parameters and a contraction parameter c.

2. We next substitute these redefinitions into the action and transformation rules and take the limit that $c \to 0$. [3]

We illustrate these two steps by first showing how by taking a Carroll limit the spacetime Poincaré transformations become Carroll transformations. We consider the Lorentz spacetime transformation rules in D dimensions $\delta X^A = -\Lambda^A{}_B X^B$, decompose the index A into $A = (0, a)$ with $a = 1, , 2, \cdots D-1$ and make the following redefinitions:

$$X^0 = \frac{t}{\tilde{c}}\,, \qquad X^a = x^a\,, \qquad \Lambda^{ab} = \lambda^{ab}\,, \qquad \Lambda^{0a} = \frac{1}{\tilde{c}}\lambda^{0a} \tag{1}$$

so that we obtain

$$\delta t = -\lambda^0{}_a x^a\,, \qquad \delta x^a = -\lambda^a{}_b\, x^b - \frac{1}{\tilde{c}^2}\lambda^a{}_0\, t \tag{2}$$

In particular, this shows that the Carroll boosts only work in one direction and in this way form a reducible but indecomposable representation:

$$t \;\; \to \;\; x^a \;\; \to \;\; 0\,. \tag{3}$$

We will assume that the Carroll limit of the spacetime coordinates always works in this way and from now on concentrate on the internal Carroll boosts only. It turns out that there are two different ways of defining a Carroll limit which we will discuss below separately.

2.1 Electric Carroll Scalars

We consider the following Lagrangian describing a D-dimensional relativistic real scalar Φ with mass M :

$$\mathcal{L} = \frac{\tilde{c}^2}{2}(\partial_t\Phi)^2 - \frac{1}{2}\partial_a\Phi\partial^a\Phi - \frac{M^2}{2\tilde{c}^2}\Phi^2\,. \tag{4}$$

Besides the coordinate redefinitions mentioned above we now make the further redefinitions

$$\Phi = \frac{\phi}{\tilde{c}}\,, \qquad M = m\tilde{c}^2\,. \tag{5}$$

[3] We find it convenient to define a different contraction parameter $\tilde{c} = 1/c$ and to take $\tilde{c} \to \infty$.

Taking $\tilde{c} \to \infty$, we obtain the following electric Carroll scalar Lagrangian :

$$\mathcal{L}_{\text{electric scalar}} = \frac{1}{2}\,(\partial_t\phi)^2 - \frac{m^2}{2}\phi^2\,. \tag{6}$$

Under the internal Carroll boosts we have

$$\partial_a\phi \quad\to\quad \partial_t\phi \quad\to\quad 0\,, \tag{7}$$

which shows that the electric Carroll Lagrangian is invariant under these internal Carroll boost transformations. Due to the absence of a term with spatial derivatives this scalar field theory corresponds to an electric Carroll particle which has non-zero energy but cannot move [1, 2].

2.2 Magnetic Carroll Scalars

There is a different way of taking the Carrol limit where one uses a so-called Hubbard-Stratonovich transformation that allows us to control the leading divergence in an expansion of the action and consider the sub-leading terms as the outcome of the limit. This transformation works as follows. [4] If the leading order divergence is given by a complete square of the generic form X^2, one can write an equivalent expression using an auxiliary field λ as follows :

$$\tilde{c}^2X^2 + O(\tilde{c}^0) \qquad\Leftrightarrow\qquad -\frac{1}{\tilde{c}^2}\lambda^2 + 2\lambda X + O(\tilde{c}^0)\,. \tag{8}$$

The equivalence, for finite $\tilde{c}$, can be shown by substituting back the solution of the auxiliary field given by

$$\lambda = \tilde{c}^2X\,. \tag{9}$$

After taking the limit $\tilde{c} \to \infty$ the first term in the alternative expression vanishes and the auxiliary field becomes a Lagrange multiplier with the result that the limit is given by the sub-leading terms.

Alternatively, one can avoid such a Hubbard-Stratonovich transformation by not starting with a scalar field in a second-order formulation like we did in the electric case but, instead, by starting from a Hamiltonian formulation of this Lagrangian. On top of this we also take an opposite sign of the mass term, for reasons given below, so that we obtain the following Lagrangian:

$$\mathcal{L} = \Pi\partial_t\Phi - \frac{1}{2\tilde{c}^2}\Pi^2 - \frac{1}{2}\partial_a\Phi\partial^a\Phi + \frac{M^2}{2\tilde{c}^2}\Phi\,. \tag{10}$$

Making the redefinitions

$$\Pi = \pi\,, \qquad\qquad \Phi = \phi\,, \qquad\qquad M = m\tilde{c} \tag{11}$$

and taking $\tilde{c} \to \infty$ we obtain the following magnetic Carroll scalar Lagrangian [4, 5, 15] :

$$\mathcal{L}_{\text{magnetic scalar}} = \pi\partial_t\phi - \frac{1}{2}\partial_a\phi\partial^a\phi + \frac{m^2}{2}\phi^2\,. \tag{12}$$

This Lagrangian is invariant under internal Carroll boosts due to the following exact sequence:

$$\pi \quad\to\quad \partial_a\phi \quad\to\quad \partial_t\phi \quad\to 0\,. \tag{13}$$

[4]The transformation given below can be generalized to the cases where the leading divergence is given by terms of the form XY or $X^2 + Y^2$ for certain X and Y. We will not need these generalizations here.

Due to the Lagrange multiplier term this action corresponds to a so-called magnetic Carroll particle with zero energy, which, however, can move.

The reason that we had to change the sign of the mass term in the magnetic case, so that we are dealing with a tachyon, is that we needed consistency with the dispersion relation whose original relativistic expression reads:

$$M^2 = E^2 - \vec{p}^2 \,. \tag{14}$$

When taking the electric limit we get $\vec{p} = 0$ and $m^2 = E^2$. However, in the magnetic case we have $E = 0$ and we would obtain the inconsistent relation $m^2 = -\vec{p}^2$ unless we replace M by iM.

This closes our discussion of the Carroll scalars. We next turn our attention to the fermionic case.

3. Carroll Fermions

Recently, several works have appeared dealing with Carroll fermions. Several of these works use different limiting techniques applied to Lagrangians, Hamiltonians or equations of motion [14, 18, 21, 24, 27]. An interesting bottom-up construction of Carroll fermion field theories appeared in [21, 22] (see also [26]). In this approach, one does not start from a $c \to 0$ limit of relativistic fermions, but instead constructs spinor representations of the homogeneous Carroll group, starting from degenerate Clifford algebras [28–30].

One obstacle in defining two different (electric and magnetic) limits of a Dirac fermion, as we did for scalars above, is that the Dirac lagrangian is already written in a first-order form. Here we show that nevertheless two different limits can be defined by making use of a special projection operator acting on the spinor indices. Furhermore, we will show that in even dimensions we can construct a minimal magnetic formulation using a single Dirac fermion, by making use of a higher-dimensional generalization of the four-dimensional Γ_5 matrix.

Before taking limits of Lagrangians, we first consider a relativistic complex Dirac spinor Ψ in a D-dimensional Minkowski spacetime with the standard Lorentz transformation rule

$$\delta\Psi(x) = \Xi^A\partial_A\Psi(x) - \frac{1}{4}\Lambda_{AB}\Gamma^{AB}\Psi(x) \tag{15}$$

where

$$\delta X^A \equiv X'^A - X^A = -\Xi^A\,, \qquad\qquad \Xi^A = \Lambda^A{}_B X^B\,. \tag{16}$$

To obtain a Carroll fermion we decompose $A = (0, a)$, redefine the coordinates as in the scalar case together with the following redefinitions of the parameters and the Dirac fermion:

$$\Lambda^{ab} = \lambda^{ab} \text{ and } \xi^a = \lambda^a{}_b x^b\,, \qquad \Lambda^{0a} = \frac{1}{\tilde{c}}\lambda^{0a} \text{ and } \xi^0 = \lambda^0{}_a x^a\,, \qquad \Psi = \psi\,.$$

After taking the limit that $\tilde{c} \to \infty$ we obtain in this way a transformation rule without internal Carroll boosts:

$$\delta\psi = \xi^0\frac{\partial\psi}{\partial t} + \xi^a\frac{\partial\psi}{\partial x^a} - \frac{1}{4}\lambda^{ab}\Gamma_{ab}\psi\,.$$

To obtain non-trivial internal Carroll boosts we decompose the Dirac spinor Ψ while preserving the spatial rotations. For this purpose we can use any of the matrices $\Gamma^0, \Gamma^0\Gamma_\star$ or $\Gamma_\star$ where $\Gamma_\star$, to be defined below, indicates the generalization of Γ_5 to any even dimension. It turns out that the option $\Gamma^0\Gamma_\star$ gives similar answers as when we use Γ^0 in terms of some redefined gamma matrices. The option $\Gamma_\star$ does not work for our purposes since it cannot distinguish between spatial rotations and boosts. We therefore take the Γ^0 option and define the following projected Dirac fermion $\Psi_\pm$:

$$\Psi_\pm = \frac{1}{2}\big(1 \pm i\Gamma^0\big)\,\Psi\,. \tag{17}$$

We next redefine the two projections differently as follows:

$$\Psi_\pm = \tilde{c}^{\pm 1/2+\epsilon}\,\frac{1}{2}\big(1 \pm i\Gamma^0\big)\,\psi_\pm\,, \tag{18}$$

where ϵ is an arbitrary free parameter. Calulating the transformation rules of the two projections we find that after taking the limit $\tilde{c} \to \infty$ one of the two projections transform under internal Carroll boosts as follows:

$$\delta\psi_+ = \xi^0\frac{\partial\psi_+}{\partial t} + \xi^a\frac{\partial\psi_+}{\partial x^a} - \frac{1}{4}\lambda^{ab}\Gamma_{ab}\psi_+\,, \tag{19}$$

$$\delta\psi_- = \xi^0\frac{\partial\psi_-}{\partial t} + \xi^a\frac{\partial\psi_-}{\partial x^a} - \frac{1}{4}\lambda^{ab}\Gamma_{ab}\psi_- \frac{1}{2}\lambda^{0a}\Gamma_{0a}\psi_+\,. \tag{20}$$

This shows that the projected spinors $\psi_\pm$ form a reducible but indecomposable representation of the homogeneous Carroll group :

$$\psi_- \;\to\; \psi_+ \;\to\; 0\,. \tag{21}$$

We now discuss the electric and magnetic Carroll fermions separately.

3.1 Electric Carroll fermions

The electric Carroll fermions are the easiest to define. Starting from the relativistic Dirac Lagrangian

$$\mathcal{L}_{\text{relativistic Dirac}} = \bar{\Psi}\Gamma^\mu\partial_\mu\Psi - \frac{M}{\tilde{c}}\bar{\Psi}\Psi \tag{22}$$

and taking the Carroll limit defined by the redefinitions

$$X^0 = t/\tilde{c}\,, \qquad X^a = x^a\,, \qquad \Psi = \psi\,, \qquad M = \tilde{c}^2 m \tag{23}$$

we obtain the following electric Carroll Dirac Lagrangian [18]

$$\mathcal{L}_{\text{electric Carroll Dirac}} = \bar{\psi}\Gamma^0\dot{\psi} - m\bar{\psi}\psi + \text{h.c.}\,. \tag{24}$$

Here we have defined $\dot{\psi} \equiv \frac{\partial\psi}{\partial t}$. The verification of the invariance of this Lagrangian under internal Carroll boosts can be done using the sequences

$$\partial_a\psi \;\to\; \partial_t\psi \;\to\; 0 \qquad \text{and} \qquad \psi \;\to\; 0\,. \tag{25}$$

Note that the electric Carroll Lagrangian (24) can be truncated consistently with the Carroll symmetry as follows:

$$\psi_+ = 0 \;\text{ or }\; \psi_- = 0 \qquad \text{or} \qquad \psi_{\rm L} = 0 \;\text{ or }\; \psi_{\rm R} = 0\,. \tag{26}$$

3.2 Magnetic Carroll fermions

To make contact with recent other work on magnetic Carroll fermions we start from a non-minimal off-diagonal Lagrangian and only at a later stage give the truncation (in even dimensions) to a minimal model [31]. To be precise, we consider the following off-diagonal Lagrangian for two Dirac spinors Ψ and $\mathbf{X}$:

$$\mathcal{L}_{\text{off-diagonal}} = \bar{\mathbf{X}}\Gamma^A \partial_A \Psi - i\frac{M}{\tilde{c}}\bar{\mathbf{X}}\Psi + \text{h.c.}\,. \tag{27}$$

To define a magnetic Carroll limit we consider the projected spinors $\Psi_\pm$ and $X_\pm$ and make, besides the usual coordinate redefinitions and mass rescaling $M = \tilde{c}^2 m$, the following redefinitions of the fermion fields :

$$\Psi_+ = \sqrt{\tilde{c}}\,\tilde{c}^{\epsilon}\,\psi_+\,, \qquad \Psi_- = \frac{1}{\sqrt{\tilde{c}}}\,\tilde{c}^{\epsilon}\,\psi_-\,, \tag{28}$$

$$\mathbf{X}_+ = \frac{1}{\sqrt{\tilde{c}}}\,\tilde{c}^{\epsilon}\,\chi_+\,, \qquad \mathbf{X}_- = \sqrt{\tilde{c}}\,\tilde{c}^{\epsilon}\,\chi_-\,. \tag{29}$$

Here ϵ is an overall scaling parameter that in the present case needs to be $\epsilon = -1/2$. Substituting these redefinitions back into the off-diagonal Lagrangian (27) and taking the limit $\tilde{c} \to \infty$ one finds a non-minimal off-diagonal Lagrangian. To explain why (in even dimensions) a truncation to a minimal Lagrangian is possible, it is instructive to substitute the same redefinitions into the relativistic transformation rules. After taking $\tilde{c} \to \infty$ we obtain the following result:

$$\delta\psi_+ = \xi^0\dot{\psi}_+ + \xi^a\partial_a\psi_+ - \frac{1}{4}\lambda_{ab}\Gamma^{ab}\psi_+ \quad \text{and similar for } \chi_-\,, \tag{30}$$

$$\delta\psi_- = \xi^0\dot{\psi}_- + \xi^a\partial_a\psi_- - \frac{1}{4}\lambda_{ab}\Gamma^{ab}\psi_- - \frac{1}{2}\lambda_{0a}\Gamma^{0a}\psi_+ \quad \text{and similar for } \chi_+\,. \tag{31}$$

These transformation rules suggest the following truncations in even dimensions:

$$\chi_\pm = \Gamma_\star\psi_\mp\,,$$

where we have defined the generalization $\Gamma_\star$ of the 4D Γ_5 matrix as follows:

$$\Gamma_\star = (-\mathrm{i})^{\frac{D}{2}+1}\Gamma^0\Gamma^1\cdots\Gamma^{D-1}\,. \tag{32}$$

We thus obtain the following minimal magnetic Carroll Dirac Lagrangian :

$$\begin{aligned}\mathcal{L}_{\text{magnetic Carroll Dirac}} &= 2\bar{\psi}_-\Gamma^0\Gamma_\star\dot{\psi}_+ + 2\bar{\psi}_+\Gamma^0\Gamma_\star\dot{\psi}_- + 2\bar{\psi}_+\Gamma^a\Gamma_\star\partial_a\psi_+ + \\ &\quad +im\,(\bar{\psi}_+\Gamma_\star\psi_- + \bar{\psi}_-\Gamma_\star\psi_+)\end{aligned}$$

To verify the internal Carroll boost symmetry of this Lagrangian one may use the following sequences:

$$\partial_a\psi \;\to\; \partial_t\psi \;\to\; 0 \quad \text{and} \quad \psi_- \;\to\; \psi_+ \;\to\; 0\,. \tag{33}$$

4. Conclusions

We note that the results on Carroll limits discussed here can be generalized to similar Galilei limits of both scalars [32] and fermions [24]. This leads to similar notions of electric and magnetic Galilei scalars and fermions.

An undesirable feature of our limit technique is that it requires the existence of a Dirac fermion. This excludes for instance applications to supergravity in ten and eleven spacetime dimensions. This obstacle can be circumvented by generalizing our particle limit to extended objects and by requiring that the projection operator is invariant under the transverse spatial rotations only. This can be achieved by replacing the Γ_0 occurring in the particle projection operator by a gamma matrix with indices in all directions longitudinal to the extended object [33]. In this way one can define in ten and eleven dimensions, where the corresponding supergravity theories do not contain Dirac spinors, suitable string and membrane projection operators as follows:

$$P_\pm = \frac{1}{2}(1 \pm \Gamma_{01}) \ : \text{strings} \qquad \text{and} \qquad P_\pm = \frac{1}{2}(1 \pm \Gamma_{012}) \ : \text{membranes}\,. \tag{34}$$

These projection operators are consistent with Majorana-Weyl spinors in 10D and Majorana spinors in 11D.

The rescalings with $\tilde{c}$ we made when defining our Carroll limits guarantee that our final expressions are invariant under global scalings of the fields. This implies that the Carroll symmetry is extended to a conformal Carroll symmetry and that the actions we constructed, for generic dimensions, are invariant under a (finite-dimensional) conformal Carroll algebra. This leads to interesting connections with the BMS symmetry that plays such a prominent role in flat space holography and celestial holography. In this context, it is of interest to note that the free Carroll fermion models we constructed can be used to construct infinite dimensional algebras of the type $w_{1+\infty}$ in the same way as this has been done before in the relativistic case, see e.g. [34, 35]. It would be interesting to see whether there exists electric and magnetic Carroll versions of these infinite-dimensional algebras.

Last but not least, given that we have electric (magnetic) Carroll scalars and fermions, it is natural to consider combinations that exhibit electric (magnetic) Carroll supersymmetry. We hope to report on this in a future work [36] (see also [24]).

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