An LMI-based approach to the design of super-twisting observers with application to a lipase production process from olive oil by *Candida rugosa*

Christian G. Feudjio Letchindjio
Automatic Control Laboratory
University of Mons, Belgium
christiangabin.feudjioletchindjio@umons.ac.be

Daniel Coutinho
Department of Automation and Systems
Universidade Federal de Santa Catarina, Brazil
daniel.coutinho@ufsc.br

Alejandro Vargas
Instituto de Ingeniería
Universidad Nacional Autónoma de México, Mexico
avargasc@ii.unam.mx

Alain Vande Wouwer
Automatic Control Laboratory
University of Mons, Belgium
alain.vandewouwer@umons.ac.be

***Abstract*—This paper presents a systematic methodology for designing super-twisting observers. Thanks to a state transformation, the estimation error dynamics can be cast in a Linear Parameter Varying (LPV) framework and a design procedure based on LMIs can be developed ensuring practical stability and finite time convergence. This approach is applied to the design of 3 super-twisting observers for estimating the reaction rates in a lipase production process from olive oil by *Candida rugosa*. The estimated rates can be used in an asymptotic observer to reconstruct the component concentrations.**

***Index Terms*—state estimation, sliding mode observers, robust control, LMIs, biotechnology.**

I. INTRODUCTION

Although they have been widely applied to monitoring of bioprocesses [1], [2], an inherent drawback of classical state estimation algorithms, such as Extended Kalman filter, is their dependency on the model accuracy. As bioprocess models are usually complex, nonlinear and partially known (in their parameters or even structure), the need for more robust strategies appears naturally.

The use of super-twisting observers (STOs) for kinetic rate estimation has been successfully investigated in [3]–[5]. However, to the best of our knowledge, no systematic design procedure for the gains of the STOs is available yet, and the design usually proceeds based on the user expertise, in an ad-hoc manner. The objective of this study is twofold: (a) to present a robust design STO procedure, leading to a computational algorithm and (b) to develop a monitoring system for a lipase production process from olive oil by *Candida rugosa*, based on the design of 3 STOs for the several reaction rates.

This paper is organized as follows. The next section introduces the methodology for STO gain computation, while section III describes the bioprocess case study and section IV the development of a STO-based monitoring scheme. Finally some conclusions are drawn.

II. STO GAIN DESIGN

Let us consider a generic second-order system:

$$\dot{\xi}_1(t) = b(t,y,u)\xi_2(t) + h_1(t,y) + f_1(t,\xi), \quad \xi_1(0) = \xi_{1,0}, \tag{1a}$$

$$\dot{\xi}_2(t) = h_2(t,y) + f_2(t,\xi), \quad \xi_2(0) = \xi_{2,0}, \tag{1b}$$

$$y(t) = \xi_1(t), \tag{1c}$$

where $\xi := [\,\xi_1 \quad \xi_2\,]^T \in \Xi \subset \mathbb{R}^2$ is the state vector; $u(t) \in \mathcal{U} \subset \mathbb{R}$ and $y(t) \in \mathcal{Y} \subset \mathbb{R}$ are respectively the input and output signals assumed to be measurable; and Ξ, $\mathcal{U}$ and $\mathcal{Y}$ are given compact sets in the state-, input- and output-space, respectively. h_1 and h_2 encompass the part of dynamic that is perfectly known (measurable) and f_1 and f_2 capture the uncertainty in the model.

Assumption 1. *The parameter $b(t,y,u)$ is a bounded positive measurable function of (t,y,u) such that $0 < \underline{b} \le b(t,y,u) \le \overline{b}$ for all $t \ge 0$, $y \in \mathcal{Y}$ with*

$$\mathcal{Y} := \{y \in \mathbb{R} : y = C\xi,\ \xi \in \Xi\},\ C = [\,1 \quad 0\,], \tag{2}$$

and $u \in \mathcal{U}$, where $\underline{b}, \overline{b}$ are known positive constants.

The following observer is considered:

$$\begin{aligned}
\dot{\hat{\xi}}_1 &= -k_1\phi_1(\tilde{\xi}_1) + b(t,y,u)\hat{\xi}_2 + h_1(t,y) + \hat{f}_1(t,\hat{\xi}), \quad \hat{\xi}_1(0) = \hat{\xi}_{1,0},\\
\dot{\hat{\xi}}_2 &= -k_2\phi_2(\tilde{\xi}_1) + h_2(t,y) + \hat{f}_2(t,\hat{\xi}), \quad \hat{\xi}_2(0) = \hat{\xi}_{2,0},\\
\tilde{\xi}_1 &= \hat{\xi}_1 - y
\end{aligned} \tag{3}$$

where $\hat{\xi} := [\,\hat{\xi}_1 \quad \hat{\xi}_2\,]^T \in \hat{\Xi} \subset \mathbb{R}^2$ is the STO state vector; $\hat{f}_1(t,\hat{\xi})$ and $\hat{f}_2(t,\hat{\xi})$ are $f_1(t,\xi)$ and $f_2(t,\xi)$ estimates, k_1 and k_2 are the observer gains. The correction terms ϕ_1 and ϕ_2

are switching functions to be specified later in this paper. The estimation error reads:

$$\tilde{\xi} = \begin{bmatrix} \tilde{\xi}_1 & \tilde{\xi}_2 \end{bmatrix}^T, \qquad \tilde{\xi}_i = \hat{\xi}_i - \xi_i, \qquad i = 1, 2. \quad (4)$$

For illustration, the following definitions for ϕ_1 and ϕ_2 are considered for the classical super-twisting observer

$$\phi_1(\tilde{\xi}_1) = \sqrt{|\tilde{\xi}_1|}\,\mathrm{sign}(\tilde{\xi}_1), \qquad \phi_2(\tilde{\xi}_1) = \frac{1}{2}\,\mathrm{sign}(\tilde{\xi}_1). \quad (5)$$

Then, the estimation error dynamics follow:

$$\begin{cases} \dot{\tilde{\xi}}_1 &= -k_1\phi_1(\tilde{\xi}_1) + b(t,y,u)\tilde{\xi}_2 + \delta_1(t), \quad \tilde{\xi}_1(0) &= \tilde{\xi}_{1,0}, \\ \dot{\tilde{\xi}}_2 &= -k_2\phi_2(\tilde{\xi}_1) + \delta_2(t), \quad \tilde{\xi}_2(0) &= \tilde{\xi}_{2,0}, \end{cases} \quad (6)$$

where $\tilde{\xi} \in \tilde{\Xi} \subset \mathbb{R}^2$, $\tilde{\Xi}$ is a given compact set containing $\tilde{\xi} = 0$,

$$\delta_i(t) = \hat{f}_i(t, \hat{\xi}) - f_i(t, \xi), \qquad i = 1, 2. \quad (7)$$

Assumption 2. *There exist bounded functions $g_1(t), g_2(t)$ and positive scalars α_1, α_2 such that*

$$\delta_i(t) = g_i(t)\phi_i(\tilde{\xi}_1),\ |g_i(t)| \leq \alpha_i,\ i = 1, 2,\ \forall\, \hat{\xi} \in \hat{\Xi},\ \xi \in \Xi. \quad (8)$$

The change of variables $\Phi : \tilde{\Xi} \to \mathbb{R}^2$

$$\zeta = \Phi(\tilde{\xi}) = \begin{bmatrix} \phi_1(\tilde{\xi}_1) \\ \tilde{\xi}_2 \end{bmatrix} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}. \quad (9)$$

allows for the error dynamics to be described almost everywhere by the state-space representation:

$$\dot{\zeta} = \frac{\partial \phi_1}{\partial \tilde{\xi}_1}\big(A(g_1, g_2, b) - KC\big)\zeta \quad (10)$$

where C is as in (2), and

$$A(g_1, g_2, b) = \begin{bmatrix} g_1(t) & b(t,y,u) \\ g_2(t) & 0 \end{bmatrix}, \qquad K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}. \quad (11)$$

Notice in (10) that $\dot{\zeta}$ is properly defined in $\mathcal{Z} \setminus \mathcal{T}$ with $\mathcal{T} := \{\zeta \in \mathbb{R}^2 : \zeta_1 = 0\}$ and $\mathcal{Z} \subset \mathbb{R}^2$ being a compact set containing $\zeta = 0$ and defining the admissible ζ-space.

We then choose the following Lyapunov function candidate:

$$V(\zeta) = \zeta^T P \zeta,\ P = P^T > 0, \quad (12)$$

(12) is monotonic decreasing and the estimation error dynamics is locally stable [6] if $\dot{V}(\zeta) < 0$, for all $\zeta \in \mathcal{Z} \setminus \mathcal{T}$.

Afterwards, we introduce a set of LMIs allowing the computation of the observer gain matrix K and ensuring both local asymptotic stability and a finite-time convergence of the STO.

Theorem 1. *Consider the system* (6) *with* (5)*, and the assumptions* **A1** *and* **A2***. Suppose there exist matrices $P = P' > 0$ and L, and a scalar $\lambda_3 > 0$ satisfying the following LMI constraints:*

$$A(g_1, g_2, b)^T P + P A(g_1, g_2, b) - LC - C^T L^T + \lambda_3 I < 0,\ \forall\, (g_1, g_2, b) \in \mathcal{V} \quad (13)$$

where $\mathcal{V}$ is the set of all vertices of the meta set $[-\alpha_1, \alpha_1] \times [-\alpha_2, \alpha_2] \times [\underline{b}, \overline{b}]$. Then, the origin of the error dynamics (6) *with $K = P^{-1}L$ is locally asymptotically stable and $\tilde{\xi}_1(t)$ and $\tilde{\xi}_2(t)$ converge to zero in a finite time $T_c \leq T_c^*$ with:*

$$T_c^* = \frac{2}{\gamma}\sqrt{V(0)}, \qquad \gamma = \frac{\sqrt{\lambda_1}\lambda_3}{\lambda_2}, \quad (14)$$

where $V(0) = V(\zeta_0)$ is the initial value of the Lyapunov function defined in (12) *in some vicinity of $\zeta = 0$, and λ_1 and λ_2 are respectively the smallest and largest eigenvalues of P.*

The proof of the above theorem is provided in [7].

However, this result is not yet directly implementable as ζ_0, thus $V(0)$, is unknown a priori. Let us define :

$$\mathcal{R}(\zeta) := \{\zeta \in \mathcal{Z} : V(\zeta) \leq 1\}. \quad (15)$$

Where the set $\mathcal{R}(\zeta)$ provides an estimate of the stability region. $\mathcal{R}(\zeta)$ will be positive invariant if $V(\zeta)$ is monotonic decreasing along the trajectories of system (10). Then, in addition to (13), we have to guarantee that $\mathcal{R}(\zeta) \subset \mathcal{Z}$. However, the error dynamics is defined in the $\tilde{\xi}$-space and thus it is more convenient to define $\mathcal{Z}$ in terms of the bounds on the $\tilde{\xi}$-space. Then, let the $\tilde{\xi}$-space be defined by the following rectangle

$$\tilde{\Xi} := \{\tilde{\xi} \in \mathbb{R}^2 : |\tilde{\xi}_i| \leq \beta_i,\ i = 1, 2\}, \quad (16)$$

where $\beta_i \in \mathbb{R}_+$. Since $\zeta_1^2 = |\tilde{\xi}_1|$ and $\zeta_2 = \tilde{\xi}_2$, the resulting region in the ζ-space can be defined as follows

$$\mathcal{Z} := \{\zeta \in \mathbb{R}^2 : \zeta_1^2 \leq \beta_1,\ |\zeta_2| \leq \beta_2\}. \quad (17)$$

In light of (15) and (17), notice that $\mathcal{R}(\zeta) \subset \mathcal{Z}$ can be cast as follows:

$$\left.\begin{array}{c} \beta_1 - \zeta^T e_1 e_1^T \zeta \geq 0 \\ 1 \pm \beta_2^{-1} e_2^T \zeta \geq 0 \end{array}\right\}\ \forall\ \zeta : V(\zeta) - 1 \leq 0, \quad (18)$$

where $e_1 = \begin{bmatrix} 1\, 0 \end{bmatrix}^T$ and $e_2 = \begin{bmatrix} 0\, 1 \end{bmatrix}^T$. These are satisfied if the following inequalities hold [8]

$$\tau\beta_1 - 1 \geq 0,\ P - \tau e_1 e_1^T \geq 0,\ \begin{bmatrix} \beta_2^2 & e_2^T \\ e_2 & P \end{bmatrix} \geq 0, \quad (19)$$

where τ is a positive scalar to be determined.

In order to parameterize T_c^* only in terms of λ_3 and no longer as a function of the stability region size (λ_1, λ_2), it will be considered an ellipsoidal set $\mathcal{R}_0(\zeta)$ of admissible initial conditions defined as follows:

$$\mathcal{R}_0(\zeta) := \{\zeta \in \mathbb{R}^2 : \zeta' P_0 \zeta \leq 1\}, \quad (20)$$

such that $\mathcal{R}_0(\zeta) \subseteq \mathcal{R}(\zeta)$, where $P_0 > 0$ is a given matrix defining the size of $\mathcal{R}_0(\zeta)$. Thus, it is possible to impose T_c^* and deduce a lower bound on λ_3 following inequality:

$$\lambda_3 \geq \frac{2\lambda_{02}}{\sqrt{\lambda_{01}}T_c^*}, \quad (21)$$

where λ_{01} and λ_{02} are the smallest and largest eigenvalues of P_0.

Collorary 1. *Consider the system* (6) *with* (5)*, and the assumptions* **A1** *and* **A2***. Let β_1 and β_2 be given positive scalars defining $\tilde{\Xi}$. Let P_0 be a given symmetric positive definite matrix and T_c^* be a given positive scalar. Suppose there exist matrices $P = P^T > 0$ and L, and positive scalars τ and λ_3 satisfying* (13)*,* (19) *and* (21)*. Then, the origin of the error dynamics* (6) *with $K = P^{-1}L$ is regionally asymptotically* *stable. Moreover, for all* $\tilde{\xi}(0) \in \{\tilde{\xi} \in \mathbb{R}^2 : \zeta^T(\tilde{\xi})P_0\zeta(\tilde{\xi}) \leq 1\}$ *and* $t \geq 0$ *with* $P_0 - P > 0$*, the state trajectory satisfies* $\tilde{\xi}(t) \in \{\tilde{\xi} \in \mathbb{R}^2 : \zeta^T(\tilde{\xi})P\zeta(\tilde{\xi}) \leq 1\}$ *and converges to zero in a finite time* $T_c \leq T_c^*$.

A. Practical Stability

Assumption **A4** imposes that the perturbation $\delta_1(t)$ vanishes at the origin. When $\delta_1(t)$ is persistent, from the observability properties analysis, it is known that the error system will be only *practically* stable (i.e., $\|\tilde{\xi}(t)\|$ converges to a small neighborhood of the origin) [5]. In this case, the condition

$$\delta_1(t) = g_1(t)\phi_i(\tilde{\xi}_1), \quad |g_1(t)| \leq \alpha_1, \quad \tilde{\xi}_1 = \hat{\xi}_1 - \xi_1, \quad \forall\, \hat{\xi} \in \hat{\Xi}, \quad \xi \in \Xi \tag{22}$$

can be satisfied for values of $\tilde{\xi}_1$ outside a small neighborhood of zero. More precisely, suppose there exists a positive scalar w_1 such that (22) holds for all $\tilde{\xi}_1$ satisfying $|\tilde{\xi}_1| \geq w_1$.

Hence, to ensure the practical stability, the line segment $\tilde{\xi}_1 \in [-w_1, w_1]$ has to belong to the system stability region. Equivalently, in the ζ-space, the set

$$\mathcal{D}(\zeta) := \left\{\zeta \in \mathbb{R}^2 : \zeta^T e_1 e_1^T \zeta \leq w_1\right\}, \tag{23}$$

should belong to $\mathcal{R}(\zeta)$, which is a standard LMI problem (see e.g. [8]). Then, the error system origin is regionally practically stable (i.e., the invariance condition $\mathcal{D}(\zeta) \subset \mathcal{R}(\zeta)$ is satisfied) if the following holds:

$$1 - w_1 e_1^T P e_1 \geq 0. \tag{24}$$

From a practical point of vue, for the design of a CSTO, it can be firstly taken $\alpha_2 = 2d_2$ (where d_2 is such that $|\delta_2(t)| \leq d_2$). Furthermore, it is assumed that a bound α_1 on $|g_1(t)|$ is known *a priori* so that it can be considered $w_1 = (d_1/\alpha_1)^2$ in the LMI solution of Corollary 1 with (24).

B. Algorithm for STO gain computation

The design procedure resumes:

Algorithm 1 STO design algorithm

1: Identify 2^{nd} order system dynamics as (1).
2: Cast error dynamics following (6) and identify $b(t, y, u), \delta_1$ and δ_2.
3: Compute/provide an estimation of $d_2 \geq |\delta_2(t)|$ and deduce $\alpha_2 = 2d_2$.
4: Compute/provide an estimation of $\underline{b}, \bar{b}$.
5: Set a value for α_1.
6: Deduce $\mathcal{V}$, the set of all vertices of the meta set $[-\alpha_1, \alpha_1] \times [-\alpha_2, \alpha_2] \times [\underline{b}, \bar{b}]$
7: Compute/provide an estimation of $d_1 \geq |\delta_1(t)|$
8: Set P_0 to fix $\mathcal{R}_0(\zeta)$. ▷ start with $P_0 = \varpi\, I_{2\times 2}$.
9: Set T_c^* and deduce lower bound on λ_3 (21).
10: Following *Corollary* 1, solve LMIs (13), (19) and deduce $K = P^{-1}L$.
11: **if** LMIs Infeasible **then** either
12:     **goto** *8:* increase the value of ϖ .
13:     **goto** *9:* and increase T_c^*.

Ensure: Fulfillment of (24)

14: **if** Not **then**
15:     **goto** *5:* and increase α_1.

For this application, LMIs are solved with ***YALMIP*** as parser on ***Matlab*** with ***sdpt3*** as solver. A more detailed development of the STO design procedure can be found in [7].

III. Olive oil production by *Candida rugosa*

The proposed methodology is now applied to the design of STOs for kinetic rate estimation in a lipase production process from olive oil by *Candida rugosa*, which model is given by [9], [10]:

$$\begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\\ \dot{E}\\ \dot{X}\\ \dot{O}\\ \dot{P}\end{bmatrix} = \begin{bmatrix}-k_1 & 0 & 0\\ 1 & -k_3 & 0\\ k_2 & 0 & -k_6\\ 0 & 0 & k_7\\ 0 & 1 & 1\\ 0 & -k_4 & -k_8\\ 0 & k_5 & k_9\end{bmatrix}\begin{bmatrix}\varphi_1\\ \varphi_2\\ \varphi_3\end{bmatrix} - D\begin{bmatrix}S_1\\ S_2\\ S_3\\ E\\ X\\ O\\ P\end{bmatrix} + \begin{bmatrix}DS_{1in}\\ DS_{2in}\\ DS_{3in}\\ 0\\ 0\\ 0\\ 0\end{bmatrix} - \begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0\\ Q_{O_2}\\ Q_{CO_2}\end{bmatrix} \tag{25}$$

where S_1 is the primary substrate concentration (olive oil), S_2 and S_3 secondary substrates (respectively glycerol and fatty acids), E, X , O and P being respectively the enzyme, biomass, oxygen and CO_2 concentrations. In a compact matrix-vector notation, the model can be expressed as

$$\dot{\xi} = K\varphi(\xi) - D\xi + F - Q \tag{26}$$

with K the stoichiometric coefficients matrix, $\varphi(\xi) = \begin{bmatrix}\varphi_1 & \varphi_2 & \varphi_3\end{bmatrix}^T$ the vector of reaction rates, D the dilution rate, $\xi = \begin{bmatrix}S_1 & S_2 & S_3 & E & X & O & P\end{bmatrix}^T$ the state vector, $F = \begin{bmatrix}DS_{1in} & DS_{2in} & DS_{3in} & 0 & 0 & 0 & 0\end{bmatrix}^T$ the vector of feed rates and $Q = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & Q_{O_2} & Q_{CO_2}\end{bmatrix}^T$ the gaseous flow rate vector. The reaction rates are given by:

$$\varphi_1(\xi) = \varphi_1(S_1, E, X) = \mu_1^* \frac{S_1}{K_{m1} + S_1} \frac{E}{K_{m2} + E} X = \mu_1(\xi)X \tag{27}$$

$$\varphi_2(\xi) = \varphi_2(S_2, O, X) = \mu_2^* \frac{S_2}{K_{m3} + S_2} \frac{O}{K_{m4} + O} X = \mu_2(\xi)X \tag{28}$$

$$\begin{aligned}\varphi_3(\xi) &= \varphi_3(S_2, S_3, O, X) = \mu_3^* \frac{S_3}{(K_{m5} + S_3)(K_{m6} + S_2)} \frac{O^2}{K_{m7} + O^2} X\\ &= \mu_3(\xi)X\end{aligned} \tag{29}$$

The numerical parameter values are listed in Tables I-II.

TABLE I
KINETIC PARAMETERS VALUES

Parameter	Model	CSTOs	Unit
μ_1^*	0.0208		h^{-1}
μ_2^*	0.125		h^{-1}
μ_3^*	0.833		$g/(L\,h)$
K_{m1}	2		g/L
K_{m2},K_{m4},K_{m6}	0.2		g/L
K_{m3},K_{m5}	1		g/L
K_{m7}	2		g^2/L^2
$(k_La)_{O_2}$,$(k_La)_{CO_2}$	0.208		g/L
O_{sat}	0.5		h^{-1}
P_{sat}	15		h^{-1}
$\|\Psi_{S_2}\|$		$0.0002 - 0.002$	$g/(L\,h)$
$\|\Psi_{S_3}\|$		$0.005 - 0.06$	$g/(L\,h)$
$\|\Psi_X\|$		$0.001 - 0.025$	$g/(L\,h)$

IV. Software sensor design

A. Kinetic rate estimation - STO design

Assuming S_2, S_3 and X can be measured on-line, the following functions are introduced:

$$\Psi_{S_2} = \left[\mu_1 - k_3\mu_2\right] X \tag{30}$$
$$\Psi_{S_3} = \left[k_2\mu_1 - k_6\mu_3\right] X \tag{31}$$
$$\Psi_X = \left[\mu_2 + \mu_3\right] X \tag{32}$$

3 STOs will be designed to estimate those intermediate variables from which the specific reaction rates μ_1, μ_2, μ_3 could be inferred back-solving system (30)-(31)-(32) as depicted in (33). Considering the worst-case scenario where very little knowledge about the kinetic laws is available, the STOs are designed based on upper and lower bounds for the functions Ψ_i.

$$\left\{\begin{array}{lcl} \hat{\mu}_1 & = & \frac{k_3\hat{\Psi}_{S_3}+k_6\hat{\Psi}_{S_2}+k_3k_6\hat{\Psi}_X}{(X(k_6+k_2k_3))} \\ \hat{\mu}_2 & = & \frac{\hat{\Psi}_{S_3}-k_2\hat{\Psi}_{S_2}+k_6\hat{\Psi}_X}{(X(k_6+k_2k_3))} \\ \hat{\mu}_3 & = & \frac{-\hat{\Psi}_{S_3}+k_2\hat{\Psi}_{S_2}+k_2k_3\hat{\Psi}_X}{(X(k_6+k_2k_3))} \end{array}\right. \tag{33}$$

Following the first step of the algorithm given in subsection II-B, the following subsystems are defined:

$$\left\{\begin{array}{ll} \dot{S}_j & = \Psi_{S_j} + D(S_{jin} - S_j) \\ \dot{\Psi}_{S_j} & = \frac{\partial\Psi_{S_j}}{\partial t} = \left[\frac{\partial\Psi_{S_j}}{\partial X}\right].\left[\frac{\partial X}{\partial t}\right] \quad = \eta_{jx}X - \eta_{jd}D \end{array}\right. \tag{34}$$

$$\left\{\begin{array}{ll} \dot{X} & = \Psi_X - DX \\ \dot{\Psi}_X & = \frac{\partial\Psi_X}{\partial t} = \left[\frac{\partial\Psi_X}{\partial X}\right].\left[\frac{X}{\partial t}\right] \quad = \eta_x X - \eta_d D \end{array}\right. \tag{35}$$

with associated STOs (3) :

$$\left\{\begin{array}{ll} \dot{\hat{S}}_j & = -k_{1_{S_j}}\phi_1(\tilde{S}_j) + \hat{\Psi}_{S_j} + D(S_{jin} - \hat{S}_j) \\ \dot{\hat{\Psi}}_{S_j} & = -k_{2_{S_j}}\phi_2(\tilde{S}_j) + \hat{\eta}_{jx}\hat{X} - \hat{\eta}_{jd}D \end{array}\right. \tag{36}$$

$$\left\{\begin{array}{ll} \dot{\hat{X}} & = -k_{1_X}\phi_1(\tilde{X}) + \hat{\Psi}_X - D\hat{X} \\ \dot{\hat{\Psi}}_X & = -k_{2_X}\phi_2(\tilde{X}) + \hat{\eta}_x\hat{X} - \hat{\eta}_d D \end{array}\right. \tag{37}$$

where the $\eta-$functions are described in Table III and $j \in \{2,3\}$. The estimation error dynamics are now developed so as to identify the functions $b(t,y,u), \delta_1$ and δ_2. To this end, $\tilde{x} = \hat{x} - x_n$, with $x = x_n \pm x_\nu$ where x is the measured value of a given state, x_ν is the noise contribution and x_n the nominal(true) value. In the ideal case $x_\nu = 0$. Thus,

$$\begin{array}{l} \dot{\tilde{S}}_j = -k_{1_{S_j}}\phi_1(\tilde{S}_j) + \hat{\Psi}_{S_j} + D(S_{jin} - \hat{S}_j) - \Psi_{S_j} - D(S_{jin} - S_j) \\ \rightarrow \dot{\tilde{S}}_j = -k_{1_{S_j}}\phi_1(\tilde{S}_j) + b_S\tilde{\Psi}_{S_j} + \delta_{1_{S_j}} \end{array} \tag{38}$$

with $b_S = 1$, $\delta_{1_{S_j}} = -D(\tilde{S}_j \pm S_{j\nu})$ from which we deduce: .

$$d_{1_{S_j}} = sup|D(\Delta S_j + |S_{j\nu}|)| \geq |\delta_{1_{S_j}}|. \tag{39}$$

Similarly,

$$\begin{array}{l} \dot{\tilde{X}} = -k_{1_X}\phi_1(\tilde{X}) + \hat{\Psi}_X - D\hat{X} - \Psi_X + DX \\ \rightarrow \dot{\tilde{X}} = -k_{1_X}\phi_1(\tilde{X}) + b_X\tilde{\Psi}_X + \delta_{1_X} \end{array}$$

with $b_X = 1$, $\delta_{1_X} = -D(\tilde{X} \pm X_\nu)$ from which we deduce:

$$d_{1_X} = sup|D(\Delta X + |X_\nu|)| \geq |\delta_{1_X}|. \tag{40}$$

Finally

$$\begin{array}{l} \delta_2 = \hat{\eta}_x\hat{X} - \hat{\eta}_D D - \eta_x X + \eta_D D \\ \rightarrow \delta_2 = (\tilde{\eta}_x - \eta_x)(\tilde{X} + X_n) - \eta_x(X_n \pm X_\nu) - \tilde{\eta}_d D \end{array}$$

From which we deduce:

$$d_2 = sup|(\Delta\eta_x + |\eta_x|)(\Delta X + X_n) + |\eta_x|(X_n + |X_\nu|) + \Delta\eta_d D| \geq \delta_2. \tag{41}$$

Operator Δ is defined as:

$$\Delta. = \frac{|\sup\{.\} - \min\{.\}|}{2} \geq |\tilde{.}| \tag{42}$$

e.g: $\Delta\eta_x = \frac{|\sup\{\eta_x\} - \min\{\eta_x\}|}{2} \geq |\tilde{\eta}_x|$

B. State estimation - Asymptotic observer design

In [11], a state partition and a linear change of coordinates allow the unmeasured states dynamics to be expressed as functions of yield coefficients and measured states. The corresponding asymptotic observer uses S_2, S_3 and X measurements to reconstruct the unmeasured states S_1, E, O, P which estimates are then used by a high gain observer for kinetic rate estimation. In the present study, no state transformation is required as an asymptotic observer can be obtained straightforwardly, based on the STO rate estimates:

$$\begin{bmatrix} \dot{\hat{S}}_1 \\ \dot{\hat{E}} \\ \dot{\hat{O}} \\ \dot{\hat{P}} \end{bmatrix} = \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & 0 & k_7 \\ 0 & -k_4 & -k_8 \\ 0 & k_5 & k_9 \end{bmatrix} \begin{bmatrix} \hat{\varphi}_1 \\ \hat{\varphi}_2 \\ \hat{\varphi}_3 \end{bmatrix} - D \begin{bmatrix} \hat{S}_1 \\ \hat{E} \\ \hat{O} \\ \hat{P} \end{bmatrix} + \begin{bmatrix} DS_{1in} \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \hat{Q}_{O_2} \\ \hat{Q}_{CO_2} \end{bmatrix} \tag{43}$$

TABLE II
Yield coefficients values

Yield Coefficients	Value
k_1	3
k_2	0.3
k_3	4.54
k_4	1.33
k_5	0.34
k_6	0.5
k_7	0.19
k_8	0.72
k_9	1.24

TABLE III
$\eta-$function description

Subsystem	η_{iX}	η_{id}
S_2/Ψ_{S_2}	$\frac{\Psi_{S_2}\Psi_X}{X^2}$	Ψ_{S_2}
S_3/Ψ_{S_3}	$\frac{\Psi_{S_3}\Psi_X}{X^2}$	Ψ_{S_3}
X/Ψ_X	$\frac{\Psi_X^2}{X^2}$	Ψ_X

TABLE IV
$d-$FUNCTIONS VALUES

Subsystem	d_1	d_2	α_1
S_2/Ψ_{S_2}	0.0743	0.0890	1
S_3/Ψ_{S_3}	0.0605	0.0042	0.1
X/Ψ_X	0.0303	0.0245	1

Proof. The estimation error $\tilde{\xi} = \hat{\xi} - \xi$ dynamics follow:

$$\begin{aligned} \dot{\tilde{\xi}} &= \dot{\hat{\xi}} - \dot{\xi} \\ &= K\left(\hat{\varphi}(\hat{\xi}) - \varphi(\xi)\right) - D\left(\hat{\xi} - \xi\right) - \left(\hat{Q} - Q\right) \\ &= K\tilde{\varphi} - D\tilde{\xi} - \tilde{Q} \end{aligned} \quad (44)$$

where $\tilde{\varphi} = \hat{\varphi}(\hat{\xi}) - \varphi(\xi)$ and $\tilde{Q} = \hat{Q} - Q$. Assuming STO convergence in finite time, $\hat{\varphi}(\hat{\xi}) \to \varphi(\xi)$ such that $\tilde{\varphi} \to 0$ and (44) becomes:

$$\dot{\tilde{\xi}} = -\lambda_i \tilde{\xi} \qquad \forall i \in \{1 \dots 4\} \quad (45)$$

where $\lambda = \begin{bmatrix} D & D & (D + (k_L a)_{O_2}) & (D + (k_L a)_{CO_2}) \end{bmatrix}^T$. □

In the ideal case (without noise), there is an exponential decay of the estimation error, with a faster convergence for O and P.

C. Simulation Results

Simulations are performed using parameters values from [9], [10] and assuming intervals for Ψ_i as depicted in Tables I & II while Fig. 1 shows the Dilution rate pattern [11] . For this simulation we assume no noise on measurements $\to$ $X_\nu = S_\nu = 0$, [7] illustrates noisy scenarios. Initial conditions are respectively $x_0 = [2\ 3\ 4\ 0.01\ 0.1\ 0\ 0]$ and $\hat{x}_0 = [1.8\ 3\ 4\ 0.02\ 0.1\ 0.3\ 1]$

The η and Ψ-functions evolutions with respect to biomass concentration are plotted in Fig 2 from which the $\delta_{1i, i\in\{S_2,S_3,X\}}$ and $\delta_{2i, i\in\{S_2,S_3,X\}}$ values are deduced following (39) - (40)- (41) and presented in Table IV. Finally, STO gains are obtained following *Algorithm* 1 with $T_c^* = 10$ hours : $K_{S_2} = \begin{bmatrix} 3.1431 & 1.3798 \end{bmatrix}^T$, $K_{S_3} = \begin{bmatrix} 5.4720 & 0.4233 \end{bmatrix}^T$ and $K_X = \begin{bmatrix} 2.3891 & 0.7402 \end{bmatrix}^T$.

Fig. 4 and Fig 5 illustrate the good performances of STOs as a fast convergence of the $\hat{\Psi}_i$ and thus of $\hat{\mu}_i$ to the real kinetics values is observed. This estimation outperforms the one presented in [11] where a bias appeared in the kinetic estimations with the same operating conditions (no noise and same dilution rate pattern). Finally, Fig 6 shows the asymptotic observer estimation of the unmeasured states (with a faster convergence for O and P).

V. Conclusion

In this study, a systematic design procedure for super-twisting observers (STOs) is applied to monitoring a lipase production process from olive oil by *Candida rugosa*. The kinetic laws can be estimated using 3 STOs, and the results are

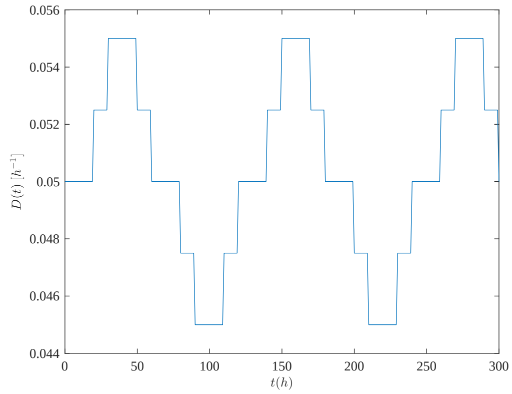

Fig. 1. Time evolution of dilution rate.

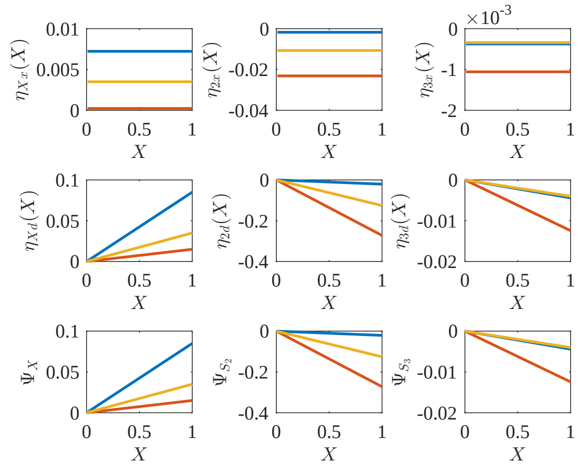

Fig. 2. Evolution of η and Ψ-function with respect to the biomass concentration. Max in 'blue', min in 'red', Δ in 'orange'

fused in an asymptotic observer reconstructing the unmeasured component concentrations. The main advantage of STOs is their robustness to unmodelled dynamics and their (usually fast) convergence in finite time. The existence of a systematic computational procedure for the observer gains will hopefully ease their application.

Acknowledgment

The authors are grateful to FNRS for supporting a research stay of the second author at UMONS, and gratefully acknowledge the support of FNRS and CONACYT (project 245954) in the framework of a joint research program between UMONS and UNAM.

References

[1] P. Bogaerts and A. Vande Wouwer, "Software sensors for bioprocesses," *ISA transactions*, vol. 42, no. 4, pp. 547–558, 2003.

[2] G. Goffaux and A. Vande Wouwer, "Bioprocess state estimation: some classical and less classical approaches," in *Control and Observer Design for Nonlinear Finite and Infinite Dimensional Systems*. Springer, 2005, pp. 111–128.

[3] S. Nuñez, H. De Battista, F. Garelli, A. Vignoni, and J. Picó, "Second-order sliding mode observer for multiple kinetic rates estimation in bioprocesses," *Control Eng Pract*, vol. 21, pp. 1259–1265, 2013.

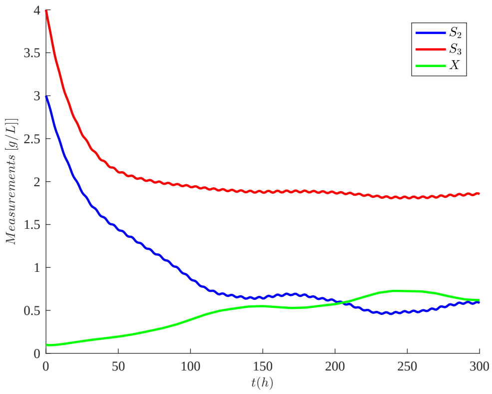

Fig. 3. Time evolution of measured states S_2, S_3, X

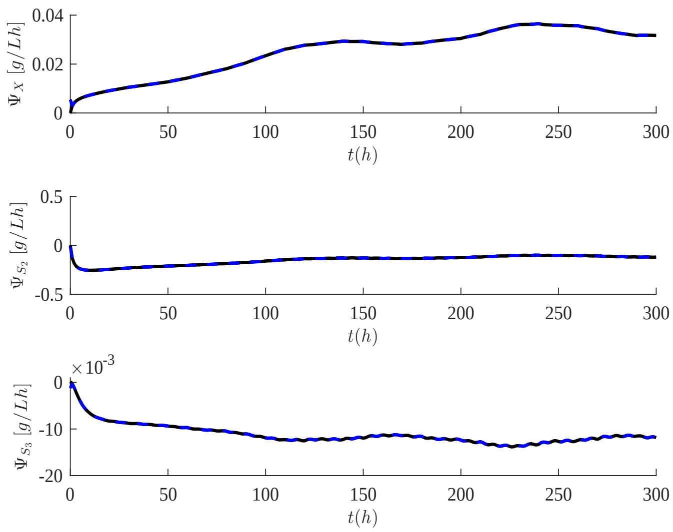

Fig. 4. Time evolution of Ψ_{S_2}, Ψ_{S_3}, Ψ_X. Model in 'black', STO estimation in 'blue'

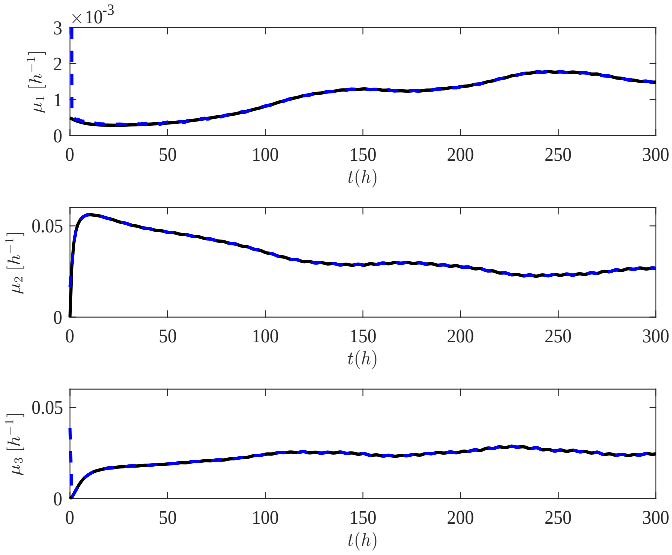

Fig. 5. Time evolution of μ_1, μ_2, μ_3. Model in 'black', STO estimation in 'blue'

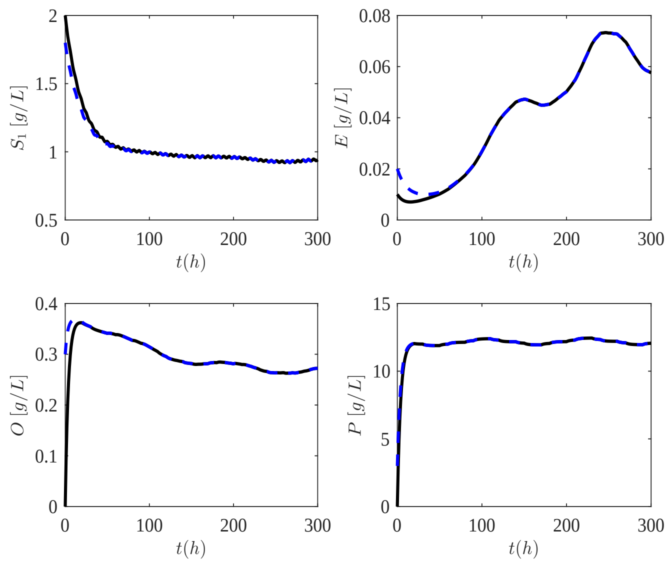

Fig. 6. Evolution of states estimates provided by the Asymptotic Observer. Model in 'black' , estimation in 'blue'

[4] J. A. Moreno and I. Mendoza, "Application of super-twisting-like observers for bioprocesses," in *Variable Structure Systems (VSS), 2014 13th International Workshop on*. IEEE, 2014, pp. 1–6.

[5] A. Vargas, J. A. Moreno, and A. Vande Wouwer, "A weighted variable gain super-twisting observer for the estimation of kinetic rates in biological systems," *J Process Contr*, vol. 24, pp. 957 – 965, 2014.

[6] A. Bacciotti and L. Rosier, *Liapunov Functions and Stability in Control Theory*. Berlin Heidelberg: Springer-Verlag, 2001.

[7] D. Coutinho, A. Vargas, C. Feudjio, M. Benavides, and A. Vande Wouwer, "A robust approach to the design of super-twisting observers – application to monitoring microalgae cultures in photo-bioreactors," *Computers and Chemical Engineering*, vol. submitted, 2018.

[8] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia – PA: Society for Industrial and Applied Mathematics (SIAM), 1994.

[9] L. Chen and G. Bastin, "Structural identifiability of the yield coefficients in bioprocess models when the reaction rates are unknown," *Mathematical Biosciences*, vol. 132, no. 1, pp. 35–67, 1996.

[10] O. Bernard and G. Bastin, "Identification of reaction networks for bioprocesses: determination of a partially unknown pseudo-stoichiometric matrix," *Bioprocess and Biosystems Engineering*, vol. 27, no. 5, pp. 293–301, 2005.

[11] D. Selişteanu, S. Tebbani, M. Roman, E. Petre, and V. Georgeanu, "Microbial production of enzymes: Nonlinear state and kinetic reaction rates estimation," *Biochemical engineering journal*, vol. 91, pp. 23–36, 2014.