Exceptional Points of Degeneracy in Coupled Chirowaveguides

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Abstract – **Exceptional points of degeneracy are critical junctures wherein eigenvalues and eigenvectors coalesce, resulting in unique dispersion features. In coupled waveguides, they occur via co-directional or contra-directional coupling. The former requires gain-loss modulation, akin to $\mathcal{PT}$-symmetric gratings, whilst the latter relies on negative phase velocity. We illustrate forks in modal dispersion arising due to negative refraction induced by giant chirality. Such an approach circumvents the manufacturing challenges of balancing photon creation and absorption, while not requiring simultaneous negativity of permittivity and permeability. Meta-media with giant and controllable chirality offer exciting opportunities for light manipulation.**

I. Introduction

Exceptional points of degeneracy (EPsD) are critical junctures in a system's parameter space, wherein eigenvalues and eigenvectors coincide [1]. These points hold significance in non-Hermitian systems adhering to $\mathcal{PT}$-symmetry, whereby real spectra can emerge. In waveguiding systems, EPsD result in the convergence of multiple eigenmodes into a single mode, leading to remarkable dispersion relations nearby the merging point(s).

In a photonic configuration comprising two coupled waveguides, potential methods for achieving EPsD involve co-directional and contra-directional coupling. The first method relies on non-Hermitian Hamiltonians, particularly to those conforming to $\mathcal{PT}$-symmetry, with conventional coupling between waveguides exhibiting balanced gain and loss. By contrast, as elaborated in [1], the second mechanism does not require the presence of gain or dissipation modulation, but rather a coupling of a standard waveguide with one exhibiting negative phase velocity.

Implementing $\mathcal{PT}$-gratings poses challenges due to intricate photon creation and absorption requirements. Recent research explores alternatives, like coupling a conventional dielectric waveguide with one possessing a negative refractive index [2]. Here, we adopt Pendry's chiral route to negative refraction [3], offering an alternative method for achieving EPsD without relying on $\mathcal{PT}$-symmetry and introducing an additional degree of freedom for manipulating light. Meta-media advancements have enabled significant progress in accessing giant chirality values across various spectral realms [4], showcasing techniques for chirality control via, e.g., a pneumatic force.

II. Lorentz Reciprocity and Coupled-Wave Theory Description

The constitutive relations of reciprocal bi-isotropic media, as per Tellegen's formalism, read: $\mathbf{d} = \epsilon\mathbf{E} + i\kappa\mathbf{h}$ and $\mathbf{b} = -i\kappa\mathbf{E} + \mu\mathbf{h}$, with all parameters being relative. When supplementing Maxwell's curl relations with these connections for a monochromatic excitation, the supported eigenstates are two orthogonal circular polarizations, with corresponding wavenumbers $k_{\pm} = k_0\,(n \pm \kappa)$, $n = (\epsilon\mu)^{1/2}$. For extreme chirality, i.e., $|\kappa| > n$, the direction of phase propagation and handedness for one circular state changes, while the Poynting vector does not.

When lightwaves are confined in a dielectric chiral slab, surrounded, say, by vacuum, the aforementioned dispersion relation of unbounded media does not hold. Nevertheless, upon inspection of the dispersion relation of a chiral waveguide, as seen in Eq. (16) of [5], it is straightforward to show that when $|\kappa| = n$, one effective refractive index becomes zero. Numerical calculations confirm that such a dispersion curve changes sign at *all* zeros.

While weak chirality allows for approximating the hybrid modes without longitudinal field components, giant chirality disrupts such a transverse character. If an achiral dielectric waveguide is an unperturbed geometry and a bi-isotropic slab with giant chirality is the perturbed one, Lorentz reciprocity for the m, n-modes implies

$$\mathrm{sign}\,(n)\,\delta_{mn}\sum_{m}\left[\frac{\partial a_m}{\partial z}+i\delta k_{mn}a_m\right]\exp\,(i\delta k_{mn}z)\;=\sum_{m}a_m\exp\,(i\delta k_{mn}z)I_{mn}\,; \tag{1}$$

δ_{mn} is the Kronecker delta, and δk_{mn} is a detuning parameter; I_{mn} expresses modes-overlapping integrals [6].

Considering two modes, we may recast Eq. (1) as

$$\frac{\mathrm{d}a_1}{\mathrm{d}z}=C_{21}\exp\,(i\delta k_{21}z)a_2\;\;\text{and}\;\;\frac{\mathrm{d}a_2}{\mathrm{d}z}=C_{12}\exp\,(-i\delta k_{21}z)a_1\,. \tag{2}$$

Leveraging the symmetries of I_{mn}, one can show that $C_{nm}=-C^*_{mn}$ for co-directional coupling and $C_{nm}=C^*_{mn}$ for contra-directional coupling. Hence, the condition for the occurrence of EPsD is $\delta k_{21}=\pm 2\left(C_{12}C_{21}\right)^{1/2}$. In a system devoid of gain/loss, δk_{21} is purely real; therefore $C_{12}C_{21}\in\mathbb{R}^+$. Such a requirement can only be met in instances of contra-directional coupling, thereby corroborating the conclusions of [1].

III. $\mathcal{PT}$-Symmetric-Like Behaviour Without Gain/Loss Modulation

We numerically simulate a single chiral slab surrounded by vacuum, as depicted in Fig. 1(a). Most modes of the waveguide with giant chirality exhibit backward propagation, characterized by a negative slope of $\omega(k)$ and a negative z-component of the Poynting vector (n.b., causality is, of course, respected upon considering appropriate excitation sources). Thus, a forward-propagating mode will be coupled with one of the backward modes to generate $\mathcal{PT}$-like bifurcation points. Upon inspection of Figs. 1(b) and (c), the signature "forks" of $\mathcal{PT}$-symmetric gratings arise, with an EPD occurring at $\lambda\approx 0.9$ µm. There, two eigenmodes converge, as illustrated in Fig. 1(d), wherein a forward-propagating mode (blue) and a backward-propagating mode (green) merge at the EPD. Such behavior is attained in a single *homogeneous* chirowaveguide when the chirality exceeds the refractive index.

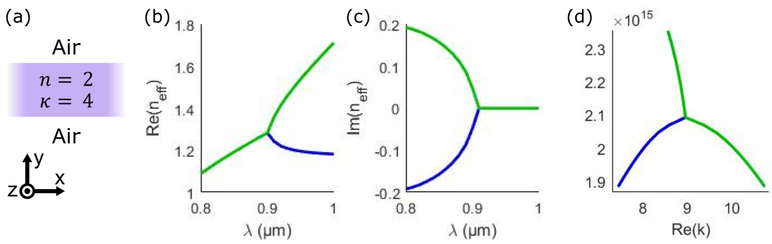

Fig. 1: (a) Schematic of a 150 nm-thick chirowaveguide, with $n=2$ and $\kappa=2n$, embedded in vacuum. (b) Real, (c) imaginary parts of the refractive indices of two interfering modes, and (d) dispersion $\omega(k)$ of these modes.

IV. Avoided Crossings in Coupled Achiral/Chiral Dielectric Slabs

Although Eq. (1) pertains to a single chirowaveguide, it can be extended to describe coupled waveguides. Consider an achiral waveguide brought into proximity, but not directly adjacent to, a chiral waveguide. The achiral waveguide sustains a forward-propagating eigenmode, strongly coupled to a backward-propagating mode within the chirowaveguide with negative phase velocity. Figs. 2(b) and (c) display the real and imaginary components of the effective indices, respectively. The insets highlight that the blue and red modes predominantly localize within the achiral waveguide. The interaction between the backward-propagating chiral mode (green) and the forward-propagating achiral modes (blue and red) generates a broken $\mathcal{PT}$-like zone around each chiral-achiral crossing.

The width and magnitude of the $\mathcal{PT}$-broken zone increase as the waveguide separation decreases, resulting in enhanced coupling. When the gap is 100 nm, the two zones merge into a single $\mathcal{PT}$-broken zone. The forward-propagating mode (blue in Figs. 2(d) and (e)) gradually evolves from lower-order achiral-dominated mode to higher-order achiral-dominated mode by hybridizing with the $\mathcal{PT}$-broken modes during the phase transition (cf. the insets in Fig. 2(d)). The $\omega(k)$ dispersion curves reveal an interesting trend: in Fig. 3(a), modes of the single achiral waveguide are forward-propagating, while the chiral waveguide presents a backward-propagating mode.

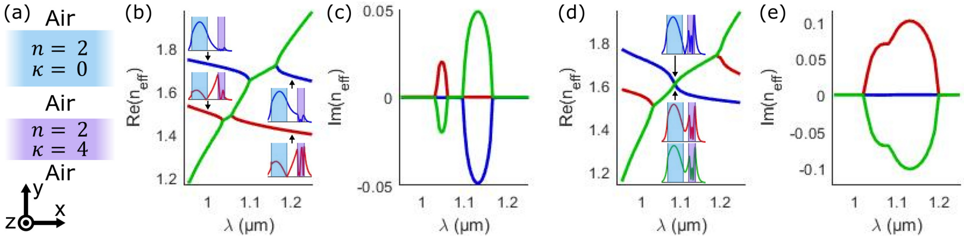

Fig. 2: (a) A 300 nm-thick dielectric slab coupled to a 130 nm-thick chirowaveguide, separated by 200 nm. (b) Real and (c) imaginary parts of the refractive indices of three interacting modes. The insets in (b) show E_x profiles of the blue and red modes at $\lambda = 1$ μm (left) and 1.2 μm (right). (d) Real and (e) imaginary parts of the refractive indices of the same modes for a 100 nm gap. The corresponding insets refer to $\lambda = 1.084$ μm.

However, the forward or backward natures of the coupled modes are more intertwined. For low coupling (200 nm gap, Fig. 3(b)), forward propagation dominates in the $\mathcal{PT}$-broken zones, while the modes maintain their isolated propagation direction outside these zones. In the case of strong coupling (100 nm gap, Fig. 3(c)), the $\mathcal{PT}$-broken modes alternate between forward propagation away from the achiral-dominated (blue) mode crossing and backward propagation close to this crossing, while also maintaining their isolated propagation direction asymptotically.

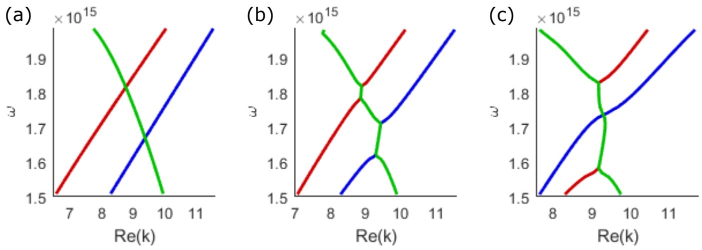

Fig. 3: (a) $\omega(k)$ curves of two modes from a 300 nm-thick achiral waveguide (blue and red) and a mode from a 130 nm-thick chiral waveguide (green). (b,c) Interacting modes with separations of (b) 200 nm and (c) 100 nm.

V. Conclusion

We achieve EPsD through negative refraction from giant chirality, bypassing challenges of $\mathcal{PT}$-symmetric gratings, without necessitating simultaneous negativity of permittivity and permeability. Meta-media advancements enabling giant and controllable chirality open exciting avenues for light manipulation and dispersion engineering.

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