Active Learning of Mealy Machines with Timers

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In short: finite Mealy machines augmented with **clocks** that can be reset or used in guards along transitions and states.

BUT timed Mealy machines are hard to construct and understand.

We focus on systems that can be represented with **timers**: **Mealy machines with timers**.

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▶ Timed Mealy machines are more expressive;	▶ Mealy machines with timers are more restrictive;
▶ Well-known model;	▶ We previously studied some properties of Mealy machines with timers;[1]
▶ Learning timed Mealy machines is challenging.	▶ This work: learning algorithm.

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A **Mealy machine with timers** (MMT) is a tuple $\mathcal{M} = (X, I, O, Q, q_0, \delta)$ where

- X is the set of **timers**;
- I is the set of **inputs**; the set of all **actions** is:

$$I \cup \{to[x] \mid x \in X\};$$

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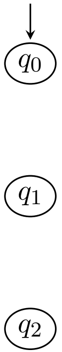

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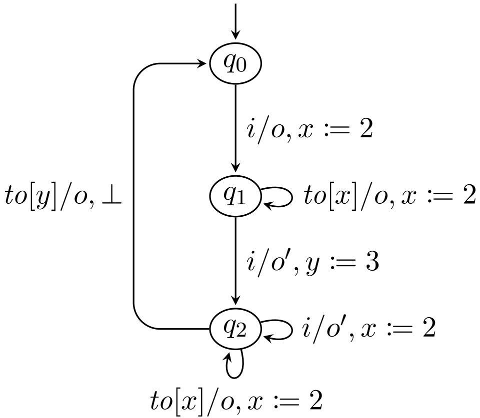
- δ is the transition function.



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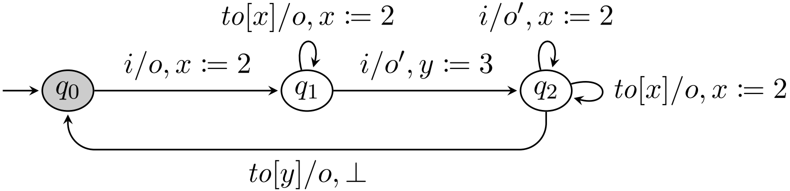
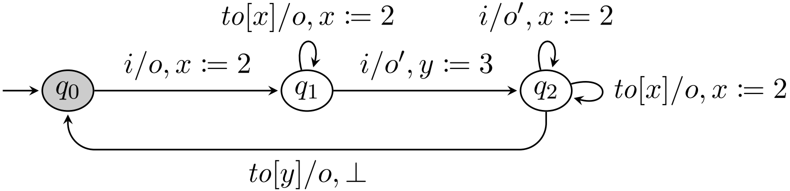
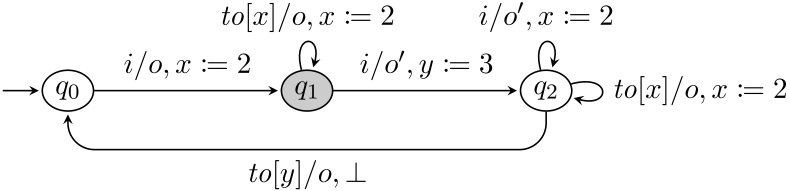


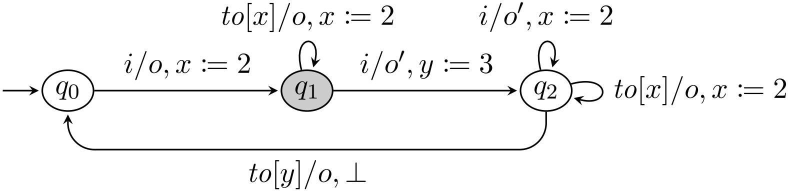
$(q_0, \emptyset)$



$$(q_0, \emptyset) \xrightarrow{1} (q_0, \emptyset)$$



$$(q_0, \emptyset) \xrightarrow{1} (q_0, \emptyset) \xrightarrow{i/o} (q_1, x = 2)$$



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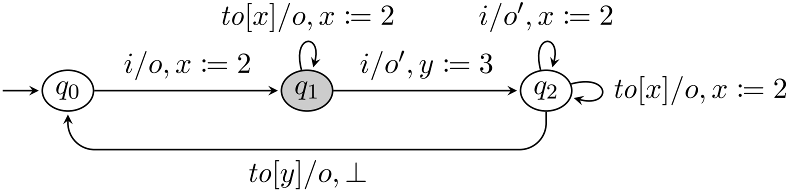

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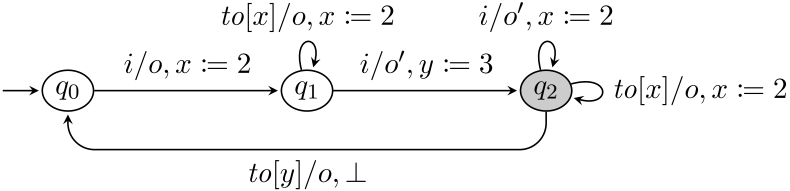

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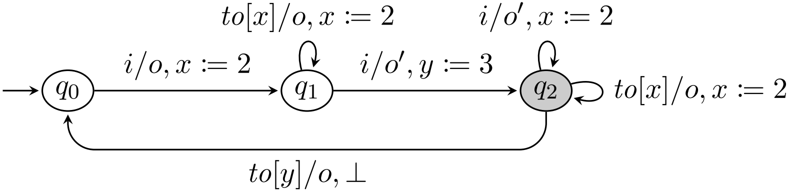

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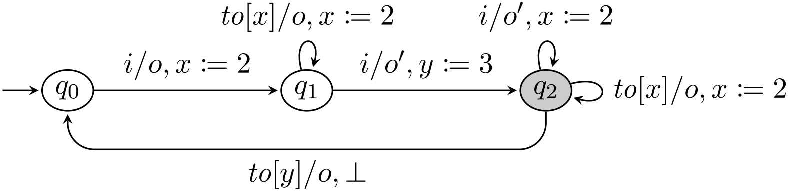

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$$\xrightarrow{i/o'} (q_2, x = 2, y = 1) \xrightarrow{0.5} (q_2, x = 1.5, y = 0.5).$$

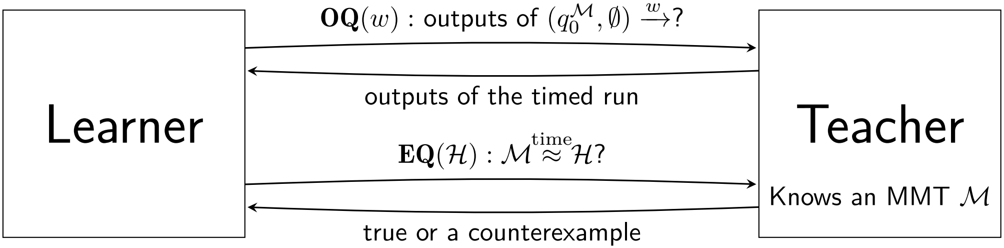

Figure 2: Adaptation of Angluin's framework[2] to MMTs.

[2] Angluin, "Learning Regular Sets from Queries and Counterexamples", 1987; Shahbaz and Groz, "Inferring mealy machines", 2009.

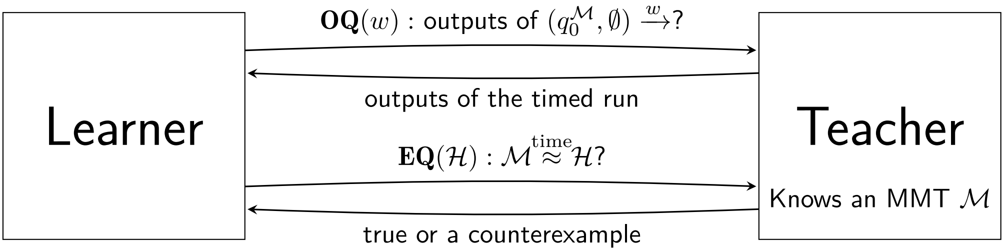

Figure 2: Adaptation of Angluin's framework[2] to MMTs.

Both queries are in the **timed** world... Cumbersome to use!

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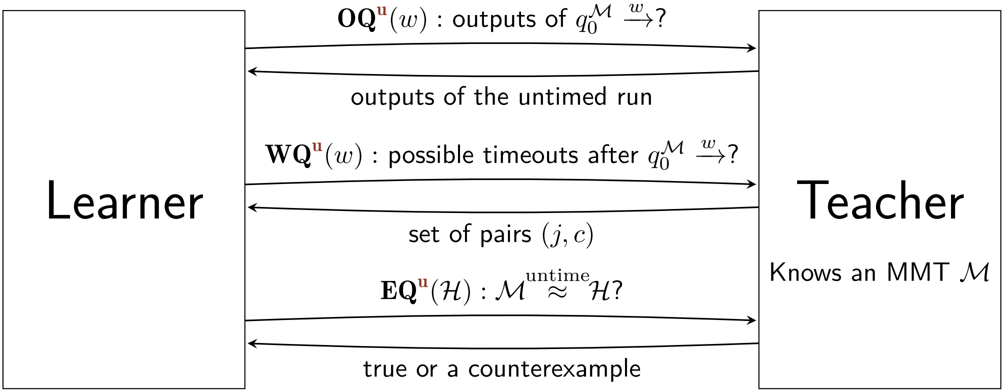

Figure 3: Untimed adaptation of Angluin's framework to MMTs.

We stay in the **untimed** world!

Proposition 1. *The untimed queries can be implemented via a polynomial number of timed queries.*

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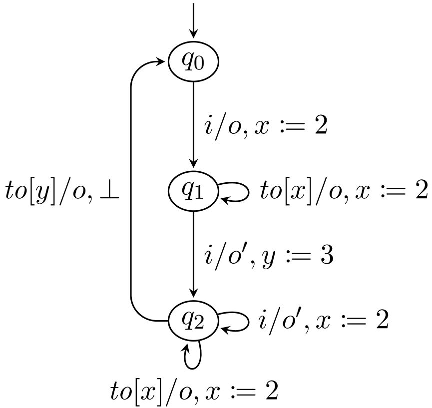
Proposition 2. *It is possible to construct an MMT in which the second condition is satisfied.*


q_0
$i/o, x := 2$
q_1
$to[x]/o, x := 2$
$to[y]/o, \bot$
$i/o', y := 3$
q_2
$i/o', x := 2$
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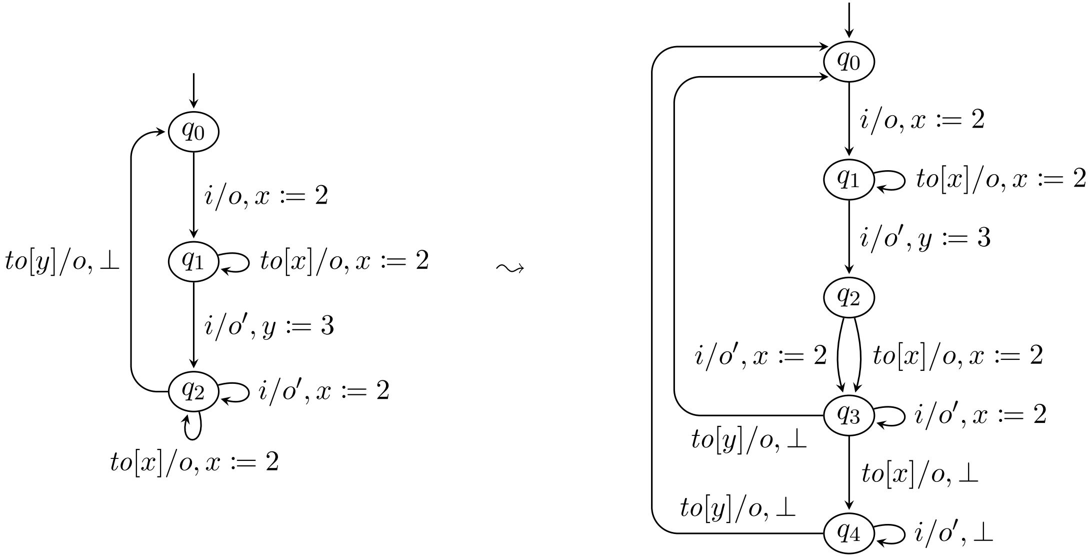
q_0
$i/o, x := 2$
q_1
$to[x]/o, x := 2$
$i/o', y := 3$
q_2
$i/o', x := 2$
$to[x]/o, x := 2$
$to[y]/o, \bot$
$\rightsquigarrow$
q_0
$i/o, x := 2$
q_1
$to[x]/o, x := 2$
$i/o', y := 3$
q_2
$i/o', x := 2$
$to[x]/o, x := 2$
q_3
$i/o', x := 2$
$to[y]/o, \bot$
$to[x]/o, \bot$
$to[y]/o, \bot$
q_4
$i/o', \bot$

We adapt $L^{\#}$ (active learning algorithm for Mealy machines[3]) to MMTs: $L^{\#}_{\text{MMT}}$.

[3] Vaandrager et al., "A New Approach for Active Automata Learning Based on Apartness", 2022.

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Theorem 3. *Let $\mathcal{M}$ be a "good" MMT and ℓ be the length of the longest counterexample returned by the teacher. Then,*

- *the $L^{\#}_{\text{MMT}}$ algorithm eventually terminates and returns an MMT $\mathcal{N}$ such that $\mathcal{M} \overset{\text{time}}{\approx} \mathcal{N}$ and whose size is **polynomial** in $|Q^{\mathcal{M}}|$ and **factorial** in $|X^{\mathcal{M}}|$, and*
- *in time and number of untimed queries **polynomial** in $|Q^{\mathcal{M}}|$, $|I|$, and ℓ, and **factorial** in $|X^{\mathcal{M}}|$.*

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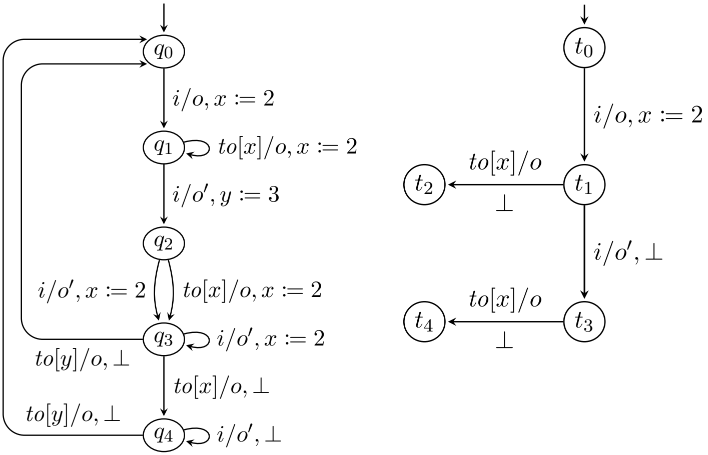
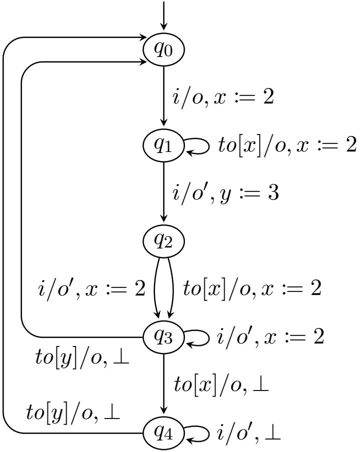
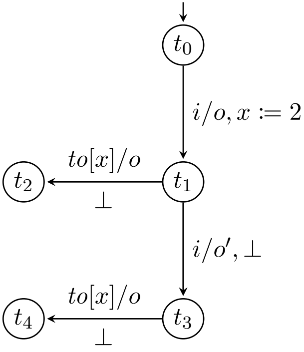


We want to add $i \cdot i \cdot i$ and the potential timeouts.





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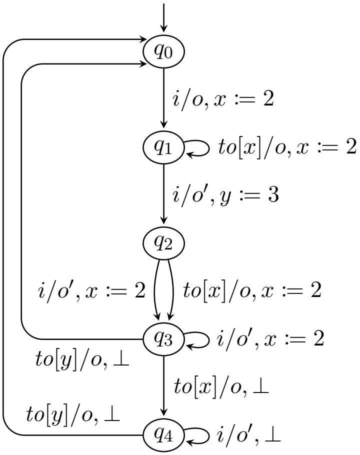
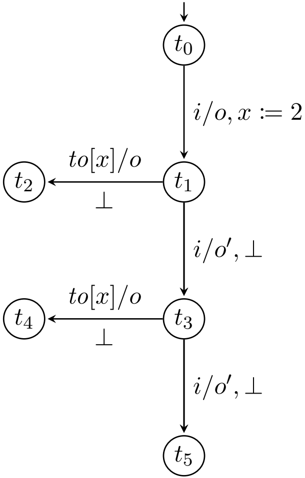
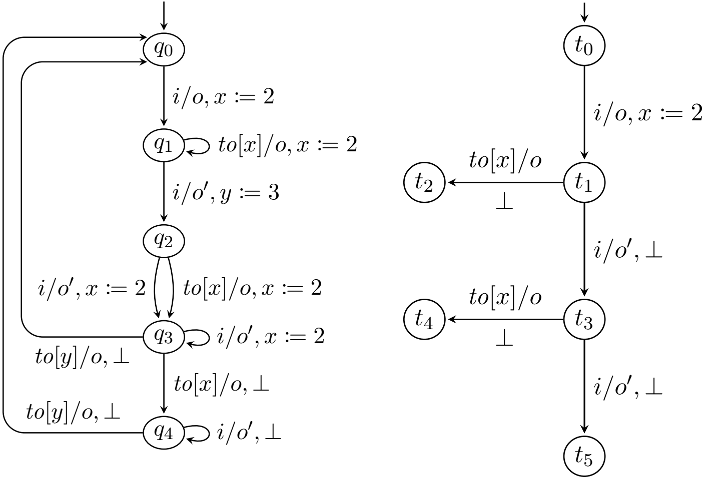
- $\mathbf{OQ^u}(i \cdot i \cdot i) \rightsquigarrow o \cdot o' \cdot o'$.





We want to add $i \cdot i \cdot i$ and the potential timeouts.

- $\mathbf{OQ^u}(i{\cdot}i{\cdot}i) \rightsquigarrow o{\cdot}o'{\cdot}o'$.
- So, $t_3 \xrightarrow[\bot]{i/o'} t_5$.



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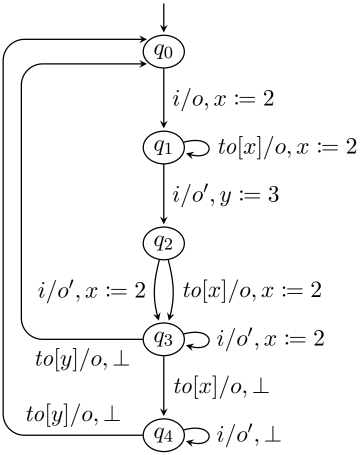
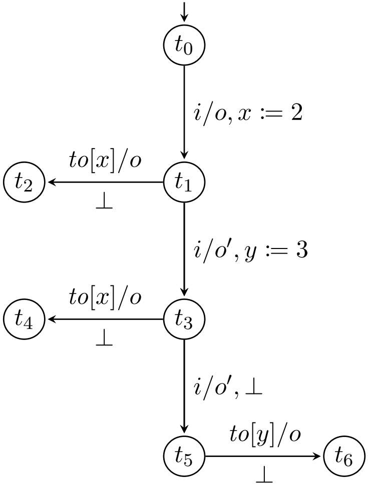
- $\mathbf{WQ^u}(i \cdot i \cdot i) \rightsquigarrow \{(2, 3), (3, 2)\}$.





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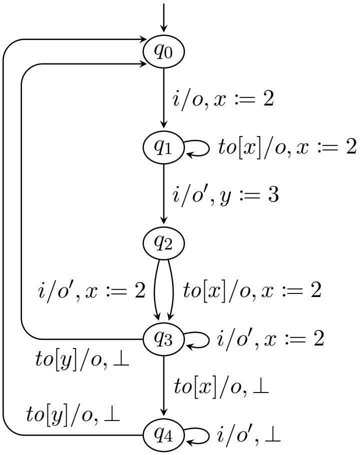
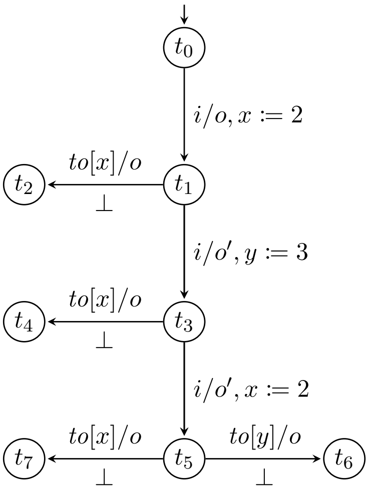
- $\mathbf{OQ}^{\mathbf{u}}(i \cdot i \cdot i) \rightsquigarrow o \cdot o' \cdot o'$.
- So, $t_3 \xrightarrow[\bot]{i/o'} t_5$.
- $\mathbf{WQ}^{\mathbf{u}}(i \cdot i \cdot i)$ $\rightsquigarrow \{(2,3),(3,2)\}$.
- So, $t_1 \xrightarrow{i} t_3$ starts a timer at constant 3.





We want to add $i \cdot i \cdot i$ and the potential timeouts.

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- So, $t_3 \xrightarrow[\bot]{i/o'} t_5$.
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- So, $t_1 \xrightarrow{i} t_3$ starts a timer at constant 3.
- And $t_3 \xrightarrow{i} t_5$ starts a timer at constant 2.

We implemented $L^{\#}_{\mathsf{MMT}}$ in Rust[4] and ran some experiments.

Model	$\|Q\|$	$\|I\|$	$\|X\|$	$\|\mathbf{WQ^u}\|$	$\|\mathbf{OQ^u}\|$	$\|\mathbf{EQ^u}\|$	Time[msecs]
AKM	4	5	1	22	35	2	684
CAS	8	4	1	60	89	3	1344
Light	4	2	1	10	13	2	302
PC	8	9	1	75	183	4	2696
TCP	11	8	1	123	366	8	3182
Train	6	3	1	32	28	3	1559
Running example	3	1	2	11	5	2	1039
FDDI 1-station	9	2	2	32	20	1	1105
Oven	12	5	1	907	317	3	9452
WSN	9	4	1	175	108	4	3291

[4] `https://gitlab.science.ru.nl/bharat/mmt_lsharp`.

Still work to be done:

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- Simplify the learning algorithm as much as possible.

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Thank you!

For all details, see

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Part I – Appendix

Appendix

References I

- Angluin, Dana. "Learning Regular Sets from Queries and Counterexamples". In: *Inf. Comput.* 75.2 (1987), pp. 87–106. DOI: 10.1016/0890-5401(87)90052-6.
- Bruyère, Véronique, Bharat Garhewal, et al. "Active Learning of Mealy Machines with Timers". In: *CoRR* abs/2403.02019 (2024). DOI: 10.48550/arXiv.2403.02019. arXiv: 2403.02019.
- Bruyère, Véronique, Guillermo A. Pérez, et al. "Automata with Timers". In: *Formal Modeling and Analysis of Timed Systems - 21st International Conference, FORMATS 2023, Antwerp, Belgium, September 19-21, 2023, Proceedings*. Ed. by Laure Petrucci and Jeremy Sproston. Vol. 14138. Lecture Notes in Computer Science. Springer, 2023, pp. 33–49. DOI: 10.1007/978-3-031-42626-1_3.
- Shahbaz, Muzammil and Roland Groz. "Inferring mealy machines". In: *International Symposium on Formal Methods*. Springer. 2009, pp. 207–222.

References II

Vaandrager, Frits W. et al. "A New Approach for Active Automata Learning Based on Apartness". In: *Tools and Algorithms for the Construction and Analysis of Systems - 28th International Conference, TACAS 2022, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2022, Munich, Germany, April 2-7, 2022, Proceedings, Part I*. Ed. by Dana Fisman and Grigore Rosu. Vol. 13243. Lecture Notes in Computer Science. Springer, 2022, pp. 223–243. DOI: 10.1007/978-3-030-99524-9_12.