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# Cubic interactions for massless and partially massless spin-1 and spin-2 fields

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Abstract: We perform a complete classification of the consistent two-derivative cubic couplings for a system containing an arbitrary number of massless spin-1, massless spin-2, and partially massless (PM) spin-2 fields in $D$-dimensional (anti-)de Sitter space. In addition to previously known results, we find a unique candidate mixing between spin-1 and PM spin-2 fields. We derive all the quadratic constraints on the structure constants of the theory, allowing for relative "wrong-sign" kinetic terms for any of the fields. In the particular case when the kinetic terms in each sector have no relative signs, we find that the unique consistent non-trivial theory is given by multiple independent copies of conformal gravity coupled to a Yang-Mills sector in $D = 4$. Our results strengthen the well-known no-go theorems on the absence of mutual interactions for massless and PM spin-2 fields.

Keywords: Classical Theories of Gravity, Gauge Symmetry, BRST Quantization, de Sitter space

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## Contents





## 1 Introduction and summary

De Sitter (dS) and anti-de Sitter (AdS) spaces are natural and well-motivated arenas to study theories of higher-spin gauge fields. Vasiliev theory [1–3] provides a striking example of how the obstructions one encounters in attempts to construct higher-spin theories in flat space can be evaded in AdS (see [4, 5] for reviews). On the other hand, observational evidence indicates that our universe was very approximately a dS space during the pre-hot Big Bang epoch and will evolve towards a dS space in the future. The high energy scales that could potentially be probed by cosmological experiments therefore motivate a good understanding of the dynamics of higher-spin particles in dS. Although this mostly concerns massive fields rather than gauge fields, one may envisage a Goldstone equivalence regime in which the massive theory is described by a tower of higher-spin gauge fields, or speculate about higher spin analogs of the Brout-Englert-Higgs or dynamical symmetry breaking mechanisms — see [6] and [7], respectively. Additionally, and more to the point of this paper, (A)dS space allows for the existence of exotic "massive" particles described by gauge fields, that one calls partially massless (PM) fields [8–11]. While bosonic PM fields are non-unitary in AdS, in dS they precisely saturate the Higuchi bound [12, 13], meaning that they would have behaved as light fields during a putative phase of cosmic inflation, with interesting observational imprints [14–16].

Similarly to their Fronsdal cousins [17], interactions of higher spin PM fields are highly constrained by their gauge structure, and in fact no fully satisfactory examples of interacting higher-spin theories with PM fields are known (see however [18, 19] for an intriguing proposal of a Vasiliev-like PM theory, and [20] for a proposal in three dimensions). While a few interesting studies have tackled the problem in the case of PM particles with spin $s > 2$ [21–23], the most detailed analyses have been focused on PM fields of spin $s = 2$ [24–31]. One reason is that this is of course the most technically tractable case, but there is also the more physical motivation that PM spin-2 fields may have some connection with theories of massive gravity [32–37]. Cubic couplings involving PM gravitons have in particular been subject of several studies [16, 31, 38, 39]. It is by now well established that three-point vertices for PM spin-2 particles are forbidden, at least if one assumes up to two-derivative interactions *and* the absence of "ghost-like" fields, i.e. fields with wrong-sign kinetic term. This no-go result has been extended in ref. [40] to also include the gravitational coupling with a massless spin-2 field.

Although the assumption of unitarity or, more specifically in the present context, absence of negative-norm states may seem essential, it is important to bear in mind that a complete non-linear theory that couples a PM graviton with a massless graviton in fact exists: conformal gravity [41–43]. More in detail, conformal gravity in $D = 4$ dimensions, upon linearizing the theory about an (A)dS background, yields a massless and a PM spin-2 modes, one of which is necessarily ghostly.[1] Although the physical viability of conformal gravity is perhaps questionable due to this latter property, it is nevertheless a model of obvious theoretical interest; see [45, 46] for reviews, including its supersymmetric extension. It is therefore a tantalizing possibility that other non-unitary theories that include PM fields may exist. This is further motivated by the results of refs. [47–49] on the classification of supersymmetric multiplets that include PM fields in four dimensions: PM supersymmetric representations are always non-unitary;[2] in particular, the simplest multiplet contains a PM spin-2 and a massless spin-1 field in its bosonic sector, and these must be relatively ghostly. The fermionic sector contains a massive and a massless spin-3/2 fields. Cubic couplings for PM spin-2 and massive spin-3/2 particles have been recently classified in [53] (see also [54] for earlier work). Interestingly, the system containing relatively ghostly PM spin-2 and a massless spin-1 fields has been shown to be conformally invariant in $D = 4$ dimensions [55].

In line with these considerations we mention two works that have studied the consequences of relaxing the assumption of positivity of kinetic terms in the construction of two-derivative cubic vertices involving PM spin-2 fields. The first work [44] considered the inclusion of massive and massless spin-2 fields, finding the cubic vertex of even-dimensional conformal gravity as a consistent solution, among other candidates. The second work [31] restricted its attention to only PM fields, showing that there exists a cubic vertex which is unique and consistent at the full non-linear level. Once again, these results necessitate non-positive-definite kinetic (or "internal") matrices, but are otherwise consistent from the point of view of the gauge structure. Remarkably, the model of [31] is the only consistent, interacting theory for a multiplet of PM spin-2 fields, although non-unitary at the classical level.

In the analysis of ref. [44] the additional spin-2 fields were introduced in an ad-hoc fashion so as to ensure consistency of the PM gravitational coupling. This motivates us to revisit the

[1] This may be generalized to any higher even dimension $D$: the spectrum contains $D/2$ spin-2 modes, one of which is massless, one is PM, and the rest are massive [44].

[2] Note that unitarity could be restored in dS background, see [50–52] for more explanations.



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problem of classifying interactions for massless and PM spin-2 fields, assuming a more general starting point in which the number of fields and the signature of their internal metrics are arbitrary. Furthermore, again prompted by the results on supersymmetric representations as well as by the related findings of refs. [53, 55], we also consider the inclusion of massless spin-1 fields in the spectrum. Let us summarize our main results:

- We provide a complete classification of the consistent first-order deformations, in the sense of the Noether procedure, of the free theory describing an arbitrary collection of massless spin-1, massless spin-2 and PM spin-2 fields in rigid $D$-dimensional (A)dS space. Our classification is completely general in regards to deformations of the gauge algebra, but is restricted to at most four derivatives in the deformations of the gauge symmetries and to at most two derivatives in the cubic vertices. A further restriction is that we focus on parity-even deformations.

  Our results confirm the classification of ref. [44] for massless and PM spin-2 fields. We also find a unique candidate vertex mixing massless spin-1 and PM spin-2 particles. This vertex is of the Chapline-Manton type, i.e. it is Abelian yet induces a non-linear gauge transformation of the spin-1 fields. Moreover, it only exists in $D = 4$ dimensions.

- We analyze the consistency of the candidate deformations at the next order in perturbation, starting with the Jacobi identities for the candidate gauge algebras. We derive all the quadratic constraints on the structure constants obtained in the previous step. The results are valid for any choice of the internal metrics that define the kinetic terms of the fields, thus accommodating any choice of healthy/ghostly field content.

  We consider the most general solution of the constraints under the assumption that each field sector (spin-1, massless spin-2 and PM spin-2) contains no relative 'healthy/ghostly' signs in the kinetic terms, although distinct sectors may do so. We find the answer to be given by multiple, independent copies of $D = 4$ conformal gravity minimally coupled with a Yang-Mills (or possibly Abelian) spin-1 sector.

  This outcome again confirms the conclusions of ref. [44] with regards to the spin-2 fields, although with a more general starting point. Our results also imply that the doubled spectra model identified in that reference in fact corresponds to two non-interacting copies of conformal gravity and is therefore not a new theory.[3] Moreover, we can rule out the consistency of the non-geometric vertex identified in [44],[4] at least under the aforementioned assumptions. An additional corollary is that distinct massless graviton species cannot mutually interact through the exchange of massless spin-1 or PM spin-2 particles, thus further generalizing the well-known no-go theorem of ref. [58].

  Finally, we discuss some solutions to the quadratic constraints in the more general set-up with non-sign-definite internal metrics. Although our analysis is not exhaustive, we are able to exhibit particular solutions for which all the candidate vertices remain consistent.

[3]Note that the consistent deformations of (multi-)conformal (or Weyl) gravity were investigated in [56].

[4]This vertex actually is the contraction of the massless spin-2 field $h_{\mu\nu}$ with the PM spin-2 current $J^{\mu\nu}$ first identified in [57] in the context of PM spin-2 self interactions.

| | Field variable | Curvature | Indices |
|---|---|---|---|
| Massless spin-2 | $h^I_{\mu\nu}$ | $K^I_{\mu\nu\rho\sigma}$ | $I, J, \ldots \in \{1, \ldots, n_g\}$ |
| PM spin-2 | $k^\Delta_{\mu\nu}$ | $\mathcal{F}^\Delta_{\mu\nu\rho}$ | $\Delta, \Sigma, \ldots \in \{1, \ldots, n_{\rm PM}\}$ |
| Massless spin-1 | $A^a_\mu$ | $F^a_{\mu\nu}$ | $a, b, \ldots \in \{1, \ldots, n_v\}$ |

**Table 1.** Field content and notations for the system considered in this paper.

Our analysis makes use of the Becchi-Rouet-Stora-Tyutin-Batalin-Vilkovisky (BRST-BV) [59–62] reformulation of the Noether procedure along the cohomological lines of [63, 64]. This is an ideally well-suited technique to deal with the usual ambiguities related to field and gauge parameter redefinitions, essentially recasting and generalizing the procedure of [65] in the form of a well-defined cohomological problem in the presence of antifields. Our results are thus guaranteed to be general and unambiguous within the stated assumptions. We briefly review the method and formulate its application to our system in section 2. In section 3 we present the results of the first-order deformation analysis, continuing in section 4 with the study of the second-order consistency and derivation of quadratic constraints. Finally in section 5 we investigate the resolution of the quadratic constraints and briefly summarize our finding in section 6.

## 2 BRST-BV formulation

The spectrum of fields considered in this paper consists of an arbitrary collection of $n_g$ tensor fields $h^I_{\mu\nu}$ describing massless spin-2 fields, $n_{\rm PM}$ tensor fields $k^\Delta_{\mu\nu}$ describing PM spin-2 fields, and $n_v$ vector fields $A^a_\mu$ describing massless spin-1 fields. The tensors $h^I_{\mu\nu}$ and $k^\Delta_{\mu\nu}$ are symmetric in their lower indices. See table 1 for a summary of our notations.

Our starting point is the non-interacting action for the free propagation of the fields on a rigid $D$-dimensional (A)dS space,

$$\begin{aligned} S_0\left[h^I_{\mu\nu}, k^\Delta_{\mu\nu}, A^a_\mu\right] = \int {\rm d}^D x\sqrt{-g}\Big[\mathfrak{g}_{IJ}\Big( & -\frac{1}{2}\nabla^\rho h^{I\mu\nu}\nabla_\rho h^J_{\mu\nu} + \nabla_\rho h^{I\mu\nu}\nabla_\mu h^{J\rho}_\nu - \nabla_\mu h^I \nabla_\nu h^{J\mu\nu} \\ & + \frac{1}{2}\nabla^\mu h^I \nabla_\mu h^J - \left(\frac{D-1}{\sigma L^2}\right) h^{I\mu\nu} h^J_{\mu\nu} + \frac{1}{2}\left(\frac{D-1}{\sigma L^2}\right) h^I h^J\Big) \\ & + \mathfrak{g}_{\Delta\Omega}\left(-\frac{1}{4}\mathcal{F}^{\Delta\lambda\mu\nu}\mathcal{F}^\Omega_{\lambda\mu\nu} + \frac{1}{2}\mathcal{F}^{\Delta\lambda}\mathcal{F}^\Omega_\lambda\right) - \frac{1}{4}\,\mathfrak{g}_{ab}\, F^{a\mu\nu} F^b_{\mu\nu}\Big]\,. \end{aligned} \tag{2.1}$$

Here $L$ is the radius of the (A)dS space and $\sigma$ is a sign, $+1$ for AdS and $-1$ for dS. Spacetime (greek) indices are moved with the (A)dS metric $g_{\mu\nu}$ and $\nabla$ is the corresponding metric-compatible covariant derivative. We write $h^I := g^{\mu\nu}h^I_{\mu\nu}$ and $k^\Delta := g^{\mu\nu}k^\Delta_{\mu\nu}$. As explained in the Introduction, we have allowed for arbitrary (constant) 'internal' field space metrics $\mathfrak{g}_{IJ}$, $\mathfrak{g}_{\Delta\Omega}$ and $\mathfrak{g}_{ab}$. Through trivial field redefinitions these may be brought to the form ${\rm diag}(+, \ldots, +, -, \ldots, -)$, and a 'unitary' theory corresponds to the case with positive definite metrics.[5] All the internal 'color' indices are raised and lowered with these metrics. The above

[5]We say 'unitary' with some abuse of terminology, since actually PM spin-2 fields are anyway non- unitary in AdS, irrespective of the signature of $\mathfrak{g}_{\Delta\Omega}$ [9].

action features the tensors $\mathcal{F}^{\Delta}_{\lambda\mu\nu} := 2\,\nabla_{[\lambda}k^{\Delta}_{\mu]\nu}$ and $\mathcal{F}^{\Delta}_{\lambda} := g^{\mu\nu}\mathcal{F}^{\Delta}_{\lambda\mu\nu}$ , as well as $F^{a}_{\mu\nu} := 2\,\nabla_{[\mu}A^{a}_{\nu]}$ . The action $S_0$ is invariant under the following gauge transformations

$$\begin{aligned} \delta_0 h^I_{\mu\nu} &= 2\,\nabla_{(\mu}\epsilon^I_{\nu)}\,, \\ \delta_0 k^{\Delta}_{\mu\nu} &= \nabla_\mu\nabla_\nu\epsilon^{\Delta} - \frac{\sigma}{L^2}g_{\mu\nu}\epsilon^{\Delta}\,, \\ \delta_0 A^a_\mu &= \nabla_\mu\epsilon^a\,. \end{aligned} \tag{2.2}$$

The gauge parameters $\epsilon^I_\mu$, $\epsilon^\Delta$ and $\epsilon^a$ are arbitrary functions and the gauge symmetries are irreducible, i.e. , there is no gauge-for-gauge transformations. The following linearized curvatures or field strengths are invariant under the above gauge transformations:

$$\begin{aligned} K^I_{\mu\nu\rho\sigma} :=& -\frac{1}{2}\left(\nabla_\rho\nabla_{[\mu}h^I_{\nu]\sigma} - \nabla_\sigma\nabla_{[\mu}h^I_{\nu]\rho} + \nabla_\mu\nabla_{[\rho}h^I_{\sigma]\nu} - \nabla_\nu\nabla_{[\rho}h^I_{\sigma]\mu}\right) \\ &+ \frac{\sigma}{L^2}\left(g_{\rho[\mu}h^I_{\nu]\sigma} - g_{\sigma[\mu}h^I_{\nu]\rho}\right)\,, \\ \mathcal{F}^{\Delta}_{\lambda\mu\nu} :=& 2\,\nabla_{[\lambda}k^{\Delta}_{\mu]\nu}\,, \qquad\qquad F^a_{\mu\nu} := 2\,\nabla_{[\mu}A^a_{\nu]}\,. \end{aligned} \tag{2.3}$$

The Noether procedure, or its generalization given in [65], consists in the construction of interactions, perturbatively in a set of deformation parameters, under the requirement of maintaining the number of gauge symmetries. No other restrictions are made a priori, although eventually we will set limits on the number of derivatives that may appear in the gauge transformations and in the Lagrangian, therefore ensuring locality. We also assume covariance of the deformation under the (A)dS background isometry algebra, as explained in [66], to which we refer the reader for more details. As stated earlier, we actually consider the reformulation (and further generalization) of the deformation procedure of [65] using the BRST-BV cohomological approach spelled out in [63, 64].

We begin by defining an enlarged field content through the introduction of ghosts $\xi^I_\mu$, $\chi^\Delta$ and $C^a$ respectively associated with the gauge parameters $\epsilon^I_\mu, \epsilon^\Delta$ and $\epsilon^a$. We also introduce antifields and antighosts, collectively denoted by $\{\Phi^*_\Xi\} := \{h^{*\mu\nu}_I, k^{*\mu\nu}_\Delta, A^{*\mu}_a, \xi^{*\mu}_I, \chi^*_\Delta, C^*_a\}$. These are canonically conjugate to the fields and ghosts, collectively denoted $\{\Phi^\Xi\} := \{h^I_{\mu\nu}, k^\Delta_{\mu\nu}, A^a_\mu,\ \xi^I_\mu, \chi^\Delta, C^a\}$, through the BV antibracket

$$(A,B) := \frac{\delta^R A}{\delta\Phi^\Xi}\frac{\delta^L B}{\delta\Phi^*_\Xi} - \frac{\delta^R A}{\delta\Phi^*_\Xi}\frac{\delta^L B}{\delta\Phi^\Xi}\,, \tag{2.4}$$

for any local functionals $A$ and $B$, and where we use De Witt's condensed notations for summations over repeated indices that imply integration over spacetime. Note that ghosts and antifields are Grassmann-odd variables in the present context with only bosonic fields and gauge symmetries.

The fundamental object of interest in this formalism is the BV functional $W[\Phi^\Xi, \Phi^*_\Xi]$, which encodes all information about the interaction vertices and gauge structure of the theory. At the free field level it reads

$$W_0 = S_0 + \int \mathrm{d}^D x\sqrt{-g}\left[h^{*\mu\nu}_I\left(2\,\nabla_{(\mu}\xi^I_{\nu)}\right) + k^{*\mu\nu}_\Delta\left(\nabla_\mu\nabla_\nu\chi^\Delta - \frac{\sigma}{L^2}\bar{g}_{\mu\nu}\chi^\Delta\right) + A^{*\mu}_a\nabla_\mu C^a\right]\,. \tag{2.5}$$

In our conventions, the antifields $h_I^{*\mu\nu}$ , $k_\Delta^{*\mu\nu}$ and $A_a^{*\mu}$ are tensors and not tensorial densities.

The consistency of the theory hinges on the invariance of the action under gauge symmetries and the existence of a consistent algebraic structure for the latter. In the BRST-BV formalism this is compactly enforced by the classical master equation

$$(W, W) = 0 \,, \tag{2.6}$$

and it is easy to verify, through the use of the free-theory Noether identities, that $W_0$ indeed satisfies this equation.

The reformulation of the Noether procedure continues with the perturbative expansion of the BV functional,

$$W = W_0 + W_1 + W_2 + \dots \,, \tag{2.7}$$

where, in our set-up, $W_1$ is cubic in the fields and antifields, $W_2$ is quartic, and so on. The master equation is to be solved in perturbation theory so as to determine the most general $W_1$, $W_2$ and so on. Thus one determines first $W_1$ by solving $(W_0, W_1) = 0$, next $W_2$ by solving $(W_0, W_2) = -\frac{1}{2}(W_1, W_1)$, and so on.

It has proved extremely useful to recast the procedure in the form of a cohomological problem [63, 64, 67, 68]. To this end one defines the BRST differential $s$, here given by $s\,\bullet := (W_0, \bullet)$ as we are interested in deformations of a free theory.[6] The first-order deformation $W_1$ should therefore satisfy $sW_1 = 0$. Since $s$ is nilpotent, $s^2 = 0$, it follows that any $s$-exact contribution $sB$ to $W_1$ (where $B$ is a local functional) is a trivial solution of the master equation to that order. In fact, such a solution must be discarded since it can be shown to correspond to a deformation generated from the free theory by trivial redefinition of the fields and gauge parameters. To sum up, it can be shown that a non-trivial deformation $W_1$ should satisfy $sW_1 = 0$ and should not be of the form $W_1 = sB$ for a local function $B$, so that non-trivial cubic deformations are characterized by the cohomology of $s$ in the space of local functionals with ghost number zero. The ghost number (denoted by gh) is a useful grading for the purpose of organizing the classification of solutions to the master equation. In the same vein it is helpful to also define the gradings called *pure ghost number* (puregh for short), and *antifield number* (antifld for short). The rationale for introducing these numbers will become clearer in the following, and we refer the reader to [58, 69] for more complete explanations. The values of these gradings for the variables considered in this paper are given below in table 2.

Another useful ingredient is the decomposition of the BRST differential into $s = \gamma + \delta$. Here $\gamma$ is a differential which acts on the fields in the form of a gauge transformation in terms of the ghosts; the differential $\delta$ acts on the antifields to produce the linear equations of motion and corresponding Noether identities in terms of the fields and antifields, respectively. The actions of $\gamma$ and $\delta$ are explicitly shown in table 2 for the system under consideration. Once the actions of $\gamma$ and $\delta$ are known through table 2 on the fields $\Phi^\Xi$ and antifields $\Phi^*_\Xi$ , we extend their actions on the jet space of the fields, antifields and all their derivatives by asking $\gamma$ and $\delta$ to be derivations that anticommute with the total exterior differential $d$. In our

[6] In particular, note that in our conventions we have $s\,\Phi^\Xi(x) = -\frac{1}{\sqrt{-g}}\,\frac{\delta^R W_0}{\delta \Phi^*_\Xi(x)}$ and $s\,\Phi^*_\Xi(x) = \frac{1}{\sqrt{-g}}\,\frac{\delta^R W_0}{\delta \Phi^\Xi(x)}$ .

| | $\lvert\bullet\rvert$ | gh | puregh | antifld | $\gamma\,\bullet$ | $\delta\,\bullet$ |
|---|---|---|---|---|---|---|
| $h^I_{\mu\nu}$ | 0 | 0 | 0 | 0 | $2\nabla_{(\mu}\xi^I_{\nu)}$ | 0 |
| $k^\Delta_{\mu\nu}$ | 0 | 0 | 0 | 0 | $\nabla_\mu\nabla_\nu\chi^\Delta - \frac{\sigma}{L^2}g_{\mu\nu}\chi^\Delta$ | 0 |
| $A^a_\mu$ | 0 | 0 | 0 | 0 | $\nabla_\mu C^a$ | 0 |
| $\xi^I_\mu$ | 1 | 1 | 1 | 0 | 0 | 0 |
| $\chi^\Delta$ | 1 | 1 | 1 | 0 | 0 | 0 |
| $C^a$ | 1 | 1 | 1 | 0 | 0 | 0 |
| $h^{*\mu\nu}_I$ | 1 | $-1$ | 0 | 1 | 0 | $\mathcal{E}^{\mu\nu}_I$ |
| $k^{*\mu\nu}_\Delta$ | 1 | $-1$ | 0 | 1 | 0 | $\mathcal{E}^{\mu\nu}_\Delta$ |
| $A^{*\mu}_a$ | 1 | $-1$ | 0 | 1 | 0 | $\mathfrak{g}_{ab}\,\nabla_\nu F^{b\nu\mu}$ |
| $\xi^{*\mu}_I$ | 0 | $-2$ | 0 | 2 | 0 | $-2\nabla_\nu h^{*\mu\nu}_I$ |
| $\chi^*_\Delta$ | 0 | $-2$ | 0 | 2 | 0 | $\nabla_\mu\nabla_\nu k^{*\mu\nu}_\Delta - \frac{\sigma}{L^2}k^*_\Delta$ |
| $C^*_a$ | 0 | $-2$ | 0 | 2 | 0 | $-\nabla_\mu A^{*\mu}_a$ |

**Table 2.** Gradings of fields and antifields, and the action of $\gamma$ and $\delta$ for the system studied in this paper. Here $\lvert\bullet\rvert$ denotes the Grassmann parity. For brevity we omit the explicit expressions of the equations of motion for the spin-2 fields, $\mathcal{E}^{\mu\nu}_I$ and $\mathcal{E}^{\mu\nu}_\Delta$, which may be straightforwardly inferred from $S_0$ (see also appendix A).

conventions for the antifields, note that one has $(A^a_\mu(x), A^{*\nu}_b(y)) = \frac{1}{\sqrt{-g}}\,\delta^a_b\delta^\mu_\nu\delta^D(x-y)$ where $\delta^D(x-y)$ is the Dirac delta density obeying $\int d^Dx\,\delta^D(x-y)f(y) = f(x)$. We also note that, on top of the relations $\gamma^2 = 0 = \delta^2$, one has $\gamma\delta = -\delta\gamma$. In our context the cohomology of $\gamma$ is easy to work out and will be extensively used in our analysis:

$$H(\gamma) \cong \left\{ f\left( [K^I_{\mu\nu\rho\sigma}], \left[\mathcal{F}^\Delta_{\lambda\mu\nu}\right], \left[F^a_{\mu\nu}\right], \xi^I_\mu, \nabla_{[\mu}\xi^I_{\nu]}, \chi^\Delta, \nabla_\mu\chi^\Delta, C^a, [\Phi^*_\Xi] \right) \right\}\,, \tag{2.8}$$

where $f$ is an arbitrary function of the arguments shown, and by square brackets we mean the variable and all its (A)dS covariant derivatives.

## 3 First order deformations: cubic vertices

In this section we consider the classification of non-trivial solutions to the master equation at first order in the deformation procedure,

$$sW_1 = 0\,. \tag{3.1}$$

Following refs. [56, 58, 67, 68], it proves useful to expand the BV functional $W_1$ in terms of local functionals with definite antifield number,

$$W_1 = \int \mathrm{d}^Dx\sqrt{-g}\,(a_0 + a_1 + a_2)\,, \tag{3.2}$$

where $\text{antifld}(a_n) = n$. As we are focusing on cubic deformations, it is easy to see that one cannot write terms with antifield number higher than 2 and ghost number zero. Each $a_n$ plays a specific role in the gauge structure of the theory: $a_2$ encodes the deformations of the gauge algebra (in particular, $a_2 = 0$ means that the algebra is Abelian); $a_1$ characterizes the deformations of the gauge transformations; and $a_0$ is nothing but the set of cubic vertices that we seek.

From the master equation (3.1) we infer the following descent equations:

$$\gamma a_2 = 0\,, \tag{3.3}$$
$$\delta a_2 + \gamma a_1 = \nabla_\mu j_1^\mu\,, \tag{3.4}$$
$$\delta a_1 + \gamma a_0 = \nabla_\mu j_0^\mu\,. \tag{3.5}$$

Here $j_n^\mu$ is some local vector with antifld$(j_n^\mu) = n$. Note that our perturbative assumption implies that a $j_2^\mu$ term does not exist with the required quantum numbers (antifield number 2 and ghost number 1). In fact, it is a general result, see [58, 68], that from the equation $\gamma a_k + \nabla_\mu j^\mu = 0$ with $k > 0$, one can redefine away $j^\mu$.

### 3.1 Gauge algebra

We start by determining the general solution of eq. (3.3). Non-trivial solutions correspond to the elements of the cohomology of $\gamma$ at antifield number 2 and ghost number zero, and must be scalars under the (A)dS isometries and parity-even. A complete basis of solutions is given by

$$\begin{aligned}
a_2^{(\mathrm{EH})} &= \xi_I^{*\mu}\xi^{J\nu}\nabla_{[\mu}\xi^k_{\nu]}g^I{}_{JK}, & a_2^{(\mathrm{YM})} &= \frac{1}{2}C_a^*C^bC^cf^a{}_{bc},\\
a_2^{(\mathrm{PM1})} &= \chi_\Delta^*\chi^\Omega\chi^\Gamma\, m^\Delta{}_{\Omega\Gamma}, & a_2^{(\mathrm{PM2})} &= \chi_\Delta^*\nabla_\mu\chi^\Omega\nabla^\mu\chi^\Gamma\, n^\Delta{}_{\Omega\Gamma},\\
a_2^{(1)} &= \xi_I^{*\mu}\xi^J_\mu\chi^\Delta\, f_{(1)}{}^I{}_{J\Delta}, & a_2^{(2)} &= \xi_I^{*\mu}\xi^J_\mu C^a\, f_{(2)}{}^I{}_{Ja},\\
a_2^{(3)} &= \xi_I^{*\mu}\nabla_{[\mu}\xi^J_{\nu]}\nabla^\nu\chi^\Delta\, f_{(3)}{}^I{}_{J\Delta}, & a_2^{(4)} &= \xi_I^{*\mu}C^a\nabla_\mu\chi^\Delta\, f_{(4)}{}^I{}_{a\Delta},\\
a_2^{(5)} &= \xi_I^{*\mu}\chi^\Delta\nabla_\mu\chi^\Omega\, f_{(5)}{}^I{}_{\Delta\Omega}, & a_2^{(6)} &= \chi_\Delta^*\chi^\Omega C^a\, f_{(6)}{}^\Delta{}_{\Omega a},\\
a_2^{(7)} &= \chi_\Delta^*\nabla^\mu\chi^\Omega\xi^I_\mu\, f_{(7)}{}^\Delta{}_{\Omega I}, & a_2^{(8)} &= \chi_\Delta^*C^aC^b\, f_{(8)}{}^\Delta{}_{ab},\\
a_2^{(9)} &= \chi_\Delta^*\xi^I_\mu\xi^{J\mu}\, f_{(9)}{}^\Delta{}_{IJ}, & a_2^{(10)} &= \chi_\Delta^*\nabla_{[\mu}\xi^I_{\nu]}\nabla^{[\mu}\xi^{\nu]J}\, f_{(10)}{}^\Delta{}_{IJ},\\
a_2^{(11)} &= C_a^*C^b\chi^\Delta\, f_{(11)}{}^a{}_{b\Delta}, & a_2^{(12)} &= C_a^*\chi^\Delta\chi^\Omega\, f_{(12)}{}^a{}_{\Delta\Omega},\\
a_2^{(13)} &= C_a^*\nabla_\mu\chi^\Delta\nabla^\mu\chi^\Omega\, f_{(13)}{}^a{}_{\Delta\Omega}, & a_2^{(14)} &= C_a^*\nabla^\mu\chi^\Delta\xi^I_\mu\, f_{(14)}{}^a{}_{\Delta I},\\
a_2^{(15)} &= C_a^*\xi^I_\mu\xi^{J\mu}\, f_{(15)}{}^a{}_{IJ}, & a_2^{(16)} &= C_a^*\nabla_{[\mu}\xi^I_{\nu]}\nabla^{[\mu}\xi^{\nu]J}\, f_{(16)}{}^a{}_{IJ}\,.
\end{aligned} \tag{3.6}$$

The structure constants appearing in these expressions satisfy some symmetrization constraints due to the Grassmann parity of the ghosts,

$$\begin{aligned}
&f^a{}_{bc} = f^a{}_{[bc]}, \qquad m^\Delta{}_{\Omega\Gamma} = m^\Delta{}_{[\Omega\Gamma]}, \qquad n^\Delta{}_{\Omega\Gamma} = n^\Delta{}_{[\Omega\Gamma]}, \qquad f_{(8)ab} = f_{(8)[ab]}\,,\\
&f_{(9)}{}^\Delta{}_{IJ} = f_{(9)}{}^\Delta{}_{[IJ]}, \qquad f_{(10)}{}^\Delta{}_{IJ} = f_{(10)}{}^\Delta{}_{[IJ]}, \qquad f_{(12)}{}^a_{\Delta\Omega} = f_{(12)}{}^a{}_{[\Delta\Omega]}\,,\\
&f_{(13)}{}^a{}_{\Delta\Omega} = f_{(13)}{}^a{}_{[\Delta\Omega]}, \qquad f_{(15)}{}^a{}_{IJ} = f_{(15)}{}^a{}_{[IJ]}, \qquad f_{(16)}{}^a{}_{IJ} = f_{(16)}{}^a{}_{[IJ]}\,.
\end{aligned} \tag{3.7}$$

In the list (3.6), $a_2^{(\mathrm{EH})}$ is the massless multi-graviton, called Einstein-Hilbert deformation [58], $a_2^{(\mathrm{YM})}$ is the usual Yang-Mills deformation [68], and $a_2^{(\mathrm{PM1})}$, $a_2^{(\mathrm{PM2})}$ are the unique candidate deformations of the PM spin-2 gauge algebra without additional fields [31]. The rest of the terms correspond to mixings among the gauge variations for different field types. We emphasize that this list is complete insofar as cubic deformations are concerned, without any restriction on the number of derivatives.

Not all among the candidates in (3.6) are admissible at first order in the deformation procedure. Indeed the second descent equation, eq. (3.4), states that $\delta a_2$ must be $\gamma$-exact modulo total derivatives. We find the following linear combination of candidates to be unobstructed at this order:

$$
\begin{aligned}
a_2 = \kappa\, a_2^{(\mathrm{EH})} + g_{(\mathrm{YM})}\, a_2^{(\mathrm{YM})} + \kappa_{(2)}\, a_2^{(2)} + \kappa_{(5)}\, a_2^{(5)} + \kappa_{(6)}\, a_2^{(6)} + \kappa_{(7)}\, a_2^{(7)} \\
+ \kappa_{(13)} \left( a_2^{(13)} - \frac{\sigma}{L^2} a_2^{(12)} \right) + \kappa_{(16)} \left( a_2^{(16)} - \frac{\sigma}{L^2} a_2^{(15)} \right) ,
\end{aligned}
\tag{3.8}
$$

where $\kappa$, $g_{\mathrm{YM}}$ and $\kappa_{(i)}$ are arbitrary coupling coefficients,[7] in addition to the following linear constraints on the structure constants:

$$
f_{(12)}{}^{a}{}_{\Delta\Omega} = f_{(13)}{}^{a}{}_{\Delta\Omega} \,, \quad f_{(15)}{}^{a}{}_{IJ} = f_{(16)}{}^{a}{}_{IJ} \,, \quad f_{(5)}{}^{I}{}_{\Delta\Omega} = f_{(5)}{}^{I}{}_{(\Delta\Omega)} \,. \tag{3.9}
$$

### 3.2 Gauge symmetries

We have already identified in eq. (3.8) the deformation $a_2$ which is not unobstructed at this order in the procedure. A particular solution of the descent equation for $a_1$ is given by

$$
\begin{aligned}
\hat{a}_1 = \kappa\, \hat{a}_1^{(\mathrm{EH})} + g_{(\mathrm{YM})}\, \hat{a}_1^{(\mathrm{YM})} + \kappa_{(2)}\, \hat{a}_1^{(2)} + \kappa_{(5)}\, \hat{a}_1^{(5)} + \kappa_{(6)}\, \hat{a}_1^{(6)} + \kappa_{(7)}\, \hat{a}_1^{(7)} \\
+ \kappa_{(13)}\, \hat{a}_1^{(12-13)} + \kappa_{(16)}\, \hat{a}_1^{(15-16)} ,
\end{aligned}
\tag{3.10}
$$

where each $\hat{a}_1^{(i)}$ solves $\delta a_2^{(i)} + \gamma \hat{a}_1^{(i)} =$ (total derivative), with the $a_2^{(i)}$ as given in (3.8). The explicit expressions are given by

$$
\begin{aligned}
\hat{a}_1^{(\mathrm{EH})} &= h_I^{*\mu\nu} \left( h^J_{\mu\sigma} \nabla_\nu \xi^{K\sigma} - \nabla_\mu h^K_{\nu\sigma} \xi^{J\sigma} + \nabla_\sigma h^K_{\mu\nu} \xi^{J\sigma} \right) g^{I}{}_{JK} \,, \\
\hat{a}_1^{(\mathrm{YM})} &= A_a^{*\mu} A^b_\mu C^c f^a{}_{bc} \,, \\
\hat{a}_1^{(2)} &= h_I^{*\mu\nu} \left( \frac{1}{2} h^J_{\mu\nu} C^a - A^a_\mu \xi^J_\nu \right) f_{(2)}{}^{I}{}_{Ja} \,, \\
\hat{a}_1^{(5)} &= h_I^{*\mu\nu} k^\Delta_{\mu\nu} \chi^\Omega f_{(5)}{}^{I}{}_{\Delta\Omega} \,, \\
\hat{a}_1^{(6)} &= k_\Delta^{*\mu\nu} \left( k^\Omega_{\mu\nu} C^a - \nabla_\mu A^a_\nu \chi^\Omega - 2 A^a_\mu \nabla_\nu \chi^\Omega \right) f_{(6)}{}^{\Delta}{}_{\Omega a} \,, \\
\hat{a}_1^{(7)} &= k_\Delta^{*\mu\nu} \Big( \nabla_\mu k^\Omega{}_{\nu\sigma} \xi^{I\sigma} + 2 k^\Omega_{\mu\sigma} \nabla_\nu \xi^{I\sigma} - \nabla_\mu h^I_{\nu\sigma} \nabla^\sigma \chi^\Omega \\
&\qquad + \frac{1}{2} \nabla_\sigma h^I_{\mu\nu} \nabla^\sigma \chi^\Omega - \frac{\sigma}{L^2} h^I_{\mu\nu} \chi^\Omega \Big) f_{(7)}{}^{\Delta}{}_{\Omega I} \,, \\
\hat{a}_1^{(12-13)} &= 2 A_a^{*\mu} k^\Delta_{\mu\nu} \nabla^\nu \chi^\Omega f_{(13)}{}^{a}{}_{\Delta\Omega} \,, \\
\hat{a}_1^{(15-16)} &= A_a^{*\mu} \Big( 2 \nabla^\nu h^\sigma{}_\mu{}^I \nabla_{[\nu} \xi^J_{\sigma]} - \frac{\sigma}{L^2} h^I_{\mu\nu} \xi^{J\nu} \Big) f_{(16)}{}^{a}{}_{IJ} \,.
\end{aligned}
\tag{3.11}
$$

Notice that the PM deformations, $a_2^{(\mathrm{PM1})}$ and $a_2^{(\mathrm{PM2})}$, are obstructed at this order [31]. To the above particular solution one must add the general solution $\bar{a}_1$ of the homogeneous equation

$$
\gamma \bar{a}_1 = \nabla_\mu \bar{\jmath}_1^\mu \,. \tag{3.12}
$$

[7]These constants are of course just for bookkeeping since one may choose to absorb them into the structure constants.

Since antigh$(\bar{a}_1) = 1 > 0$, general results [58, 68] show that one can absorb away the right-hand side, yielding $\gamma\bar{a}_1 = 0$. These correspond to Abelian, but non-linear, deformations of the gauge symmetries, sometimes referred to as Chapline-Manton type deformations in analogy to the theories studied in [70–72]. At cubic order, $\bar{a}_1$ contains one power of the curvatures or any number of derivatives thereof (cf. eq. (2.8)). In order to have a bounded number of solutions, it is therefore necessary at this stage to set a limit on the number of derivatives. Keeping in mind that our aim is to obtain general cubic vertices with at most two derivatives, we are thus led to consider at most four derivatives in $a_1$ (because $\gamma$ increases the number of derivatives by at most two, cf. table 2). With this restriction, the classification of Abelian solutions $\bar{a}_1$ is straightforward, although quite cumbersome. The full list may be found in [73].

As before, our main objective is to determine which among all these candidate $a_1$ are unobstructed in the next descent equation, eq. (3.5). At this stage we note that our assumption on the number of derivatives allowed in the space of solutions presents an issue: it may occur that a candidate $a_1$ is unobstructed, but the corresponding vertex $a_0$ contains strictly more than two derivatives. However, one must check that the higher-derivative terms are not spurious, in the sense that the solution is actually equivalent to a two-derivative vertex via a field redefinition. In order not to miss such solutions in our classification, it proves useful to include in our list of $a_1$ trivial terms which are $\gamma$-exact, since then the corresponding $a_0$ will be $\delta$-exact (i.e. it can be removed by a field redefinition). The list of $\gamma$-exact $a_1$ candidates with the required number of derivatives is also given in [73].

### 3.3 Cubic vertices

In this subsection we present the list of solutions for the cubic vertices consistent with the last descent equation. At this stage further constraints appear on the structure constants of the gauge algebra (and also, in some instances, on the spacetime dimension $D$), and we also indicate in each case the corresponding deformations of the gauge symmetries and algebra consistent with the existence of a two-derivative vertex.

**Einstein-Hilbert coupling.** The deformation $a_1 = \hat{a}_1^{(\mathrm{EH})}$ is a consistent solution of eq. (3.5) provided the structure constants satisfy

$$g_{IJK} = g_{(IJK)} \qquad \left(g_{IJK} := g^L{}_{JK}\mathfrak{g}_{IL}\right) . \tag{3.13}$$

The corresponding vertex $a_0$ is the multi-graviton Einstein-Hilbert solution [58],

$$\begin{aligned}
a_0^{(\mathrm{EH})} = g_{IJK}\Big[&\frac{1}{2}h^{I\mu\nu}\nabla_\mu h^J_{\rho\sigma}\nabla_\nu h^{K\rho\sigma} - \frac{1}{2}h^{I\mu\nu}\nabla_\mu h^J\nabla_\nu h^K + h^{I\mu\nu}\nabla_\mu h^J\nabla^\sigma h^K_{\sigma\nu}\\
&+ h^{I\mu\nu}\nabla_\mu h^J_{\nu\sigma}\nabla^\sigma h^K - h^{I\mu\nu}\nabla_\sigma h^J_{\mu\nu}\nabla^\sigma h^K + \frac{1}{4}h^I\nabla_\mu h^J\nabla^\mu h^K + h^{I\mu\nu}\nabla_\sigma h^J_{\mu\nu}\nabla_\rho h^{K\rho\sigma}\\
&- \frac{1}{2}h^I\nabla_\mu h^J\nabla_\nu h^{K\mu\nu} - 2h^{I\mu}{}_\nu\nabla_\mu h^J_{\rho\sigma}\nabla^\rho h^{K\sigma\nu} - h^{I\mu}{}_\nu\nabla_\sigma h^J_{\rho\mu}\nabla^\rho h^{K\sigma\nu}\\
&+ h^{I\mu}{}_\nu\nabla_\sigma h^J_{\rho\mu}\nabla^\sigma h^{K\rho\nu} + \frac{1}{2}h^I\nabla_\sigma h^J_{\mu\nu}\nabla^\mu h^{K\nu\sigma} - \frac{1}{4}h^I\nabla_\sigma h^J_{\mu\nu}\nabla^\sigma h^{K\mu\nu}\\
&+ \frac{\sigma}{L^2}\left(\frac{1}{2}h^I h^J h^K + 4h^I{}_\mu{}^\nu h^J{}_\nu{}^\rho h^K{}_\rho{}^\mu - 3h^I h^J_{\mu\nu}h^{K\mu\nu}\right)\Big] .
\end{aligned} \tag{3.14}$$



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**Yang-Mills coupling.** Similarly the deformation $a_1 = \hat{a}_1^{(\mathrm{YM})}$ is a consistent solution of the master equation provided

$$f_{abc} = f_{[abc]} \qquad \left(f_{abc} := \mathfrak{g}_{ad}\, f^d{}_{bc}\right) . \tag{3.15}$$

The vertex is given by

$$a_0^{(\mathrm{YM})} = -\frac{1}{2}\, f_{abc}\, A_\mu^a A_\nu^b F^{c\mu\nu} . \tag{3.16}$$

In the BRST-BV formalism the Yang-Mills system was first studied in [68]. See also [74] for the case of massive Yang-Mills theory.

**Partially massless spin-2 self-coupling.** We have seen already that the non-Abelian deformations of the PM spin-2 gauge algebra are obstructed. However, the Abelian term

$$\bar{a}_1^{(\mathrm{PM})} := c_{\Sigma\Delta\Omega}\, k^{*\Delta\mu\nu}\, \mathcal{F}^{\Omega}_{\sigma\mu\nu}\, \nabla^\sigma \chi^\Sigma \quad \in\ H(\gamma) \tag{3.17}$$

leads to a consistent vertex provided the constants $c_{\Sigma\Delta\Omega}$ satisfy

$$c_{\Sigma\Delta\Omega} = c_{\Sigma(\Delta\Omega)} . \tag{3.18}$$

Moreover, the spacetime dimension must be $D = 4$ for the solution to exist. The vertex is given by [31]

$$a_0^{(\mathrm{PM})} = \tfrac{1}{2}\, k^{\Delta}_{\mu\nu} J^{\mu\nu}_{\Delta} \tag{3.19}$$

in terms of the gauge-invariant current

$$\begin{aligned} J^{\mu\nu}_{\Delta} := 2\, c_{\Sigma\Delta\Omega} \Big( & \mathcal{F}^{\Omega(\mu|\rho\sigma} \mathcal{F}^{\Sigma|\nu)}{}_{\rho\sigma} - \mathcal{F}^{\Omega(\mu|} \mathcal{F}^{\Sigma|\nu)} + \mathcal{F}^{\Omega(\mu|\sigma|\nu)} \mathcal{F}^{\Sigma}_{\sigma} \\ & - \frac{1}{4} g^{\mu\nu} \mathcal{F}^{\Omega\rho\sigma\lambda} \mathcal{F}^{\Sigma}_{\rho\sigma\lambda} + \frac{1}{2} g^{\mu\nu} \mathcal{F}^{\Omega}_{\lambda} \mathcal{F}^{\Sigma\lambda} \Big) . \end{aligned} \tag{3.20}$$

A priori, this current should only satisfy the partially-massless conservation law for consistency of the vertex, but here it happens to obey the stronger constraints $\nabla_\mu J^{\mu\nu}_{\Delta} \approx 0$ (where the weak equality symbol $\approx$ stands for an equality that holds on the solutions of the free equations of motion), as well as $g_{\mu\nu}\, J^{\mu\nu}_{\Delta} = 0$ since $D = 4$; see [31] for more details.

**Spin-1 gravitational coupling.** Another solution is given by the minimal gravitational coupling of spin-1 fields. This arises from the Abelian deformation of the gauge symmetry

$$\bar{a}_1^{(v-g)} = d_{Iab} A^{*a\mu} F^b{}_{\mu\nu} \xi^{I\nu} , \tag{3.21}$$

with the requirement

$$d_{Iab} = d_{I(ab)} . \tag{3.22}$$

The vertex reads

$$a_0^{(v-g)} = -\frac{1}{2} d_{Iab} h^I_{\mu\nu} \left( F^a_{\mu\sigma} F^b{}_\nu{}^\sigma - \frac{1}{4} g_{\mu\nu} F^a_{\rho\sigma} F^{b\rho\sigma} \right) , \tag{3.23}$$

in which we indeed recognize the standard minimal coupling of gravity with the spin-1 energy-momentum tensor of Maxwell's theory.



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It may appear strange that the gravitational coupling of a field arises here from an Abelian deformation of the gauge symmetry. In fact, we can obtain the more standard Lie derivative transformation, given in our language by[8]

$$a_1^{(\mathrm{Lie})} = d_{Iab} \left( \nabla_\mu A^{*a\mu} A^b{}_\nu \xi^{I\nu} + A^{*a\mu} F^b{}_{\mu\nu} \xi^{I\nu} \right) , \tag{3.24}$$

and we observe that (3.21) and (3.24) differ by a trivial $\delta$-exact term (with a correspondingly trivial $\gamma$-exact gauge algebra term $a_2^{(\mathrm{Lie})} = d_{Iab} C^{*a} \nabla_\nu C^b \xi^{I\nu}$ signaling a trival redefinition of the gauge parameters of Maxwell's theory). Therefore $\bar{a}_1^{(v-g)}$ is indeed equivalent to the expected geometric-type deformation.

**Mixed spin-1 and partially massless spin-2 coupling.** Spin-1 and PM spin-2 fields may also interact through a geometric-looking coupling involving the spin-1 energy-momentum tensor,

$$a_0^{(v-\mathrm{PM})} = -e_{\Delta ab} k^{\Delta\mu\nu} \left( F^a_{\mu\sigma} F^b{}_\nu{}^\sigma - \frac{1}{4} g_{\mu\nu} F^a_{\rho\sigma} F^{b\rho\sigma} \right) , \tag{3.25}$$

where the constants $e_{\Delta ab}$ must satisfy

$$e_{\Delta ab} = e_{\Delta(ab)} , \tag{3.26}$$

and the spacetime dimension must be $D = 4$ for the solution to exist. The corresponding deformation of the gauge transformation is Abelian and reads

$$\bar{a}_1^{(v-\mathrm{PM})} = e_{\Delta ab} A^{*\mu a} F^b_{\mu\nu} \nabla^\nu \chi^\Delta . \tag{3.27}$$

**Partially massless spin-2 gravitational coupling.** Next we consider couplings between PM and massless spin-2 fields. We will confirm here the results of ref. [44]. We derive first the non-Abelian deformation corresponding to the minimal coupling of PM fields to gravity. It turns out that, in order to have a consistent vertex starting from $a_2^{(5)}$ and $a_2^{(7)}$, a precise linear combination of them should be taken with $f_{(5)}{}^I{}_{\Delta\Omega} = f_{(7)\Delta\Omega}{}^I$, yielding

$$a_2^{(\mathrm{PM}-g)} = 2a_2^{(7)} - \frac{(D-3)\sigma}{2L^2} a_2^{(5)} , \qquad f_{(5)}{}^I{}_{\Delta\Omega} = f_{(7)\Delta\Omega}{}^I , \tag{3.28}$$

in terms of the $a_2^{(i)}$ deformations given in eq. (3.6). The above linear combination is necessary but not sufficient to produce a consistent vertex. The corresponding particular solution $\hat{a}_1$ still needs to be completed by the addition of a suitable Abelian solution $\bar{a}_1$ solution of $\gamma\bar{a}_1 = 0$, yielding

$$a_1^{(\mathrm{PM}-g)} = 2\hat{a}_1^{(7)} + \frac{(D-3)\sigma}{L^2} \hat{a}_1^{(5)} + a_{I\Delta\Omega} \left( h^{*I\mu\nu} \mathcal{F}^\Delta_{\sigma\mu\nu} \nabla^\sigma \chi^\Omega + 2k^{*\Delta\mu\nu} \mathcal{F}^\Omega_{\sigma\mu\nu} \xi^{I\sigma} \right) , \tag{3.29}$$

---

[8] More explicitly, the Lie derivative of the vector field is

$$d_{Iab} A^{*a\mu} (-\pounds_{\xi^I} A^b_\mu) = -d_{Iab} A^{*a\mu} (\xi^{I\nu} \nabla_\nu A^b_\mu + \nabla_\mu \xi^{I\nu} A^b_\nu) .$$

This differs from $a_1^{(\mathrm{Lie})}$ in (3.24) by a total derivative. We have chosen the form given in (3.24) in order to make the equivalence with (3.21) more manifest. Notice also, incidentally, that $\hat{a}_1^{(\mathrm{EH})}$ (cf. eq. (3.11)) also agrees with the Lie derivative of the massless spin-2 field $\delta_\xi h_{\mu\nu} = h_{\mu\sigma} \nabla_\nu \xi^\sigma + h_{\nu\sigma} \nabla_\mu \xi^\sigma + \xi^\sigma \nabla_\sigma h_{\mu\nu}$ , modulo the redefinition $\xi_\mu \mapsto \xi_\mu - \frac{1}{2} h_{\mu\nu} \xi^\nu$ of the gauge parameter in the Abelian transformation law $\gamma h_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)}$.

where the $\hat{a}_1^{(i)}$ were given above in eq. (3.11), and where

$$f_{(5)}{}^I{}_{\Delta\Omega} = f_{(7)\Delta\Omega}{}^I = a^I{}_{\Delta\Omega} = a^I{}_{(\Delta\Omega)}\,. \tag{3.30}$$

More explicitly, we find

$$\begin{aligned} a_1^{(\mathrm{PM}-g)} = 2\,a_{I\Delta\Omega}\,k^{*\Delta\mu\nu}\Big(&\xi^{I\sigma}\,\nabla_\sigma k^\Omega_{\mu\nu} + 2\,k^\Omega_{\mu\sigma}\nabla_\nu\xi^{I\sigma} - \nabla_\mu h^I_{\nu\sigma}\nabla^\sigma\chi^\Omega + \frac{1}{2}\,\nabla^\sigma\chi^\Omega\,\nabla_\sigma h^I_{\mu\nu} \\ &- \frac{\sigma}{L^2}h^I_{\mu\nu}\chi^\Omega\Big) + a_{I\Delta\Omega}\,h^{*I\mu\nu}\Big(\mathcal{F}^\Delta_{\sigma\mu\nu}\nabla^\sigma\chi^\Omega + \frac{(D-3)\sigma}{L^2}k^\Delta_{\mu\nu}\chi^\Omega\Big)\,. \end{aligned} \tag{3.31}$$

Note that the first two terms correspond to the Lie-derivative transformation of the PM spin-2 field $k^\Delta_{\mu\nu}$. Lastly we provide the expression for the cubic vertex, which we write in the form

$$a_0^{(\mathrm{PM}-g)} = h^I_{\mu\nu}T_I^{\mu\nu}\,, \tag{3.32}$$

with

$$\begin{aligned} T_I{}^{\mu\nu} := a_{I\Delta\Omega}\Big[&2k^{\Delta\nu\alpha}\nabla_\alpha\nabla_\beta k^{\Omega\mu\beta} + 2k^{\Delta\mu\alpha}\nabla_\alpha\nabla_\beta k^{\Omega\nu\beta} + \nabla_\alpha k^\Omega\nabla^\alpha k^{\Delta\mu\nu} + k^\Delta\Box k^{\Omega\mu\nu} \\ &- 2k^{\Delta\mu\nu}\nabla_\alpha\nabla_\beta k^{\Omega\alpha\beta} - 2k^{\Delta\alpha\beta}\nabla_\beta\nabla_\alpha k^{\Omega\mu\nu} + 2k^{\Delta\mu\nu}\Box k^\Omega - 2\nabla_\alpha k^{\Delta\mu\nu}\nabla_\beta k^{\Omega\alpha\beta} \\ &- \frac{3}{2}k^{\Delta\nu\alpha}\Box k^{\Omega\mu}{}_\alpha - \frac{3}{2}k^{\Delta\mu\alpha}\Box k^{\Omega\nu}{}_\alpha + 2\nabla_\alpha k^{\Omega\nu\beta}\nabla_\beta k^{\Delta\mu\alpha} - \nabla_\beta k^{\Omega\nu\alpha}\nabla^\beta k^{\Delta\mu}{}_\alpha \\ &- \nabla_\alpha k^\Omega\nabla^\mu k^{\Delta\nu\alpha} + \frac{3}{2}\nabla_\beta k^{\Omega\alpha\beta}\nabla^\mu k^{\Delta\nu}{}_\alpha - \frac{1}{2}\nabla_\beta k^{\Delta\nu\alpha}\nabla^\mu k^{\Omega\beta}{}_\alpha - 2k^{\Delta\nu\alpha}\nabla^\mu\nabla_\alpha k^\Omega \\ &+ \frac{3}{2}k^{\Delta\nu\alpha}\nabla^\mu\nabla_\beta k^{\Omega\beta}{}_\alpha + \frac{3}{2}k^{\Delta\alpha\beta}\nabla^\mu\nabla_\beta k^{\Omega\nu}{}_\alpha - k^\Delta\nabla^\mu\nabla_\beta k^{\Omega\nu\beta} - \nabla_\alpha k^\Omega\nabla^\nu k^{\Delta\mu\alpha} \\ &+ \frac{3}{2}\nabla_\beta k^{\Omega\alpha\beta}\nabla^\nu k^{\Delta\mu}{}_\alpha - \frac{1}{2}\nabla_\beta k^{\Delta\mu\alpha}\nabla^\nu k^{\Omega\beta}{}_\alpha - 2k^{\Delta\mu\alpha}\nabla^\nu\nabla_\alpha k^\Omega + \frac{3}{2}k^{\Delta\mu\alpha}\nabla^\nu\nabla_\beta k^{\Omega\beta}{}_\alpha \\ &+ \frac{3}{2}k^{\Delta\alpha\beta}\nabla^\nu\nabla_\beta k^{\Omega\mu}{}_\alpha - k^\Delta\nabla^\nu\nabla_\beta k^{\Omega\mu\beta} - k^{\Delta\alpha\beta}\nabla^\mu\nabla^\nu k^\Omega{}_{\alpha\beta} + k^\Delta\nabla^\nu\nabla^\mu k^\Omega \\ &+ 2g^{\mu\nu}k^{\Delta\alpha\beta}\nabla_\beta\nabla_\alpha k^\Omega - 3g^{\mu\nu}k^{\Delta\alpha\beta}\nabla_\beta\nabla_\gamma k^{\Omega\gamma}{}_\alpha - \frac{1}{2}g^{\mu\nu}(\nabla_\beta k^\Omega)(\nabla^\beta k^\Delta) \\ &+ 2g^{\mu\nu}\nabla_\beta k^\Delta\nabla_\gamma k^{\Omega\beta\gamma} + g^{\mu\nu}k^\Delta\nabla_\gamma\nabla_\beta k^{\Omega\beta\gamma} + g^{\mu\nu}k^{\Delta\alpha\beta}\Box k^\Omega{}_{\alpha\beta} - g^{\mu\nu}k^\Delta\Box k^\Omega \\ &- \frac{1}{2}g^{\mu\nu}\nabla_\beta k^{\Omega\alpha\gamma}\nabla_\gamma k^{\Delta\beta}{}_\alpha + \frac{1}{2}g^{\mu\nu}\nabla_\gamma k^{\Omega\alpha\beta}\nabla^\gamma k^\Delta{}_{\alpha\beta} - \frac{3}{2}g^{\mu\nu}\nabla_\alpha k^{\Delta\alpha\beta}\nabla^\gamma k^\Omega{}_{\gamma\beta} \\ &+ \frac{\sigma}{L^2}\Big(4k^{\Delta\mu\nu}k^\Omega + (2D+3)k^{\Delta\mu\alpha}k^{\Omega\nu}{}_\alpha + \frac{D+7}{2}g^{\mu\nu}k^{\Delta\alpha\beta}k^\Omega{}_{\alpha\beta} + \frac{D-7}{2}g^{\mu\nu}k^\Delta k^\Omega\Big)\Big]\,. \end{aligned} \tag{3.33}$$

This current satisfies $\nabla_\mu T_I^{\mu\nu} = 0$ on the solutions of the PM free-field equations of motion (see appendix A), implying the consistency of the vertex under linearized diffeomorphisms. Less trivial is to check the consistency under PM gauge transformations since $T_I^{\mu\nu}$ is not itself gauge invariant.

**Partially massless spin-2 non-geometric coupling.** Finally we note the existence of an exotic coupling between massless and PM spin-2 fields, first identified in ref. [44], and dubbed *non-geometric* due to the fact that it is Abelian ($a_2 = 0$). The deformation of the gauge symmetry is encoded in

$$\bar{a}_1^{(\mathrm{non-geo})} = b_{I\Delta\Omega}\,k^{*\Delta\mu\nu}\Big[\mathcal{F}^\Omega_{\sigma\mu\nu}\xi^{I\sigma} - \frac{\sigma L^2}{2(D-2)}\nabla_\mu\left(\mathcal{F}^\Omega_{\rho\sigma\nu}\nabla^{[\rho}\xi^{\sigma]I}\right)\Big]\,, \tag{3.34}$$

with a restriction on the structure constants given by

$$b_{I\Delta\Omega} = b_{I(\Delta\Omega)}\,. \tag{3.35}$$

This $a_1$ can be lifted through (3.5) to furnish the vertex

$$a_0^{(\text{non-geo})} = \frac{1}{2}\, h^I_{\mu\nu} J_I^{\mu\nu}\,, \tag{3.36}$$

with the gauge-invariant current-like tensor

$$\begin{aligned} J_I^{\mu\nu} := b_{I\Omega\Sigma}\Big( & \mathcal{F}^{\Omega(\mu|\rho\sigma}\mathcal{F}^{\Sigma|\nu)}{}_{\rho\sigma} - \mathcal{F}^{\Omega(\mu|}\mathcal{F}^{\Sigma|\nu)} + \mathcal{F}^{\Omega(\mu|\sigma|\nu)}\mathcal{F}^{\Sigma}_{\sigma} \\ & - \frac{1}{4} g^{\mu\nu}\mathcal{F}^{\Omega\rho\sigma\lambda}\mathcal{F}^{\Sigma}_{\rho\sigma\lambda} + \frac{1}{2} g^{\mu\nu}\mathcal{F}^{\Omega}_{\lambda}\mathcal{F}^{\Sigma\lambda}\Big)\,. \end{aligned} \tag{3.37}$$

This current is actually identical (up to a coefficient that one may choose to factor out) to the one found in the PM self-coupling, eq. (3.20). In that context, $J_\Delta^{\mu\nu}$ has a physical interpretation as it is related to a Noether current associated with a global symmetry of the free PM theory, a property which holds only in $D = 4$ dimensions [31]. Similarly, in the present case, we have a Noether current $\mathcal{J}^\mu_{IJ} := \sqrt{-g} J_I^{\mu\nu} \bar{\epsilon}_{J\nu}$ , where by definition $\bar{\epsilon}^I_\mu$ is a Killing vector of the background (A)dS space obeying $\nabla_{(\mu}\epsilon^I_{\nu)} = 0$. Given that $\nabla_\mu J_I^{\mu\nu} \approx 0$, i.e., the (A)dS covariant divergence of $J_I^{\mu\nu}$ vanishes on the solutions of the free equations of motion (cf. appendix A), it follows immediately that $\partial_\mu \mathcal{J}^\mu_{IJ} \approx 0$, without any restriction on the spacetime dimension. The corresponding global symmetry transformation law can be read off from the above expression for $\bar{a}_1^{(\text{non-geo})}$, where the gauge parameter $\epsilon^I_\mu$ must be replaced by the Killing parameter $\bar{\epsilon}^I_\mu$.

## 4 Second order deformations: quadratic constraints

In this section we investigate the master equation at second order. We will not carry out a classification of the second-order BV functional $W_2$, but rather use the master equation as a consistency condition on the first order deformations identified in the previous section. This will lead to a set of quadratic constraints on the structure constants, i.e. generalized Jacobi identities.

As before we split the second-order BV functional by antifield number,

$$W_2 = \int \mathrm{d}^D x \sqrt{-g}\,(b_0 + b_1 + b_2)\;. \tag{4.1}$$

It is again easy to verify that the expansion stops at antifield number 2. The master equation at this order in the perturbative analysis reads

$$sW_2 = -\frac{1}{2}\,(W_1, W_1)\;, \tag{4.2}$$

or, in terms of the local functions $a_n$ and $b_n$,

$$\gamma b_2 = -\frac{1}{2}\,(a_2, a_2) + \nabla_\mu t_2^\mu\,, \tag{4.3}$$

$$\delta b_2 + \gamma b_1 = -\frac{1}{2}\,(a_1, a_1) - (a_2, a_1) + \nabla_\mu t_1^\mu\,, \tag{4.4}$$

$$\delta b_1 + \gamma b_0 = -\,(a_1, a_0) + \nabla_\mu t_0^\mu\,. \tag{4.5}$$

### 4.1 Consistency of the deformed gauge algebra

At antifield number 2, the descent equation (4.3) dictates that $(a_2, a_2)$ must be $\gamma$-exact modulo a total divergence. We recall here the full expression for the candidate $a_2$ that meets the consistency requirements at first order,

$$a_2 = a_2^{(\mathrm{EH})} + a_2^{(\mathrm{YM})} + a_2^{(\mathrm{PM}-g)}\,, \tag{4.6}$$

respectively with structure constants $g_{IJK} = g_{(IJK)}$, $f_{abc} = f_{[abc]}$ and $a^I{}_{\Delta\Omega} = a^I{}_{(\Delta\Omega)}$. Consistency of this $a_2$ with eq. (4.3) yields the following set of quadratic constraints:

$$\begin{aligned}
&0 = g^I{}_{M[J}g^M{}_{K]L}\,, \quad 0 = f^c{}_{a[b}f^a{}_{de]}\,,\\
&0 = a^{[I}{}_{\Delta\Omega}a^{J]\Delta}{}_\Theta\,, \quad 0 = a_{I\Delta[\Omega}a^I{}_{\Theta]\Sigma}\,, \quad 0 = a^{(I}{}_{\Delta\Omega}a^{J)\Delta}{}_\Theta + \tfrac{1}{4}\,g^{KIJ}a_{K\Omega\Theta}\,.
\end{aligned} \tag{4.7}$$

The first among these results states that the multi-graviton algebra is associative (in addition to being commutative and symmetric as per the constraint $g_{IJK} = g_{(IJK)}$) [58]. The second result is the usual Jacobi identity of Yang-Mills theory. The second line contains new results corresponding to restrictions on the PM gravitational coupling.

### 4.2 Consistency of the deformed gauge symmetries

Next we analyze the consistency of the gauge symmetry deformation $a_1$ with the second descent equation, eq. (4.4). Our candidate $a_1$ is given by

$$a_1 = a_1^{(\mathrm{EH})} + a_1^{(\mathrm{YM})} + a_1^{(\mathrm{PM}-g)} + \bar{a}_1^{(\mathrm{PM})} + \bar{a}_1^{(v-g)} + \bar{a}_1^{(v-\mathrm{PM})} + \bar{a}_1^{(\mathrm{non-geo})}\,. \tag{4.8}$$

We remind the reader that the last four of these terms correspond to Abelian deformations, with coefficients $c_{\Sigma\Delta\Omega}$, $d_{Iab}$, $e_{\Delta ab}$ and $b_{I\Delta\Omega}$, respectively.

Several among the obstructions (i.e. terms in (4.4) which are neither $\gamma$-exact nor $\delta$-exact modulo total derivatives) are eliminated thanks to the constraints derived previously, eq. (4.7). The remaining obstructions necessitate the following constraints on the Abelian coefficients:

$$\begin{aligned}
&0 = f^a{}_{b(c}d^I{}_{d)a}\,, \qquad 0 = f^a{}_{b(c}e^\Delta{}_{d)a}\,,\\
&0 = d_{Iab}a^{I\Delta\Omega} - 4e^{(\Delta}{}_{ac}e^{\Omega)c}{}_b\,, \qquad 0 = e^{[\Delta}{}_{ab}e^{\Omega]ac}\,,\\
&0 = e^{\Delta a}{}_{(b}d^I{}_{c)a} + 4e_{\Omega bc}a^{I\Delta\Omega}\,, \qquad 0 = e^{\Delta a}{}_{[b}d^I{}_{c]a}\,,\\
&0 = d_{(I}{}^{ab}d_{J)ac} - g_{IJK}d^{Kb}{}_c\,, \qquad 0 = d_{[I}{}^{ab}d_{J]ac}\,,\\
&0 = c_{\Delta\Sigma}{}^\Omega c_{\Theta\Pi\Omega} + 4\,a_{I\Delta\Sigma}a^I{}_{\Theta\Pi}\,,\\
&0 = a_{I\Sigma[\Pi}c_{\Delta]\Omega}{}^\Sigma\,, \qquad 0 = b_{I\Delta\Omega}b^{J\Delta}{}_\Pi\,.
\end{aligned} \tag{4.9}$$

The quadratic constraints on $c_{\Sigma\Delta\Omega}$ generalize the results derived in ref. [31] to include gravitational couplings; to our knowledge, the rest are new results.

### 4.3 Consistency of the cubic vertices

Considering finally the descent equation (4.5), we find that most obstructions are canceled upon use of the constraints derived above, eqs. (4.7) and (4.9). This is a highly non-trivial consistency check given that a priori the antibracket $(a_1, a_0)$ contains several hundreds of terms. The offending terms that remain read

$$\begin{aligned}(a_1,a_0) \supset &\left(c_{\Delta\Sigma}{}^{\Omega}c_{\Omega\Theta\Pi} + 4a_{I\Delta\Sigma}a^{I}{}_{\Theta\Pi}\right)\mathcal{F}^{\Theta\mu}{}_{\rho\sigma}\mathcal{F}^{\Pi\nu\rho\sigma}\mathcal{F}^{\Sigma\lambda}{}_{\mu\nu}\nabla_{\lambda}\chi^{\Delta} \\ &+ \left(2e^{\Omega}{}_{ab}c_{\Delta\Sigma\Omega} + d^{I}{}_{ab}a_{I\Delta\Sigma}\right)F^{a\mu\rho}F^{b}{}_{\rho}{}^{\nu}\mathcal{F}^{\Sigma\lambda}{}_{\mu\nu}\nabla_{\lambda}\chi^{\Delta}\,,\end{aligned} \tag{4.10}$$

leading to the following constraints:

$$\begin{aligned}&0 = c_{\Delta\Sigma}{}^{\Omega}c_{\Omega\Theta\Pi} + 4a_{I\Delta\Sigma}a^{I}{}_{\Theta\Pi}\,, \\ &0 = 2e^{\Omega}{}_{ab}c_{(\Delta\Sigma)\Omega} + d^{I}{}_{ab}a_{I\Delta\Sigma}\,, \quad 0 = e^{\Omega}{}_{ab}c_{[\Delta\Sigma]\Omega}\,.\end{aligned} \tag{4.11}$$

The constraints in the first line were also given previously in ref. [31] with $a_{I\Delta\Sigma} = 0$. However, in that reference, this result was assumed to hold rather than derived. Here we analyze this and the other obstruction more closely. The combination $\mathcal{F}^{\Theta\mu}{}_{\rho\sigma}\mathcal{F}^{\Pi\nu\rho\sigma}\mathcal{F}^{\Sigma\lambda}{}_{\mu\nu}\nabla_{\lambda}\chi^{\Delta}$ is clearly not $\delta$-exact since it does not vanish on the free-field equations of motion (even allowing for total derivatives). That it is also not $\gamma$-exact can be seen by considering the flat-space limit, in which case the subexpression $\mathcal{F}^{\Theta\mu}{}_{\rho\sigma}\mathcal{F}^{\Pi\nu\rho\sigma}\mathcal{F}^{\Delta\lambda}{}_{\mu\nu}$ would need to be a total derivative in order to get, upon partial integration, a second derivative acting on the ghost. That this is not the case can be verified directly by calculating its Euler-Lagrange derivative. The other obstruction in (4.10) may be analyzed in the same way.

## 5 Analysis of the results

The list of quadratic constraints given in eqs. (4.7), (4.9) and (4.11) constitute the main results of this paper. In this section we analyze the resolution of these constraints. Our aim is not to be exhaustive but rather to understand the implications of assuming versus relaxing the condition of having sign-definite internal metrics. We will show that this assumption is inconsistent with the existence of most of the cubic vertices, the unique non-trivial exception being the case of multiple independent conformal gravity sectors coupled to Yang-Mills theory in $D = 4$ dimensions.

**Massless spin-1 and PM spin-2.** For the sake of clarity, and because it is an interesting system on its own, we consider first the couplings of massless spin-1 and PM spin-2 fields. We repeat here the pertinent constraints (omitting the usual Jacobi identity for $f_{abc}$ and noting that $a_{I\Delta\Sigma} = 0$ as we ignore gravity here):

$$\begin{aligned}&0 = c_{\Delta\Sigma}{}^{\Omega}c_{\Theta\Pi\Omega}\,, \qquad 0 = c_{\Delta\Sigma}{}^{\Omega}c_{\Omega\Theta\Pi}\,, \\ &0 = e^{\Delta}{}_{ac}e^{\Omega c}{}_{b}\,, \qquad\ \ 0 = f^{a}{}_{b(c}e^{\Delta}{}_{d)a}\,,\end{aligned} \tag{5.1}$$

and recall that $D = 4$ for the coefficients $c_{\Sigma\Delta\Omega}$ and $e_{\Delta ab}$ to be a priori non-zero. Non-vanishing solutions to these constraints do not exist in the case of sign-definite metrics, i.e. when $\mathfrak{g}_{ab} = \pm\delta_{ab}$ and $\mathfrak{g}_{\Delta\Omega} = \pm\delta_{\Delta\Omega}$. The argument is the same as the one given in ref. [31]: considering $\Sigma = \Theta$ and $\Delta = \Pi$ (with no summation), the contraction $c_{\Delta\Sigma}{}^{\Omega}c_{\Theta\Pi\Omega}$ becomes a sum of squares, hence $c_{\Sigma\Delta\Omega} = 0$. Similarly $e_{\Delta ab} = 0$. An obvious corollary is that a single PM field cannot interact via cubic vertices with a unitary Yang-Mills (or Maxwell) sector.

The conclusion is different if one allows for 'healthy/ghostly' relative signs. Explicit solutions for the PM spin-2 self-coupling were given in [31]. Here we only consider the mutual vector-PM interaction. In order to have a non-vanishing $e_{\Delta ab}$ we need a non-sign-definite internal metric $\mathfrak{g}_{ab}$. The most minimal case includes one PM field and two Abelian vector fields ($f_{abc} = 0$), with $\mathfrak{g}_{ab} = \text{diag}(+1, -1)$. The unique solution (modulo a trivial overall rescaling) is given by $e_{ab} := e_{1ab} = 1 \; \forall \, a, b$. If we consider three vector fields (the simplest case that *a priori* allows for non-zero $f_{abc}$) with $\mathfrak{g}_{ab} = \text{diag}(+1, +1, -1)$, and again a single PM field, we find a family of non-trivial solutions for $e_{ab}$; however, they all lead to a vanishing $f_{abc}$ as per the last constraint in (5.1).

To have a non-zero 'ghostly Yang-Mills' coupling one needs at least four vectors. We have found the general solution in this case (still under the assumption of one PM field); it involves several free parameters and rather lengthy expressions, so for brevity we do not write here the full result and instead only give a particular solution in the model with metric $\mathfrak{g}_{ab} = \text{diag}(+1, +1, +1, -1)$:

$$\begin{aligned} &f_{123} = 1\,, \qquad f_{124} = f_{134} = \frac{1}{2}\,, \qquad f_{234} = \frac{1}{\sqrt{2}}\,, \qquad e_{11} = e_{34} = -e_{24} = 1\,, \\ &e_{12} = -e_{13} = -\frac{1}{\sqrt{2}}\,, \qquad e_{14} = \sqrt{2}\,, \qquad e_{22} = e_{33} = -e_{23} = \frac{1}{2}\,, \qquad e_{44} = 2\,. \end{aligned} \tag{5.2}$$

**Massless spin-1 and massless spin-2.** We study next the mutual couplings of massless spin-2 fields mediated by vector particles. We assume sign-definite metrics $\mathfrak{g}_{ab} = \pm\delta_{ab}$ and $\mathfrak{g}_{IJ} = \pm\delta_{IJ}$. We start by recalling the theorem of ref. [58] stating that the unique solution (modulo overall rescalings) of the constraints on the multi-graviton coefficients $g_{IJK}$ is given by $g_{IJK} = 1$ if $I = J = K$ and $g_{IJK} = 0$ otherwise.

The constraint $0 = d_{(I}{}^{ab} d_{J)ac} - g_{IJK} d^{Kb}{}_c$ (cf. eq. (4.9)) may then be written as $(d_I d_J)_{bc} = \pm\delta_{IJ}\,(d_I)_{bc}$ (no sum over $I$), where $(d_I)$ is the matrix with entries $d_{Iab}$. This result shows that these matrices are $n_g$ independent projectors. As a consequence, a basis of solutions is given by $d_{Iab} = \pm\delta_{Ia}\delta_{Ib}$ (no sum over $I$). The other relevant constraints, $0 = d_{[I}{}^{ab} d_{J]ac}$ and $0 = f^a{}_{b(c} d^I{}_{d)a}$, are then automatically satisfied. It follows that a massless spin-1 particle may only couple to one graviton (or to none), thus forbidding vector-mediated multi-graviton interactions. This also applies to Yang-Mills theory: if $d_{Iab} \neq 0$, then the constraint $0 = f^a{}_{b(c} d^I{}_{d)a}$ is satisfied, with non-zero $f_{abc}$, only if the '$a$' and '$b$' vector fields belong to the same Yang-Mills sector. Put another way, while two non-Abelian vectors may couple to distinct gravitons, they cannot be components of the same Yang-Mills multiplet, i.e. with a "common" $f_{abc}$. This precludes the possibility of multi-graviton interactions through loops of vector particles.

This outcome generalizes the no-go theorem of [58], which considered couplings mediated by scalar particles, and is also in agreement with the results of ref. [75]. The latter analysis was however restricted to the Abelian spin-1 case; the present no-go result for Yang-Mills theory appears to be new to the best of our knowledge. Once again these results are not expected to uphold in the situation with non-sign-definite internal metrics. In fact, already for pure multi-gravity non-trivial couplings are known to exist if one allows for 'ghostly' massless spin-2 fields [76, 77].

**Massless spin-2 and PM spin-2.** Focusing first on the geometric coupling between massless and PM gravitons, we have the quadratic constraints (cf. eqs. (4.7), (4.9) and (4.11))

$$
\begin{aligned}
&0 = a^{[I}{}_{\Delta\Omega}a^{J]\Delta}{}_{\Theta}\,, \qquad 0 = a_{I\Delta[\Omega}a^{I}{}_{\Theta]\Sigma}\,, \qquad a^{(I}{}_{\Delta\Omega}a^{J)\Delta}{}_{\Theta} = -\frac{1}{4}g^{KIJ}a_{K\Omega\Theta}\,,\\
&0 = c_{\Delta\Sigma}{}^{\Omega}c_{\Theta\Pi\Omega} + 4a_{I\Delta\Sigma}a^{I}{}_{\Theta\Pi}\,, \qquad 0 = a_{I\Sigma[\Pi}c_{\Delta]\Omega}{}^{\Sigma}\,,\\
&0 = c_{\Delta\Sigma}{}^{\Omega}c_{\Omega\Theta\Pi} + 4a_{I\Delta\Sigma}a^{I}{}_{\Theta\Pi}\,.
\end{aligned}
\tag{5.3}
$$

Assuming sign-definite kinetic metrics, these constraints may be analyzed in a similar fashion to the vector-graviton interaction of the previous paragraph, although here we need to be more cautious about the relative kinetic signs between massless and PM sectors. Without loss of generality, we assume the former to be $\mathfrak{g}_{IJ} = \delta_{IJ}$, and write $\mathfrak{g}_{\Delta\Sigma} = \sigma_{\rm PM}\delta_{\Delta\Sigma}$ with $\sigma_{\rm PM} = \pm 1$. The constraint $a^{(I}{}_{\Delta\Omega}a^{J)\Delta}{}_{\Theta} = -\frac{1}{4}g^{KIJ}a_{K\Omega\Theta}$ is then solved by $a_{I\Delta\Omega} = -\frac{1}{4}\sigma_{\rm PM}g_{I}\delta_{I\Delta}\delta_{I\Omega}$ (no sum over $I$), where $g_I := g_{III}$. The constraints involving the PM self-coupling $c_{\Delta\Sigma\Omega}$ may be manipulated to produce $0 = c_{[\Delta\Sigma]}{}^{\Omega}c_{\Theta\Pi\Omega}$, which implies that $c_{[\Delta\Sigma]\Omega} = 0$ if the internal metric is sign-definite. Thus we reach the conclusion that

$$
c_{\Delta\Sigma\Omega} = c_{(\Delta\Sigma\Omega)}\,. \tag{5.4}
$$

From these results it can be demonstrated that one may choose a basis in which $c_{\Delta\Sigma\Omega} = 0$ unless $\Delta = \Sigma = \Omega$. Indeed, from the constraints $0 = c_{\Delta(\Sigma}{}^{\Omega}c_{\Theta)\Pi\Omega} + 4a_{I\Delta(\Sigma}a^{I}{}_{\Theta)\Pi}$ and $0 = c_{\Delta[\Sigma}{}^{\Omega}c_{|\Omega|\Theta]\Pi}$ we infer that $c_{\Delta\Delta}{}^{\Omega}c_{\Sigma\Sigma\Omega} = 0$ (no sum over $\Delta, \Sigma$) if $\Delta \neq \Sigma$. Thus $\{c_{(\Delta)}\}_{\Delta=1}^{n_{\rm PM}}$, where $c_{(\Delta)}$ is the vector with components $c_{\Delta\Delta}{}^{\Omega}$, is an orthogonal set, and we may choose a basis with $(c_{(\Delta)})^{\Omega} \propto \delta^{\Omega}_{\Delta}$. Then, using this result and the constraints, we find (up to a sign)

$$
c_{\Delta\Delta\Delta} = \frac{\sqrt{-\sigma_{\rm PM}}}{2}g_{\Delta}\,, \tag{5.5}
$$

where $g_\Delta := g_I\delta_{I\Delta}$ is non-zero only if the '$\Delta$' PM field couples to one of the massless gravitons. Notice furthermore that we are forced to consider a 'ghostly' PM sector with $\sigma_{\rm PM} = -1$ in order to have a real solution. Finally, we can now use this result once again in the original constraint to infer that $c_{\Delta\Sigma\Omega} = 0$ when $\Delta \neq \Sigma \neq \Omega$. Note that implicit in this analysis is the assumption that $D = 4$. If $D \neq 4$ then $c_{\Delta\Sigma\Omega} = 0$ from the start, and it is then easy to prove from the above constraints that $a_{I\Delta\Sigma} = 0$ in this case.

In conclusion, a PM spin-2 field may only interact with at most one graviton, and only in four dimensions, at least if one supposes the existence of a cubic vertex as dictated by the minimal coupling prescription, while mutual interactions among different PM fields or among different massless spin-2 fields are excluded. The single PM-graviton system is consistent with the expectations spelled out in the Introduction, since we know that conformal gravity precisely includes a massless and a PM spin-2 fields, which must have opposite kinetic signs. The obstruction to mutual couplings between different conformal gravity sectors is also in agreement with the general results of [56].

The non-geometric coupling is, on the other hand, fully obstructed in the situation with sign-definite metrics, since the constraint $0 = b_{I\Delta\Omega}b^{J\Delta}{}_{\Pi}$ then implies $b_{I\Delta\Omega} = 0$, as we have explained. An immediate corollary is that a single PM field cannot interact with gravity through this Abelian vertex. The failure of the non-geometric coupling may be traced back to the absence of a mixed '$g - b$' term in the quadratic constraint (as present in the case of the geometric couplings), which is due to the absence of a non-Abelian (i.e., non-trivial

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$a_2$) deformation for the non-geometric coupling. We find it interesting, in this respect, that non-Abelian deformations are in some sense less constrained than Abelian ones.

Dropping the hypothesis of sign-definite metrics again changes this no-go result. The simplest such model requires one massless graviton and two relatively 'ghostly' PM spin-2 particles with $\mathfrak{g}_{\Delta\Omega} = \text{diag}(+1, -1)$, leading to the non-trivial solution $b_{\Delta\Omega} := b_{1\Delta\Omega} = 1\ \forall\Delta, \Omega$ (modulo an overall rescaling). More complex models with more than two fields may be similarly studied, with solutions analogous to those given in ref. [31].

**General case with sign-definite kinetic terms.** Finally we consider the general case with massless spin-1, massless spin-2 and PM spin-2 fields, focusing exclusively on the case of sign-definite internal metrics, although we allow for relative signs between different particle types. As we have seen, this assumption forbids the PM non-geometric coupling ($b_{I\Delta\Omega} = 0$). We write the internal metrics as $\mathfrak{g}_{IJ} = \delta_{IJ}$, $\mathfrak{g}_{\Delta\Sigma} = \sigma_{\text{PM}}\delta_{\Delta\Sigma}$, $\mathfrak{g}_{ab} = \sigma_v\delta_{ab}$, where $\sigma_{\text{PM}}$ and $\sigma_v$ give the relative kinetic signs of the PM and vector sectors.

Consider first, for the sake of clarity, the situation with only one field of each type, so we omit all indices in the structure coefficients (assuming the convention that all indices have been lowered with the internal metrics). Normalizing $g = 1$, we have from (4.7) $a = -\sigma_{\text{PM}}/4$ (we ignore trivial solutions, here $a = 0$). The PM self-coupling may be analyzed as in the previous paragraph, with the result $c = \sqrt{-\sigma_{\text{PM}}}/2$, so again $\sigma_{\text{PM}} = -1$ for the solution to be real (and recall that $D = 4$ is also necessary). We are left to consider the mixed constraints involving $d$ and $e$. There are four constraints in total (three in (4.9) and one in (4.11)), i.e. it is an over-determined system, yet a (unique) solution exists: $d = \sigma_v$, $e = \sigma_v/4$. Notice that the sign $\sigma_v$ remains undetermined, i.e. both 'healthy/ghostly' cases for the vector field are allowed. This outcome agrees with the expectation inferred from the existence of conformal gravity coupled to a Maxwell field, which maintains conformal invariance in four dimensions. The strength of this result lies in having established the uniqueness of the solution.

The generalization of this analysis to multiple fields is straightforward upon use of the previous results in this section. We first use the results of the massless-PM system to infer that $a_{I\Delta\Omega} = \frac{1}{4}g_I\delta_{I\Delta}\delta_{I\Omega}$ and $c_{\Delta\Sigma\Omega} = \frac{1}{2}g_I\delta_{I\Delta}\delta_{I\Sigma}\delta_{I\Omega}$, with the requirement that $\sigma_{\text{PM}} = -1$ (and we choose $\sigma_v = 1$ for concreteness), implying in particular that massless-PM spin-2 fields may only couple in independent pairs (or else remain uncoupled). The massless spin-1 gravitational coupling is also studied in the same way as above, i.e. $d_{Iab} = g_I\delta_{Ia}\delta_{Ib}$, and identical conclusions follow. The remaining non-trivial constraints involving $e_{\Delta ab}$ are

$$\begin{aligned} &0 = f^a{}_{b(c}e^{\Delta}{}_{d)a}\,, \qquad 0 = d_{Iab}a^{I\Delta\Omega} - 4e^{(\Delta}{}_{ac}e^{\Omega)c}{}_b\,, \qquad 0 = e^{[\Delta}{}_{ab}e^{\Omega]ac}\,, \\ &0 = e^{\Delta a}{}_{(b}d^I{}_{c)a} + 4e_{\Omega bc}a^{I\Delta\Omega}\,, \qquad 0 = 2e^{\Omega}{}_{ab}c_{(\Delta\Sigma)\Omega} + d^I{}_{ab}a_{I\Delta\Sigma}\,. \end{aligned} \tag{5.6}$$

Consider first the second-to-last of these equations, and fix the free index '$I$' here to correspond to a graviton which does *not* couple to a PM field. It then follows that $e_{\Delta ab} = 0$, i.e. the massless vectors cannot interact with an isolated PM field, in agreement with our previous findings. If on the other hand we have a non-trivial massless-PM spin-2 pair with $\Delta = I$ (in a suitable basis), then this constraint allows one to show that $e_{\Delta ab} \propto \delta_{I\Delta}\delta_{Ia}\delta_{Ib}$. The proportionality constant is fixed by the other constraints: $e_{\Delta ab} = \frac{1}{4}g_I\delta_{I\Delta}\delta_{Ia}\delta_{Ib}$. This establishes that distinct conformal gravity sectors cannot mutually couple through vector particles. This also applies to interactions mediated by loops of spin-1 fields belonging to the same Yang-Mills multiplet as per the constraint $0 = f^a{}_{b(c}e^{\Delta}{}_{d)a}$, using the same reasoning we used previously in the analysis of the vector-graviton system.

## 6 Conclusions

In this paper, we provided a complete classification of the consistent first-order deformations of the free theory describing an arbitrary collection of massless spin-1, massless spin-2 and partially-massless (PM) spin-2 fields in rigid $D$-dimensional (A)dS space. As our sole assumptions we requested the vertices to be parity-even, to contain no more than two derivatives and to respect the isometries of the (A)dS background of the free theory.

As far as interactions among massless and PM spin-2 fields are concerned, our results confirm the classification of ref. [44] with what it called the geometric, non-Abelian coupling (3.32)–(3.33), as well as the non-geometric Abelian coupling (3.36)–(3.37). Under our assumptions on the number of derivatives in the vertices, we also find a unique candidate vertex mixing massless spin-1 and PM spin-2 particles, see (3.25). This vertex is of the Chapline-Manton type, i.e., it is Abelian yet induces a non-linear gauge transformation of the spin-1 fields. It only exists in $D = 4$ dimensions and mimics the minimal gravitational coupling of Maxwell's fields, now for PM spin-2 fields instead of massless spin-2 fields.

In section 4 we analyzed all the consistency conditions of the candidate deformations at second order in perturbation, thereby producing the complete set of quadratic constraints on the structure constants that appear at first order in deformation, see (4.7), (4.9) and (4.11). We considered, in section 5, the most general solution of these constraints under the assumption that each field sector contains no relative healthy/ghostly signs in the kinetic terms, although distinct sectors may do so. The solution is given by multiple, independent copies of $D = 4$ conformal gravity minimally coupled with a Yang-Mills (or possibly Abelian) spin-1 sector. Our findings allow us to rule out the non-geometric vertex, at least under the aforementioned assumptions. We could also generalize the no-go theorem of ref. [58] by showing that distinct massless graviton species cannot mutually interact through the exchange of massless spin-1 (Abelian or Yang-Mills) or PM spin-2 particles.

Finally, in section 5 we also exhibited some solutions to the quadratic constraints (4.7), (4.9) and (4.11), in the general set-up with non-sign-definite internal metrics. We were able to exhibit particular solutions for which all the candidate vertices remain consistent. We speculate that, similarly to what was done for the pure PM spin-2 case studied in [31], these solutions with non-sign-definite internal metrics give rise to full theories, complete at the cubic order, consistent as far as the preservation of number of degrees of freedom is concerned. We hope to be able to report on this point in the near future.

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## A Partially massless spin-2 field equations

The classification of non-trivial cubic vertices requires knowledge of the cohomology of the differential $\delta$. It proves useful to this end to know all the consequences of the free-field equations of motion for our system, since trivial cubic vertices are precisely ones that vanish on these equations. In this appendix we analyze the free PM spin-2 theory, the cases of massless spin-1 and spin-2 being of course very well known.

The equation of motion that derives from the free PM action is

$$\mathcal{E}^{\mu\nu} := \frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta k_{\mu\nu}} = \nabla_\rho \mathcal{F}^{\rho(\mu\nu)} - g^{\mu\nu}\nabla_\rho \mathcal{F}^\rho + \nabla^{(\mu}\mathcal{F}^{\nu)} \,. \tag{A.1}$$

We omit the color index which obviously plays no role here. The Noether identity that follows from the PM gauge symmetry is

$$\nabla_\mu \nabla_\nu \mathcal{E}^{\mu\nu} - \frac{\sigma}{L^2}\,\mathcal{E} = 0 \,, \tag{A.2}$$

with $\mathcal{E} := g_{\mu\nu}\mathcal{E}^{\mu\nu}$.

The PM field strength and its derivatives satisfy several identities in terms of $\mathcal{E}^{\mu\nu}$:

$$\mathcal{F}^\mu = \nabla^\mu k - \nabla_\nu k^{\mu\nu} = -\frac{\sigma L^2}{D-2}\,\nabla_\nu \mathcal{E}^{\mu\nu} \,, \qquad \nabla_\mu \mathcal{F}^\mu = -\frac{1}{D-2}\,\mathcal{E} \,, \tag{A.3}$$

$$\nabla_\rho \mathcal{F}^{\mu\nu\rho} = \frac{2\sigma L^2}{D-2}\,\nabla^{[\mu}\nabla_\rho \mathcal{E}^{\nu]\rho} \,, \qquad \nabla_\rho \mathcal{F}^{\rho\mu\nu} = \mathcal{E}^{\mu\nu} - \frac{g^{\mu\nu}}{D-2}\,\mathcal{E} + \frac{\sigma L^2}{D-2}\,\nabla^\nu \nabla_\rho \mathcal{E}^{\mu\rho} \,, \tag{A.4}$$

$$\Box \mathcal{F}_{\mu\nu\rho} + \frac{(2D-3)\sigma}{L^2}\,\mathcal{F}_{\mu\nu\rho} = 2\nabla_{[\mu}\mathcal{E}_{\nu]\rho} + \frac{2}{D-2}\,g_{\rho[\mu}\nabla_{\nu]}\mathcal{E} \,. \tag{A.5}$$

It follows in particular that the trace and divergences of $\mathcal{F}_{\mu\nu\rho}$ all vanish on the equations of motion.

Another identity related to (A.4) is

$$\Box k_{\mu\nu} - \nabla_\mu \nabla_\nu k - \frac{\sigma}{L^2}(g_{\mu\nu}k - D k_{\mu\nu}) = \mathcal{E}_{\mu\nu} - \frac{g_{\mu\nu}}{D-2}\,\mathcal{E} + \frac{2\sigma L^2}{D-2}\,\nabla_{(\mu}\nabla^\rho \mathcal{E}_{\nu)\rho} \,. \tag{A.6}$$

On the equations of motion, and considering specifically the gauge $k = 0$ (which from (A.3), incidentally, implies also $\nabla_\nu k^{\mu\nu} = 0$), the previous equation reduces to the standard wave form (see e.g. [10]):

$$\left(\Box + \frac{\sigma D}{L^2}\right) k_{\mu\nu} = 0 \qquad\qquad (\mathcal{E}^{\mu\nu} = 0\,,\ k = 0) \,. \tag{A.7}$$



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