Higher spin physics
from quantum gravity to black hole scattering

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FREEDOM TO RESEARCH

Plan/Keywords

- **Massless higher spins:**
  - quantum gravity via higher spin gravity (HiSGRA)
  - 3d bosonization duality in vector models and higher-spins
  - new symmetry: slightly-broken higher spin symmetry
  - towards exact AdS/CFT: Chiral HiSGRA
- **Massive higher spins:**
  - black hole scattering and other EFTs
  - interactions of massive higher spins

Why higher (all) spins?

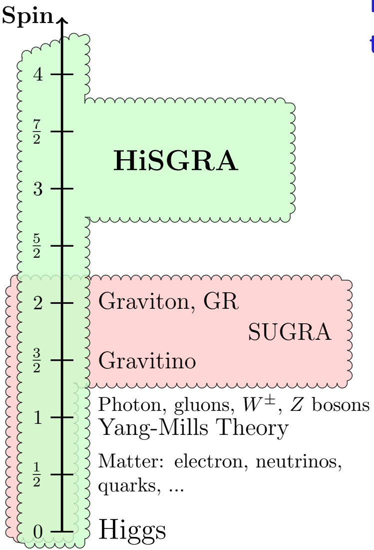
Standard context: quantum field theory, quantum gravity, ...

Standard assumption: to solve the problem without going too far from the well-established concept of particles/fields

Particles: (Wigner, 1939) explained that particles = unitary irreducible representations of the space-time symmetry group (e.g. Poincare). In $4d$ one can "observe" massive or massless particles with spin $s = 0, \frac{1}{2}, 1, \ldots$ or helicity $\lambda = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \boldsymbol{\pm 2}, \ldots$

Questions: what are the options to have interactions? Which ones incorporate gravity? Which ones are quantum consistent? Are all options accessible? ... $\ni$ our world

Spin by spin, where is higher spin?



Different spins lead to very different types of theories/physics:

- $s = 0$: Higgs
- $s = 1/2$: Matter

- $s = 1$: Yang-Mills, Lie algebras
- $s = 3/2$: SUGRA and supergeometry, graviton $\in$ spectrum
- $s = 2$ (graviton): GR and Riemann Geometry, no color (Boulanger et al)

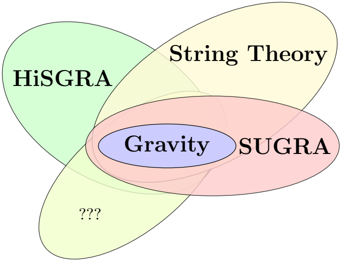
- $s > 2$: HiSGRA and String theory, ∞ states, graviton is there too!

Why massless higher spins?

- string theory
- acausality of higher der. corrections to gravity (Camanho et al.)
- divergences in (SU)GRA's
- quantum Gravity via AdS/CFT

$\to$ quantization of gravity $\to$

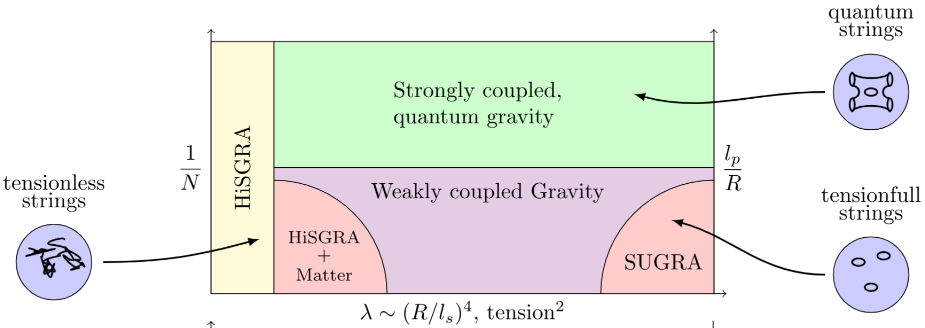
- unbounded spin $\to \infty$ many fields
- UV $\to$ massless

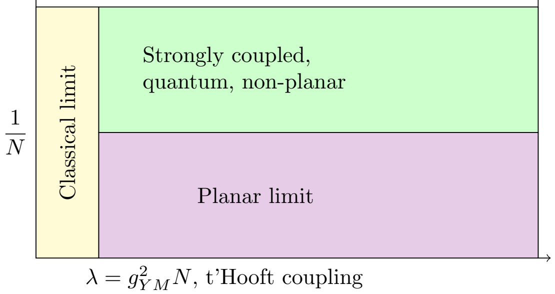


HiSGRA = the smallest extension of gravity by massless, i.e. gauge, higher spin fields. Vast gauge symmetry should render it finite ($\approx$ SUGRA).

Quantizing Gravity via HiSGRA = Classical HiSGRA?

HiSGRA from Tensionless Strings, duals of weakly coupled CFT's





Sundborg;
Klebanov, Polyakov;
Sezgin, Sundell;
Tseytlin;
Beisert, Bianchi, Morales, Samtleben;
Leigh, Petkou

S-matrix constraints from the higher spin symmetry

too small symmetry: nothing can be computed even with a theory
too big symmetry: everything is fixed even without a theory
higher spin symmetry: almost everything is fixed and very few theories

S-matrix summary

We see that **asymptotic higher spin symmetries** (HSS)

$$\delta\Phi_{\mu_1\dots\mu_s}(x) = \nabla_{(\mu_1}\xi_{\mu_2\dots\mu_s)}$$

seem to completely fix (holographic) S-matrix to be

$$S_{\text{HiSGRA}} = \begin{cases} 1^{***}, & \text{flat space, (Weinberg)} \\ \text{free CFT}, & \text{asymptotic AdS, unbroken HSS} \\ \textbf{Chern-Simons Matter}, & \textbf{asymptotic AdS}_4\textbf{, SB HSS} \end{cases}$$

Most interesting applications are to vector models, (Klebanov, Polyakov; Sezgin, Sundell; Maldacena, Zhiboedov; Giombi et al; ...)

Any CFT in $d \geq 3$ with a at least one higher-spin current J_s is a free CFT in disguise (Maldacena, Zhiboedov; Boulanger, Ponomarev, E.S., Taronna; Alba, Diab; Stanev)

HiSGRA that survived

Quantizing Gravity via HiSGRA = Constructing Classical HiSGRA

Therefore, HiSGRA can be good probes of the Quantum Gravity Problem

Local HiSGRA in 2024

3d massless, conformal and partially-massless (Blencowe; Bergshoeff, Stelle; Campoleoni, Fredenhagen, Pfenninger, Theisen; Henneaux, Rey; Gaberdiel, Gopakumar; Grumiller; Grigoriev, Mkrtchyan, E.S.; Pope, Townsend; Fradkin, Linetsky; Lovrekovic; ...), $S = S_{CS}$ for a HS extension of $sl_2 \oplus sl_2$ or $so(3,2)$

$$S = \int \omega\, d\omega + \tfrac{2}{3}\omega^3$$

4d conformal (Tseytlin, Segal; Bekaert, Joung, Mourad; Adamo, Tseytlin; Basile, Grigoriev, E.S.; ...), higher spin extension of Weyl gravity, local Weyl symmetry tames non-localities

$$S = \int \sqrt{g}\, (C_{\mu\nu,\lambda\rho})^2 + ...$$

4d massless chiral (Metsaev; Ponomarev, E.S.; Ponomarev; E.S., Tran, Tsulaia, Sharapov, Van Dongen, ...). The smallest HiSGRA with propagating fields.

IKKT model for fuzzy H_4 (Steinacker, Sperling, Fredenhagen, Tran)

The theories avoid all no-go's, as close to Field Theory as possible

Other ideas and proposals

- **Reconstruction**: invert AdS/CFT
  - Brute force (Bekaert, Erdmenger, Ponomarev, Sleight; Taronna, Sleight)
  - Collective Dipole (Jevicki, Mello Koch et al; Aharony et al)
  - Holographic RG (Leigh et al, Polchinski et al)
- **Formal HiSGRA**: constructing L_∞-extension of HS algebras, i.e. a certain odd Q, $QQ=0$, and write AKSZ sigma model (Barnich, Grigoriev)

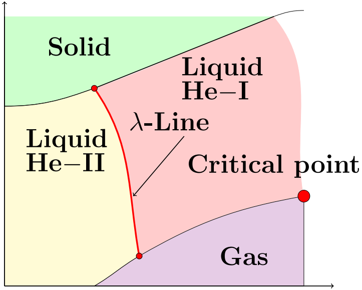
$$d\Phi = Q(\Phi)$$

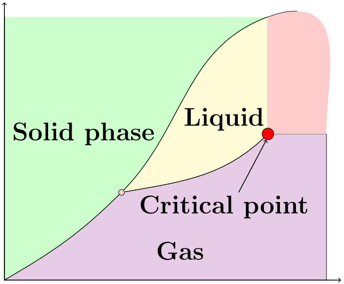
Warning: Boulanger, Kessel, E.S., Taronna

(Vasiliev; E.S., Sharapov, Bekaert, Grigoriev, E.S.; Grigoriev, E.S.; Tran; Bonezzi, Boulanger, Sezgin, Sundell; Neiman) AdS/CFT: (Sundborg, Sezgin, Sundell, Klebanov, Polyakov, Giombi, Yin, ...) Chiral HiSGRA (Sharapov, E.S., Sukhanov, Van Dongen)

Certain things do work, but the general rules are yet to be understood, e.g. non-locality, ... **More:** Snowmass paper, ArXiv: 2205.01567

Chern-Simons Matter Theories and bosonization duality





Chern-Simons Matter theories and dualities

CFT_3: Chern-Simons Matter theories, which span CFTs from vector models to ABJ(M). Let's consider the simplest 4 vector models

$$\frac{k}{4\pi}S_{CS}(A) + \text{Matter}\begin{cases}(D\phi^i)^2 & \textbf{free boson}\\ (D\phi^i)^2 + g(\phi^i\phi^i)^2 & \textbf{Wilson-Fisher (Ising)}\\ \bar{\psi}^i \not{D}\psi_i & \textbf{free fermion}\\ \bar{\psi}^i \not{D}\psi_i + g(\bar{\psi}^i\psi_i)^2 & \textbf{Gross-Neveu}\end{cases}$$

- describe some physics (Ising, quantum Hall, ...)
- break parity in general (due to Chern-Simons)
- two parameters $\lambda = N/k$, $1/N$ (λ continuous for N large)
- exhibit remarkable dualities, e.g. ***3d* bosonization duality** (Aharony, Alday, Bissi, Giombi, Jain, Karch, Maldacena, Minwalla, Prakash, Seiberg, Tong, Witten, Yacobi, Yin, Zhiboedov, ...)

Chern-Simons Matter theories and dualities

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\textbf{free boson} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"'] & \textbf{Gross-Neveu} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"] \\
\textbf{Wilson-Fisher} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] & \textbf{free fermion} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"]
\end{tikzcd}
$$

$3d$ **bosonization: these 4 families/theories are just 2 theories**
(Giombi et al; Maldacena, Zhiboedov; many checks by many people, but no proof)

The simplest gauge-invariant operators are **higher spin currents**:

$$\boldsymbol{J_s = \phi D ... D\phi} \qquad \text{and} \qquad \boldsymbol{J_s = \bar{\psi}\gamma D ... D\psi}$$

which are conserved to the leading order in $1/N$ → higher symmetry

There are many other operators, e.g. $[JJ]$, $[JJJ]$, etc., correlators thereof and anomalous dimensions, all should be the same in the duals

To see bosonization one needs all orders in λ even at large N, so it is a weak/strong duality in a sense

Since everything appears in the OPEs of Js with themselves, it is sufficient to concentrate on **higher spin currents**, i.e. to prove that

$$\langle J_{s_1} J_{s_2} ... J_{s_n} \rangle$$

are the "same" in the dual theories, which is a job for some symmetry ...

What is going on in CS-matter theories?

HS-currents are responsible for their own non-conservation:

$$\partial \cdot J_s = \sum_{s_1,s_2} C_{s,s_1,s_2}(\lambda)\frac{1}{N}[J_{s_1}J_{s_2}] + F(\lambda)\frac{1}{N^2}[JJJ]$$

which is an exact non-perturbative quantum equation. In the large-N we can use classical (representation theory) formulas for $[JJ]$.

The worst case $\partial \cdot J =$ some other operator. The symmetry is gone, the charges are not conserved, do not form Lie algebra.

In our case the non-conservation operator $[JJ]$ is made out of J themselves, but charges are still not conserved.

Slightly-broken higher spin symmetry

Maldacena, Zhiboedov applied the non-conservation equation to study the 3-point functions. The idea is to combine $\partial \cdot J = \frac{1}{\tilde{N}}[JJ]$ with the very constrained form of 3-pt correlators and $[Q, J] = J + \frac{1}{N}[JJ]$ and use large-N. The result is

$$\boldsymbol{\langle J_{s_1} J_{s_2} J_{s_3} \rangle \sim \cos^2\theta \langle JJJ \rangle_b + \sin^2\theta \langle JJJ \rangle_f + \cos\theta \sin\theta \langle JJJ \rangle_o}$$

θ is related to N, k in a complicated way.

The correlators of J_s's get fixed irrespective of what the constituents are! Sign of an ∞-dimensional symmetry ... **What is the right math?**

Slightly-broken higher spin symmetry seems to work (Alday, Zhiboedov, Turiaci, Jain et al, Li, Racobi, Silva and many others!); $\gamma(J_s)$ at order $1/N$ (Giombi, Gurucharan, Kirillin, Prakash, E.S.) confirm the duality.

Higher spin gravity dual?

Remarks on Higher Spin Gravity

AdS/CFT duals of (Chern-Simons) vector models are HiSGRA since conserved tensor J_s is dual to (massless) gauge field in AdS_4 (Sundborg; Klebanov, Polyakov; Sezgin Sundell; Leight, Petkou; Giombi, Yin, ...)

$$\partial^m J_{ma_2\dots a_s} = 0 \qquad \Longleftrightarrow \qquad \delta\Phi_{\mu_1\dots\mu_s} = \nabla_{(\mu_1}\xi_{\mu_2\dots\mu_s)}$$

Instead of tedious quantum calculations in CS-matter one could do the standard holographic computation in the HiSGRA dual, (Giombi, Yin)

However, Vasiliev's equations are incomplete (∞-many free params, non-locality) (Boulanger et al). Independently, this HiSGRA was shown to be too non-local to be constructed by field theory tools (Bekaert, Erdmenger, Ponomarev, Sleight, Taronna). It can be reconstructed (Jevicki et al; Aharony et al) from the very CFT, but no λ. Nevertheless, it has been quite useful to think of HiSGRA dual (Giombi et al)

Chiral HiSGRA

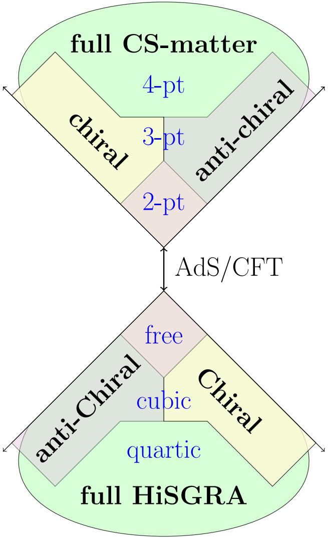
Given J_s, $s = 0, ...\infty$ we are looking for a HiSGRA in AdS_4 ...

There is a unique local HiSGRA for any value of cosmological constant with such a spectrum — Chiral HiSGRA, which was first constructed in the light-cone gauge in flat space (Metsaev; Ponomarev, E.S.). It is a HS-extension of both SDYM and SDGR. It is at least one-loop UV-finite (E.S., Tran, Tsulaia); it is integrable (Ponomarev); covariant equations (Sharapov, E.S., Sukhanov, Van Dongen); **Chiral $\in$ any $4d$ HiSGRA**

$$\boldsymbol{\mathcal{L}} = \sum_{\boldsymbol{\lambda}} \boldsymbol{\Phi}^{-\boldsymbol{\lambda}} \Box \boldsymbol{\Phi}^{+\boldsymbol{\lambda}} + \sum_{\boldsymbol{\lambda_i}} \frac{\boldsymbol{g}\, \boldsymbol{l}_{\mathsf{Pl}}^{\boldsymbol{\lambda_1+\lambda_2+\lambda_3-1}}}{\boldsymbol{\Gamma(\lambda_1+\lambda_2+\lambda_3)}} \boldsymbol{V}^{\boldsymbol{\lambda_1,\lambda_2,\lambda_3}} + \mathcal{O}(\Lambda) \overset{\text{Kontsevich}}{\underset{???}{}}$$

where the three-point amplitudes are

$$\boldsymbol{V}^{\boldsymbol{\lambda_1,\lambda_2,\lambda_3}} \sim \boldsymbol{[12]}^{\boldsymbol{\lambda_1+\lambda_2-\lambda_3}} \boldsymbol{[23]}^{\boldsymbol{\lambda_2+\lambda_3-\lambda_1}} \boldsymbol{[13]}^{\boldsymbol{\lambda_1+\lambda_3-\lambda_2}}$$



yep, there is anti-chiral as well;

The existence of Chiral HiSGRA implies: there are two closed subsectors of Chern-Simons matter theories, maybe to all orders in $1/N$, hence, Ising?

One can define them holographically, but it would be interesting to identify them on the CFT side;

There are two new CFTs!

Inter-holographic proof of the duality

The very existence of Chiral HiSGRA implies $3d$ bosonization duality at least up to the 4-point correlators of J_s (E.S.; E.S., Y.Yin)

Input: (i) chiral and anti-chiral interactions are complete at 3-pt; (ii) (anti)-chiral HiSGRA do not have free params save for coupling g.

$$\boldsymbol{V_3 = g\, V_{chiral} \oplus \bar{g}\, \bar{V}_{chiral} \qquad \leftrightarrow \qquad \langle JJJ \rangle}$$

How to glue (anti)-chiral bricks while imposing unitarity? Simple EM-duality phase rotation $\Phi_{\pm s} \to e^{\pm i\theta}\Phi_{\pm s}$ does the job and we get

$$\boldsymbol{\langle J_{s_1} J_{s_2} J_{s_3} \rangle \sim \cos^2\theta \langle JJJ \rangle_b + \sin^2\theta \langle JJJ \rangle_f + \cos\theta \sin\theta \langle JJJ \rangle_o}$$

which consists of the limiting theories in the helicity basis. **Bosonization is manifest!** Can be pushed to 4-pt to show one-parameter family of CFTs. First prediction from HiSGRA that is ahead of CFT.

Higher spin symmetry and $3d$ bosonization duality

Unbroken Higher spin symmetry

In free theories we have ∞-many conserved $J_s = \phi\partial ...\partial\phi$ tensors.

Free CFT = Associative (higher spin) algebra

Conserved tensor $\rightarrow$ current $\rightarrow$ symmetry $\rightarrow$ invariants=correlators.

$$\partial \cdot J_s = 0 \quad \Longrightarrow \quad Q_s = \int J_s \quad \Longrightarrow \quad [Q,Q] = Q \quad \& \quad [Q,J] = J$$

HS-algebra (free boson) = HS-algebra (free fermion) in $3d$.

Correlators are given by invariants (Sundell, Colombo; Didenko, E.S.; ...)

$$\langle J...J \rangle = \mathrm{Tr}(\Psi \star ... \star \Psi) \qquad\qquad \Psi \leftrightarrow J$$

where Ψ are coherent states representing J in the higher spin algebra

$$\langle JJJJ \rangle_{F.B.} \sim \cos(Q^2_{13} + Q^3_{24} + Q^4_{31} + Q^1_{43}) \cos(P_{12}) \cos(P_{23}) \cos(P_{34}) \cos(P_{41}) + ...$$

Slightly-broken Higher spin symmetry is new Virasoro?

In large-N Chern-Simons vector models (e.g. Ising) higher spin symmetry does not disappear completely (Maldacena, Zhiboedov):

$$\partial \cdot J = \frac{1}{N}[JJ] \qquad\qquad [Q, J] = J + \frac{1}{N}[JJ]$$

What is the right math? We should deform the algebra together with its action on the module, so that the currents can 'backreact':

$$\delta_\xi J = l_2(\xi, J) + l_3(\xi, J, J) + \ldots\,, \qquad\qquad [\delta_{\xi_1}, \delta_{\xi_2}] = \delta_\xi\,,$$

where $\xi = l_2(\xi_1, \xi_2) + l_3(\xi_1, \xi_2, J) + \ldots$. **This leads to L_∞-algebra.**

Correlators = invariants of L_∞-algebra and are unique (Gerasimenko, Sharapov, E.S.), **which proves $3d$ bosonization duality at least in the large-N**. Without having to compute anything one prediction is

$$\langle J \ldots J \rangle = \sum \langle \text{fixed} \rangle_i \times \text{params}$$

Gravitational waves & higher spins

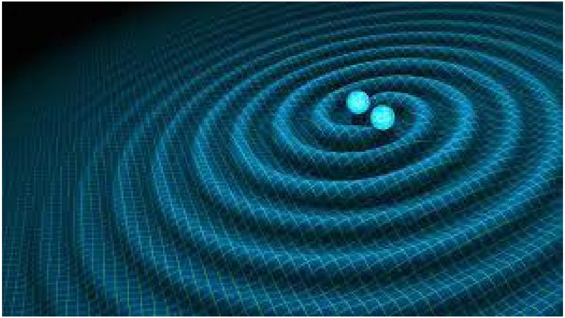
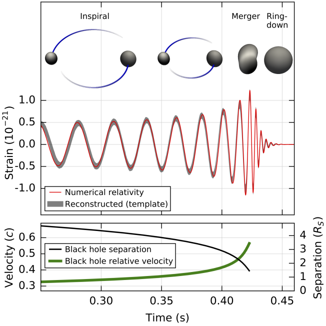


Image credit: R. Hurt/Caltech-JPL

Brief story of Gravitational Waves

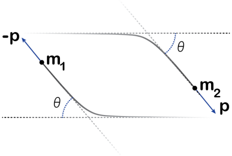
- Prediction 1916 by Einstein
- Evidence from Hulse-Taylor pulsar
- LIGO/Virgo 1st GW in 2016
  just 100 years delay
- GW is a new window to the secrets of the Universe
- Future: LISA, Einstein, ...
- Black-holes, neutron star systems
- Precision calculation are in demand

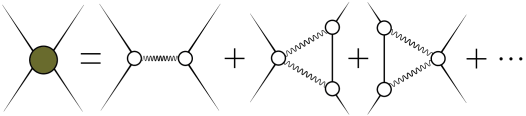


From Einstein to Amplitudes

It is hard to solve Einstein equations for this case (Buonanno, Damour). True numerics is needed only at the very last stage.

Amplitude methods has come into the game recently (Bjerrum-Bohr,...; Bern, Zeng,...; Cristofoli,...; Di Vecchia,...; Guevara,...; Maybee,...; Moynihan,...; Parra-Martinez,...; Plefka,...; Vines,...;...)





Everything compact and rotating from a distance looks like (and can be modeled as) a higher-spin massive particle

Kerr black hole is 'very simple' (Arkani-Hamed, Huang^2)

Massive Higher Spins

(massive non-elementary particles definitely exist,
but there is much less literature about them.
No no-go's, no challenge?)

Massive higher spins?

Where? Baryons known with $S \leq 15/2$; nuclei, say, Tantalum has $S = 9$; string's spectrum is full of higher spins ... now Black holes

Massive higher spins are complicated: 2nd-class constraints, Boulware-Deser ghosts, even free actions are not easy (Fierz, Pauli; Singh, Hagen; Zinoviev)

$$(\Box - m^2)\Phi_{\mu_1\dots\mu_s} = 0 \qquad\qquad \boldsymbol{\partial^\nu \Phi_{\nu\mu_2\dots\mu_s} = 0}$$

Low spins: $\boldsymbol{s = 1}$ **spontaneously broken Yang-Mills**; $\boldsymbol{s = 3/2}$; $s = 2$ massive (bi)-gravity (dRGT; Hassan, Rosen); $\boldsymbol{s = 5/2}$ (Chiodaroli, Johansson, Pichini)

Interactions:

- Zinoviev's massive gauge symmetry;
- Chiral approach (Ochirov, E.S.);
- Light-cone (Metsaev); Covariant (Buchbinder, ...)

Black holes vs. Amplitudes

Even at the cubic level there are many $s - s - h^{\pm}$ amplitudes.
(Arkani-Hamed, Huang[2]) selected a family with the best high energy behavior

$$\mathcal{A}(s,s,h^{+}) \sim \mathcal{A}(0,0,h^{+})\langle\mathbf{12}\rangle^{2s}$$
$$\mathcal{A}(s,s,h^{-}) \sim \mathcal{A}(0,0,h^{-})[\mathbf{12}]^{2s}$$

where $\mathcal{A}(0,0,h^{\pm})$ is the scalar amplitude for $h = \pm 1, \pm 2$. For $s = 0, \frac{1}{2}, 1$ these are as in "Standard model".

(Guevara, Ochirov, Vines) showed that these amplitude reproduce the classical behavior of a rotating Kerr black hole for $h = \pm 2$.

The dictionary is "roughly" $s \to \infty$ for $a = s\hbar$, $\hbar \to 0$.

Double-copy construction (Bern, Carrasco, Johansson) allows us to think of $\sqrt{\text{Kerr}}$, where $h = \pm 1$.

Chiral approach

The problem is to embed physical degrees of freedom into a Lorentz covariant field

$$\text{degrees of freedom} \rightsquigarrow \text{Lorentz (spin-)tensor}\,,\, \Phi_{A_1\ldots A_n,A'_1\ldots A'_m}$$

Any $\Phi_{A(n),A'(m)}$ of $sl(2,\mathbb{C})$ with $n+m=2s$ is good enough and the canonical choice has always been

$$\Phi_{\mu_1\ldots\mu_s} \sim \Phi_{A_1\ldots A_s,A'_1\ldots A'_s} \sim (s,s)$$

Simple idea in $4d$ (Ochirov, E.S.): let's take

$$\Phi_{A_1\ldots A_{2s}} \sim (2s,0)$$

It does not have any longitudinal unphysical modes, just $(2s+1)$.

No auxiliary fields are needed. Parity is not easy ... but everything is a consistent interaction

Chiral approach: Massive low spins

Chalmers and Siegel, '97-98, mapped "Standard model" to its chiral version, i.e. "chiralized" massive $s = 1/2, 1$

For $s = 1/2$ we just integrate out half of the Majorana spinor $\psi_A\,, \psi_{A'}$:

$$\mathcal{L}_M = i\psi^A \nabla_{AA'} \psi^{A'} + \tfrac{1}{2} m(\psi_A \psi^A + \psi_{A'} \psi^{A'})$$

to get some **nonminimal interactions** as well

$$\mathcal{L} = \psi^A (\Box - m^2)\psi_A + \psi^A F_{AB} \psi^B + R\psi^A \psi_A$$

where R is the Ricci scalar and F_{AB} is the self-dual part of the (non)-abelian gauge background field

$$F_{\mu\nu} = F_{AB}\epsilon_{A'B'} + \epsilon_{AB} F_{A'B'} = F_- + F_+$$

Wanted: Black hole Lagragian,

$$\mathcal{L}_{\text{BH}} = \langle\Phi|(\Box - m^2)|\Phi\rangle + \mathcal{L}_{\text{non-min}}$$

i.e. the simplest parity-invariant theory that couples HS to photons/gluons ($\sqrt{\textbf{Kerr}}$) or to gravitons (**Kerr**) that starts with (Arkani-Hamed, Huang^2; Guevara, Ochirov, Vines; Chung, Huang, Kim, Lee)

$$\mathcal{A}(s,s,+) \sim \langle\mathbf{1}\,\mathbf{2}\rangle^{2s}\mathcal{A}(0,0,+) \qquad \mathcal{A}(s,s,-) \sim [\mathbf{1}\,\mathbf{2}]^{2s}\mathcal{A}(0,0,-)$$

BCFW does not work (Arkani-Hamed, Huang^2; Johansson, Ochirov)

Let's use chiral approach (Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, E.S.)

How far away are we?

Wanted: Black hole Lagragian,

$$\mathcal{L}_{\textsf{BH}} = \langle\Phi|(\Box - m^2)|\Phi\rangle + \mathcal{L}_{\textsf{non-min}}$$

that starts with (Arkani-Hamed, Huang[2]; Guevara, Ochirov, Vines; Chung, Huang, Kim, Lee)

$$\mathcal{A}(s,s,+) \sim \langle\mathbf{1}\,\mathbf{2}\rangle^{2s}\mathcal{A}(0,0,+) \qquad \mathcal{A}(s,s,-) \sim [\mathbf{1}\,\mathbf{2}]^{2s}\mathcal{A}(0,0,-)$$

The minimal coupling correctly reproduces all-plus amplitudes obtained from unitarity (Aoude, Haddad, Helset; Lazopoulos, Ochirov, Shi):

$$\mathcal{A}(s,s,+,\ldots,+) \sim \langle\mathbf{1}\,\mathbf{2}\rangle^{2s}\mathcal{A}(0,0,+,\ldots,+)$$

Black hole Lagrangian

Wanted: Black hole Lagragian,

$$\mathcal{L}_{\text{BH}} = \langle \Phi | (\Box - m^2) | \Phi \rangle + \mathcal{L}_{\text{non-min}}$$

that starts with (Arkani-Hamed, Huang^2; Guevara, Ochirov, Vines; Chung, Huang, Kim, Lee)

$$\mathcal{A}(s,s,+) \sim \langle \mathbf{1}\,\mathbf{2} \rangle^{2s} \mathcal{A}(0,0,+) \qquad \mathcal{A}(s,s,-) \sim [\mathbf{1}\,\mathbf{2}]^{2s} \mathcal{A}(0,0,-)$$

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$$\mathcal{A}(s,s,+,\ldots,+) \sim \langle \mathbf{1}\,\mathbf{2} \rangle^{2s} \mathcal{A}(0,0,+,\ldots,+)$$

To restore parity at the cubic level we need

$$\mathcal{L}_{\text{non-min}} \sim \sum_{k=0}^{2s-1} \frac{1 \text{ or } (2s-k-1)}{m^{2k}} \langle \Phi | (\overleftarrow{D}\overrightarrow{D})^k \, F_- \text{ or } R_- | \Phi \rangle$$

Black hole Lagrangian

Usually, analysis at the n-th order leaves some coefs free that can be fixed at higher orders. Therefore, at the quartic order we have some ambiguities to be fixed in some way.

We used some additional assumptions, e.g. parity, unitarity, classical limit $s \to \infty$, massive higher-spin gauge symmetry, certain already known low order terms, ... to land on a Compton amplitude $A(\mathbf{1}^s, \bar{\mathbf{2}}^s, 3^-, 4^+)$

Inverting the statement, this also gives us a theory of a single massive higher-spin field interacting with photons/gluons up to the quartic order.

The Lagrangian up to quartic is known in the chiral approach (Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, E.S.)

Massless higher spin summary/comments

- Some HiSGRA do exist as local field theories, e.g. Chiral HSGRA — toy model with stringy features, no UV divergences thanks to higher spin symmetry? Also, celestial studies and flat holography: (Monteiro; Ren, Spradlin, Yelleshpur Srikant, Volovich; Ponomarev)
- Exact models of holography? Tensionless strings on $AdS_4 \times \mathbb{CP}_3$?
- Closed subsectors in Chern-Simons vector models, also small N?
- L_∞ as a new physical symmetry in vector models to explain $3d$ bosonization duality at least for large-N. It suggests an extension of deformation quantization and formality
- Relation to twistors and integrability: (Ponomarev; Adamo, Tran; Tran; Krasnov, E.S., Tran; Herfray, Krasnov, E.S.)

Massive higher spin summary/comments

- Precision calculation in GR are in demand
- An open challenge to construct theories with massive higher spins, in particular, the minimal one that describes black hole scattering
- Chiral approach facilitates introduction of interactions $\rightarrow$ four-point Compton amplitude
- Massive gauge symmetry is also helpful (Zinoviev; Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, E.S.; E.S., Tsulaia)
- Eventually all spins and masses should be needed to incorporate spin/mass-changing processes, similar to string theory

Want to read more? Snowmass paper: Higher Spin Gravity and Higher Spin Symmetry, Arxiv: 2205.01567

Thank you for your attention!

may the higher spin force be with you!

... backup slides ...

Zinoviev's approach

Instead of the tedious Hamiltonian analysis of (second class) constraints to make sure that interactions of $\Phi_{\mu_1 \ldots \mu_s}$ do not activate unphysical degrees of freedom, one can convert all second class to first class. The latter can be taken care of by gauge symmetries:

$$\text{massive spin} - s = \bigoplus_{k=0}^{k=s} \text{massless spin} - k$$

which leads to

$$\delta\Phi_k = \partial\xi_{k-1} + \xi_k + g\xi_{k-2} + \mathcal{O}(\Phi\xi)$$

For the steps along the Zinoviev approach see 'Kerr Black Holes Enjoy Massive Higher-Spin Gauge Symmetry', Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, et al; arxiv: 2212.06120 & 2308.xxxx and the talk by Lucile Cangemi

Example of spin-one

We can get an EM interaction of massive spin-one via Higgs

$$\mathcal{L} = -\tfrac{1}{2}|D_{[\mu}W_{\nu]}|^2 + |mW_\mu|^2 - iQ\boldsymbol{\varnothing W_\mu F^{\mu\nu} W_\nu} + \dots$$

for $SO(3)$, where one boson A_μ remains massless, $D = \partial - iQA$.

Instead we can start from the gauge invariant free action

$$\mathcal{L}_2 = -\frac{1}{2}|\partial_{[\mu}W_{\nu]}|^2 + |mW_\mu - \partial_\mu\varphi|^2\,,$$

that is invariant under $\delta W_\mu = \partial_\mu\xi$, $\delta\phi = m\,\xi$ (Stueckelberg).

Upon $\partial \to D$ we find that the gauge symmetry is lost, but it gets restored by adding the **non-minimal term**. $\delta W_\mu = D_\mu\xi$, $\delta\phi = m\xi$. This gives $\mathcal{A}(1,1,h)$!

Black hole interactions in Zinoviev's approach

We need to write down the most general ansatz for interactions and gauge transformations ... lots of coefficients

An equivalent approach is to use Ward identities for the massive higher-spin gauge symmetry ... which is closer to amplitudes

There is a unique solution with the lowest number of derivatives extending the "minimal coupling" $\partial \to \partial - iQA$

There is a simple generating function for on-shell cubic vertices, e.g.

$$\tilde{\mathcal{L}}^{(1)} = -\,\bar{\Phi}_\mu F^{\mu\nu}\Phi_\nu\,,$$
$$\tilde{\mathcal{L}}^{(2)} = \bar{\Phi}_{\mu\nu}F^{\nu}{}_{\rho}\Phi^{\rho\mu} + \frac{4}{m^2}\overline{D_{[\mu}\Phi_{\nu]\rho}}F^{\rho\sigma}D^{[\mu}\Phi^{\nu]}{}_{\sigma} + \frac{4}{m^2}\overline{D_{[\mu}\Phi_{\nu]\rho}}F^{\nu}{}_{\sigma}D^{[\sigma}\Phi^{\mu]\rho}\,,$$

The quartic analysis is more complicated ...

Unbroken higher spin symmetry: Higher spin algebra

Indeed, $\Box\Phi = 0$ is $so(d,2)$-invariant:

$$\delta_v \Phi = v^m \partial_m \Phi + \frac{d-2}{2d} (\partial_m v^m) \Phi$$

The latter means that $\Box \delta_v \Phi = L_v \Box \Phi = 0$ for some L_v, i.e. solutions are mapped to solutions. We can multiply such symmetries

$$\delta\Phi = \delta_{v_1} ... \delta_{v_n} \Phi$$

For example, we find hyper-translations

$$\delta\Phi = \epsilon^{a_1 \ldots a_k} \partial_{a_1} ... \partial_{a_k} \Phi$$

As a result the Lie bracket $[Q,Q]$ originates from some associative algebra, higher spin algebra, $\mathfrak{hs}$ via $a \star b - b \star a$.

Strong homotopy algebras

Strong homotopy algebra is a graded space, e.g. $V = V_{-1} \oplus V_0$ equipped with multilinear maps $l_k(x_1, ..., x_k)$ of degree-one. In our case

$$l_k(\xi, \xi, J, ..., J) \qquad\qquad l_k(\xi, J, ..., J)$$

that allow us to encode the deformed action

$$\delta_\xi J = l_2(\xi, J) + l_3(\xi, J, J) + ... , \qquad\qquad [\delta_{\xi_1}, \delta_{\xi_2}] = \delta_\xi ,$$

where $\xi = l_2(\xi_1, \xi_2) + l_3(\xi_1, \xi_2, J) + ...$ The maps obey 'Jacobi' relations

$$\sum_{i+j=n} (\pm) l_i(l_j(x_{\sigma_1}, ..., x_{\sigma_j}), x_{\sigma_{i+1}}, ..., x_{\sigma_n}) = 0$$

L_∞ **originates from** A_∞ **constructed from a certain deformation of** $\mathfrak{hs}$**, which is related to para-statistics/fuzzy sphere** (Sharapov, E.S.)

We need to construct L_∞ that 'deforms' our initial data = algebra + module, both originating from an associative algebra $A = \mathfrak{hs} \rtimes \mathbb{Z}_2$.

One can show (Sharapov, E.S.) that such L_∞ can be constructed as long as A is soft, i.e. can be deformed as an associative algebra:

$$a \circ (b \circ c) = (a \circ b) \circ c \qquad\qquad a \circ b = a \star b + \sum_{k=1} \phi_k(a,b)\, \hbar^k$$

The maps can be obtained from an auxiliary A_∞

$$\begin{aligned} m_3(a,b,u) &= \phi_1(a,b) \star u \quad \rightarrow \quad l_3 \\ m_4(a,b,u,v) &= \phi_2(a,b) \star u \star v + \phi_1(\phi_1(a,b),u) \star v \quad \rightarrow \quad l_4 \end{aligned}$$

Our algebra can be deformed thanks to para-statistics/anyons ...

Deformations of Poisson Orbifold: Weyl Algebra

Everyone knows that the Weyl algebra A_1 is rigid

$$[q,p]=i\hbar \qquad\qquad \text{no deformation of} \qquad\qquad f(q,p)\star g(q,p)$$

Suppose that $\boldsymbol{R}f(q,p)=f(-q,-p)$, i.e. we can realize it as

$$\boldsymbol{R}^2=1 \qquad\qquad \boldsymbol{R}q\boldsymbol{R}=-q \qquad\qquad \boldsymbol{R}p\boldsymbol{R}=-p$$

The crossed-product algebra $A_1\ltimes\mathbb{Z}_2$ is soft (Wigner; Yang; Mukunda; ...):

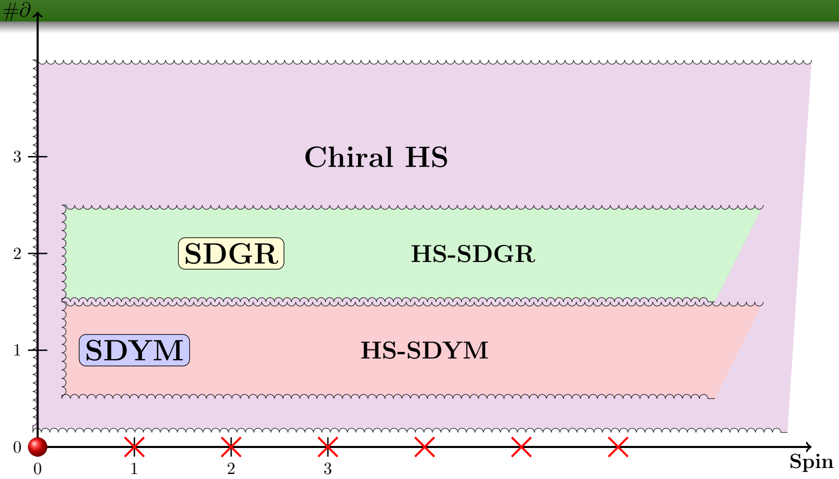
$$[q,p]=i\hbar+i\nu\boldsymbol{R}$$

Also known as para-bose oscillators. Even $\boldsymbol{R}(f)=f$ lead to $gl_\lambda=U(sp_2)/(C_2-\lambda(\lambda-1))$ (Feigin), also (Madore; Bieliavsky et al) as fuzzy-sphere, NC hyperboloid, also (Plyushchay et al) as anyons.

Orbifold $\mathbb{R}^2/\mathbb{Z}_2$ admits 'second' quantization on top of the Moyal-Weyl $\star$-product, (Pope et al; Joung, Mrtchyan; Korybut; Basile et al; Sharapov, E.S., Sukhanov)

Chiral Higher Spin Gravity

Russian doll of theories



Self-dual Yang-Mills is a useful analogy

- the theory is non-unitary due to the interactions $(\boldsymbol{A}_\mu \rightarrow \boldsymbol{\Phi}^\pm)$

$$\mathcal{L}_{\text{YM}} = \mathbf{tr}\, \boldsymbol{F}_{\mu\nu} \boldsymbol{F}^{\mu\nu}$$

$$\wr\wr$$

$$\mathcal{L}_{\text{(SD)YM}} = \boldsymbol{\Phi}^- \Box \boldsymbol{\Phi}^+ + \boldsymbol{V}^{++-} + \boldsymbol{V}^{--+} + \boldsymbol{V}^{++--}$$

- tree-level amplitudes vanish, $A_{\text{tree}} = 0$
- one-loop amplitudes coincide with $(+ + ... +)$ of QCD
- **SD theories are consistent truncations**, so anything we can compute will be a legitimate observable in the full theory
- integrability, instantons, twistors, ...

Chiral HSGRA in Minkowski

Chiral HiSGRA (Metsaev; Ponomarev, E.S.) is a 'higher spin extension' of SDYM/SDGR. It has fields of all spins $s = 0, 1, 2, 3, \ldots$:

$$\mathcal{L} = \sum_{\lambda} \Phi^{-\lambda} \Box \Phi^{+\lambda} + \sum_{\lambda_i} \frac{\kappa \, l_{\mathrm{Pl}}^{\lambda_1+\lambda_2+\lambda_3-1}}{\Gamma(\lambda_1+\lambda_2+\lambda_3)} V^{\lambda_1,\lambda_2,\lambda_3}$$

light-cone gauge is very close to the spinor-helicity language

$$V^{\lambda_1,\lambda_2,\lambda_3} \sim [12]^{\lambda_1+\lambda_2-\lambda_3} [23]^{\lambda_2+\lambda_3-\lambda_1} [13]^{\lambda_1+\lambda_3-\lambda_2}$$

Locality + Lorentz invariance + genuine higher spin interaction result in a unique completion

This is the smallest higher spin theory and it is unique.
Graviton and scalar field belong to the same multiplet

No UV Divergences! One-loop finiteness

Tree amplitudes vanish. The interactions are naively non-renormalizable, the higher the spin the more derivatives:

$$\boldsymbol{V^{\lambda_1,\lambda_2,\lambda_3} \sim \partial^{|\lambda_1+\lambda_2+\lambda_3|}\Phi^3}$$

but there are **no UV divergences!** (E.S., Tsulaia, Tran). Some loop momenta eventually factor out, just as in $\mathcal{N}=4$ SYM, but ∞-many times.

At one loop we find three factors: (1) SDYM or all-plus 1-loop QCD; (2) higher spin dressing to account for λ_i; (3) total number of d.o.f.:

$$\boldsymbol{A}^{\text{1-loop}}_{\text{Chiral}} = \boldsymbol{A}^{++\ldots+}_{\text{QCD,1-loop}} \times \boldsymbol{D}^{\text{HSG}}_{\boldsymbol{\lambda_1,\ldots,\lambda_n}} \times \sum_{\lambda} 1$$

d.o.f.$=\sum_\lambda 1 = 1 + 2\sum_{\lambda>0} 1 = 1 + 2\zeta(0) = 0$ to comply with no-go's, (Beccaria, Tseytlin) and agrees with many results in AdS, where $\neq 0$

Russian doll of theories