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A note on partially massless supergravity

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Abstract: We classify all the possible parity-invariant, non-Abelian couplings involving a partially massless spin-2 field and a pair of massless spin-3/2 fields around anti-de Sitter spacetime AdS_4. We re-derive a non-Abelian vertex recently found in [48] by Yu. M. Zinoviev following a different approach, and make explicit the structure of the non-Abelian gauge algebra, very similar to $\mathcal{N} = 2$ supergravity around AdS_4. We show that this vertex is obstructed at higher order in deformation, and that the graviton plus a vector gauge field are required to eliminate most, if not all, of the obstructions. The resulting gauge algebra produces all the structures of the super algebra $su(2, 2|1)$, suggesting that the only consistent theory that couples a partially massless spin-2 field with massless gravitini is $\mathcal{N} = 1$ conformal supergravity expanded around AdS_4.

Keywords: BRST Quantization, de Sitter space, Gauge Symmetry, Supergravity Models

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1 Introduction

The partially massless (PM) spin-2 field [1–7] is of special interest in the cosmological context, since it propagates and is unitary only on the de Sitter (dS_4) background relevant to the inflationary epoch. Since the mass of the PM spin-2 field saturates the Higuchi bound [3, 7], the PM spin-2 field, if it existed in our early Universe, would have behaved like a light field during inflation, therefore possibly leaving an imprint on the cosmic microwave background [8–10]. The mass of the PM spin-2 field is protected by gauge invariance and is uniquely fixed in terms of the cosmological constant. In fact, the present epoch of our Universe can also be approximated by a de Sitter phase, with its accelerated expansion [11, 12]. Due to the smallness of the observed positive cosmological constant [13], the mass for the PM spin-2 field is below the upper-bound on the mass of the graviton deduced from the gravitational waves detections [14–16]. We will not consider any cosmological application of the PM spin-2 field in the present paper that will remain at a purely theoretical level. Still, we hope that our study could be relevant for cosmology, in the future.

Although the partially massless spin-2 field propagates on (anti-) de Sitter (A)dS$_4$ background and not on the Minkowskian background,[1] its Stueckelberg off-shell description admits a flat limit [18–21] that shows that a partially massless spin-2 field propagates the modes of helicity ± 2 for a massless graviton, plus the modes of helicity ± 1 for a massless vector field, in a way that provides an alternative to the original Hamiltonian analysis of [6]. We stress that, in the present paper we will relax the unitarity constraint and consider the PM spin-2 field around the AdS$_4$ background. Nevertheless, whenever we consider massive spin-3/2 fields, we also consider the dS$_4$ background where the PM spin-2 field is unitary and where possible cosmological considerations could be made in the future.

There have been many studies on the self-couplings of a single PM spin-2 field which have resulted in a series of no-go theorems [20, 22–25]. Note that these no-go results are overcome by a non-unitary interacting theory for multiple PM spin-2 fields [26]. However, there have been relatively few analyses of the couplings of the PM spin-2 field with other types of fields, with the notable exceptions of [27–30] where the gravitational coupling of the PM spin-2 field was investigated, together with the interactions including massless vectors [31]. In the case of the gravitational coupling of the PM spin-2 field, the latter work strengthens the no-go results of [30] and precludes the possibility that the non-geometrical coupling considered in [30] could be pushed to higher orders in interactions. It was also shown in [31] that the vectors couple to the massless and partially massless spin-2 fields in a way that reproduces several copies of conformal gravity coupled to Yang-Mills sectors.

In a previous work [32], we considered the interactions of the partially massless spin-2 field with a massive spin-3/2 field, that together form an on-shell spectrum of particles with helicities $(\pm 2, \pm 1, \pm 3/2, \pm 1/2)$, thereby (naively) offering of possibility for supersymmetry. Two consistent couplings were discovered in [32], giving a hope that it might indeed be possible to find a locally supersymmetric theory with a PM spin-2 field in the spectrum. One of the couplings in [32] is very similar to the minimal coupling of a massive spin-3/2 field to gravity, except that now the graviton is partially massless. Actually, this spectrum of fields is too small to carry the action of rigid supersymmetry, as it was shown in [33] that the shortest partially massless supermultiplet necessitates the adjunction of a pair of massless fields with helicities $(\pm 3/2, \pm 1)$ to the PM spin-2 and the massive spin-3/2 fields in AdS$_4$. A natural question that we address in this article is the possibility for couplings among the four types of fields that constitute the shortest partially massless supermultiplet of [33]. In particular, in this work we investigate whether interactions among those fields can make local the global rigid supersymmetry carried by that multiplet. As we show in this article, the result is negative, unless one adds *two* extra gauge fields to the spectrum: the massless graviton plus another massless gravitino. Using the analyses of [34, 35], we find that the resulting spectrum of fields exactly coincides with the spectrum of $\mathcal{N} = 1$ pure conformal supergravity around AdS$_4$ [36, 37].

Similarly to the observation that conformal gravity is the only consistent way to couple the PM spin-2 field to gravity [29–31], it is therefore natural to expect that conformal supergravity

[1] See ref. [17] for other backgrounds on which a PM spin-2 field can propagate.

(see [34] for a review) should give the only non-Abelian interactions of a PM spin-2 field with massless spin-3/2 fields around AdS$_4$. The spectrum that consists of a PM spin-2 field and two massless gravitini is exactly the same as the one of pure $\mathcal{N} = 2$ supergravity around AdS$_4$, supporting the possibility that there might be consistent non-Abelian interactions among those fields, viewed as a sub-sector of conformal supergravity.

To summarize, the goal of this paper is twofold: we investigate whether it is possible to make local the global (i.e., rigid) supersymmetry of [33], and we want to determine the consistent couplings between a PM spin-2 field and a doublet of massless spin-3/2 fields around AdS$_4$. We present arguments indicating that pure $\mathcal{N} = 1$ conformal supergravity solves both problems simultaneously, thereby providing the supersymmetric extension of the results in [29–31].

More in detail, in this work we first consider a free theory in AdS$_4$ containing a partially massless spin-2 field, a massive spin-3/2 field, a massless spin-3/2 field and a massless spin-1 field which enjoys a rigid $\mathcal{N} = 1$ supersymmetry [33]. The massive sector of this multiplet was already studied in [32] using the Becchi-Rouet-Stora-Tyutin-Batalin-Vilkovisky-Stueckelberg [38–43] (BRST-BV-Stueckelberg) method developed in [44]. The work [32] led to the discovery of interaction vertices that do not vanish in the unitary gauge. Here we pursue this analysis, adding the massless spin-$(1, 3/2)$ sector, and investigate whether it is possible to make local the rigid (i.e. global) supersymmetry of [33] using the BRST-BV-Stueckelberg deformation method.

Since two spin-3/2 fields are present in the spectrum, we start with the deformations of the gauge algebra corresponding to the $\mathcal{N} = 2$ AdS$_4$ superalgebra, taking inspiration from the study of $\mathcal{N} = 2$ supergravity performed in [45]. We show that there is no non-Abelian deformation of the rigid susy algebra of [33]. At best, we are able to exhibit two Abelian couplings between the spin-3/2 sector — where we recall there is one massless and one massive field with adapted mass — and the vector gauge field, see (3.31) below. These vertices deform the gauge transformations but leave the gauge algebra Abelian. They are analogous to couplings present in $\mathcal{N} = 2$ sugra and exist only for a negative cosmological constant, i.e., in AdS$_4$. We also found a vertex on the de Sitter (dS$_4$) background, that couples a pair of real, massive spin-3/2 fields of equal mass with a vector field, see (3.37). These two massive spin-3/2 field form a charged field and the coupling to electromagnetism is the minimal one, also present in $\mathcal{N} = 2$ sugra around AdS$_4$, but in our case, the cosmological constant can be positive at the first order in interactions.

Then, we add an extra massless gravitino to the spectrum, and perform a systematic classification of the possible non-Abelian deformations of the free theory that lead to a deformation of the Lagrangian. We find that there exists a non-Abelian coupling between the PM spin-2 field and two massless gravitini in AdS$_4$. For this part of the work, we use the cohomological method of [46, 47], well-designed for gauge fields. The non-Abelian vertex we find has recently been obtained by Yu. M. Zinoviev [48] in a different formalism. We show that the gauge algebra underlying this vertex closely resembles the one of $\mathcal{N} = 2$ pure supergravity, where the diffeomorphism vector is replaced by the gradient of the PM spin-2 gauge parameter. In fact, with this relation between the translation generator and the generator of PM spin-2 transformation, we recover all the structures constants of the $\mathcal{N} = 2$ supersymmetry algebra

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on AdS$_4$, with the exception of one structure that is identically zero when the diffeomorphism vector is a gradient of a scalar. The latter term is responsible for the transformation law of the supercharges under local Lorentz transformations. Correspondingly, the non-Abelian cubic vertex that we found is very similar to the one of $\mathcal{N}=2$ pure supergravity around AdS$_4$, as a comparison with the analysis of [45] shows.

The plan of the paper is as follows. In the next section 2 we spell out our conventions and notation. We also give the free action principles for the PM spin-2 field, the massless and massive spin-3/2 fields around (A)dS$_4$, as well as the vector gauge field. In section 3, we compute the consistent interactions among the fields of the spectrum given above, using the BRST-BV-Stueckelberg method proposed in [44] and further developed in [32]. In section 4, we use the cohomological method of [46, 47] to classify all the possible non-Abelian deformation of the free theory propagating a PM spin-2 field and two massless gravitini around the AdS$_4$ background. We present the unique non-Abelian vertex that emerges from the classification and, using the results of [45], show the close relation between the corresponding gauge-algebra deformation and the gauge algebra of $\mathcal{N}=2$ pure supergravity around AdS$_4$. In section 5 we show the obstruction of the non-Abelian vertex found in the previous section at the level of the Jacobi identity, keeping the spectrum of fields unchanged. We then add a massless spin-2 field to the spectrum and show that all but one family of obstructions to the Jacobi identity can be removed thanks to the diffeomorphism algebra deformation brought in by the massless spin-2 field [49]. The theory that emerges is $\mathcal{N}=1$ conformal supergravity, here viewed as the only consistent non-Abelian theory that couples a partially massless spin-2 field to massless and massive spin-3/2 fields around AdS$_4$, thereby resolving at the same time the issue of the gauging of the rigid supersymmetry carried by the PM multiplet of [33] and the issue of the consistent interactions between a PM spin-2 field and a pair of massless spin-3/2 fields. Finally, we present our conclusions in section 6.

2 Action principles for free fields in (A)dS

In this section, we give our conventions for the (A)dS$_4$ Lorentz-covariant derivatives, and spell out the action principles for the various fields under consideration.

In our conventions and notation, $\bar{g}_{\mu\nu}$ denote the metric components of the (A)dS$_4$ background spacetime with cosmological constant

$$\Lambda = -3\sigma\lambda^2\,, \quad \sigma = \pm 1\,, \tag{2.1}$$

where λ is the inverse radius of the background and the parameter $\sigma=+1$ in the AdS$_4$ spacetime, $\sigma=-1$ in the dS$_4$ spacetime. On these two backgrounds that we consider in the present article, the components of the Riemann tensor read

$$R_{\mu\nu\rho\sigma} = -2\sigma\lambda^2\bar{g}_{\rho[\mu}\bar{g}_{\nu]\sigma}\,, \tag{2.2}$$

where we use the strength-one (anti)symmetrisation convention with (square) round brackets around the corresponding indices to be (anti)symmetrised. For example, $F_{\mu\nu} = 2\,\nabla_{[\mu}A_{\nu]} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ and $2\,\nabla_{(\mu}\xi_{\nu)} = \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu\,.$

Consequently, on a (co)vector and on a spinor, the commutator of (A)dS$_4$ Lorentz-covariant derivatives with the Levi-Civita connection is given by

$$\begin{aligned}[\nabla_\mu, \nabla_\nu]V_\rho &= -2\sigma\lambda^2\, \bar{g}_{\rho[\mu}V_{\nu]}\ , \\ [\nabla_\mu, \nabla_\nu]\chi_A &= -\frac{\sigma}{2}\,\lambda^2(\gamma_{\mu\nu})_A{}^B\chi_B\ ,\end{aligned} \tag{2.3}$$

where we indicated spinor indices with capital Latin indices. In the following, these will be omitted most of the time. The four Dirac gamma matrices are denoted $\{\gamma_a\}_{a=0,1,2,3}$, and $\gamma_\mu = \bar{e}_\mu{}^a\,\gamma_a$, $\gamma_{\mu\nu} = \gamma_{[\mu}\gamma_{\nu]}$, with $\bar{e}_\mu{}^a$ the components of the (A)dS$_4$ background vierbeins.

2.1 Partially massless spin-2 field

We consider a symmetric PM spin-2 field in the Stueckelberg formulation. Following the lines of [50], we allow both signs of the cosmological constant, which makes the AdS$_4$ case explicitly non-unitary at the classical level:

$$\begin{aligned}L_{\text{PM-St}} = &-\frac{1}{2}\nabla_\rho h_{\mu\nu}\nabla^\rho h^{\mu\nu} + \nabla_\rho h^{\mu\nu}\nabla_\mu h^\rho{}_\nu - \nabla_\mu h\nabla_\nu h^{\mu\nu} + \frac{1}{2}\nabla_\mu h\nabla^\mu h + \frac{\sigma}{4}F_{\mu\nu}F^{\mu\nu} \\ &- 2\sigma\lambda^2 h_{\mu\nu}h^{\mu\nu} + \frac{1}{2}\sigma\lambda^2\, h^2 + 2\lambda\,[h\nabla_\mu B^\mu - h^{\mu\nu}\nabla_\mu B_\nu] + 3\lambda^2 B^\mu B_\mu\ ,\end{aligned} \tag{2.4}$$

where we recall that the parameter σ takes the value $+1$ in AdS$_4$, -1 in dS$_4$, and where we denoted $h = \bar{g}^{\mu\nu}h_{\mu\nu}$.

The action with Lagrangian (2.4) is invariant under the gauge transformations given by

$$\begin{cases}\delta_0 h_{\mu\nu} &= 2\,\nabla_{(\mu}\varepsilon_{\nu)} + \lambda\,\bar{g}_{\mu\nu}\,\pi\ , \\ \delta_0 B_\mu &= \nabla_\mu\pi + 2\,\sigma\lambda\,\varepsilon_\mu\ .\end{cases} \tag{2.5}$$

We can define a tensor that is gauge invariant under the Stueckelberg gauge transformations with parameter ε_μ:

$$H_{\mu\nu} = h_{\mu\nu} - \frac{\sigma}{\lambda}\nabla_{(\mu}B_{\nu)}\ . \tag{2.6}$$

This allows to rewrite the PM spin-2 Lagrangian in the following form, where the Stueckelberg field B_μ appears only through $H_{\mu\nu}$,

$$\begin{aligned}L_{\text{PM-St}} = &-\frac{1}{2}\nabla_\rho H_{\mu\nu}\nabla^\rho H^{\mu\nu} + \nabla_\rho H^{\mu\nu}\nabla_\mu H^\rho{}_\nu - \nabla_\mu H\nabla_\nu H^{\mu\nu} + \frac{1}{2}\nabla_\mu H\nabla^\mu H \\ &- 2\sigma\lambda^2 H_{\mu\nu}H^{\mu\nu} + \frac{1}{2}\sigma\lambda^2\, H^2\ .\end{aligned} \tag{2.7}$$

If one sets $B_\mu = 0$ and $\varepsilon_\mu = -\frac{\sigma}{2\lambda}\nabla_\mu\pi$ in (2.7) and then rescale $\pi \mapsto -\sigma\lambda\pi$, we recover the usual form for the action of the PM spin-2 field [1, 3, 6], invariant under

$$\delta_0 h_{\mu\nu} = \nabla_\mu\nabla_\nu\pi - \sigma\lambda^2\bar{g}_{\mu\nu}\pi\ . \tag{2.8}$$

It is possible to define an object invariant under the complete gauge transformations (2.5),

$$\mathcal{K}_{\mu\nu\rho} = \mathcal{K}_{[\mu\nu]\rho} := 2\,\nabla_{[\mu}H_{\nu]\rho}\ , \tag{2.9}$$

Using this object, the Lagrangian (2.7) can be written in the manifestly gauge-invariant way

$$L_{\text{PM-St}} = -\frac{1}{4}\,\mathcal{K}_{\mu\nu\rho}\,\mathcal{K}^{\mu\nu\rho} + \frac{1}{2}\,\mathcal{K}^\mu\,\mathcal{K}_\mu\ , \tag{2.10}$$

where $\mathcal{K}^\mu = \bar{g}_{\nu\rho}\mathcal{K}^{\mu\nu\rho}$.

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2.2 Massless spin-3/2 field

We describe a Majorana massless spin-3/2 field in AdS$_4$ in terms of a spinor-vector $\phi_\mu = (\phi_{\mu A})$ and the Rarita-Schwinger Lagrangian

$$L_{3/2} = -\frac{1}{2}\bar{\phi}_\mu \gamma^{\mu\nu\rho}\,\nabla_\nu \phi_\rho + \frac{M}{2}\,\bar{\phi}_\mu \gamma^{\mu\nu}\,\phi_\nu\ , \quad M = \pm\lambda\ , \tag{2.11}$$

invariant under the gauge transformations

$$\delta_0 \phi_\mu = \nabla_\mu \rho + \frac{M}{2}\,\gamma_\mu\,\rho\ , \tag{2.12}$$

where ρ is a Majorana spinor. Note the two possible signs in the definition of the mass parameter M. This is because the gauge invariance condition is quadratic in M, with $M^2 = \lambda^2$. It is therefore possible to work with either of the two signs. This fact will be relevant when coupling a PM spin-2 with an even number of massless spin-3/2 fields. As a matter of a fact, the vertex we found between these fields in section 4 requires that half the number of spin-3/2 have $M = +\lambda$, the other half having $M = -\lambda$, consistently with the findings of [48]. As explained in [51], it is not possible to define a spin-3/2 field in dS$_4$ spacetime by imposing the Majorana condition and at the same time giving it a gauge transformation law of the form originally found in [52]. In simpler terms, Majorana massless spin-3/2 fields do not exist in dS$_4$ spacetime. Defining the gauge-invariant object

$$\phi_{\mu\nu} = \nabla_{[\mu}\phi_{\nu]} + \frac{M}{2}\,\gamma_{[\mu}\phi_{\nu]}\ , \tag{2.13}$$

the Lagrangian can be rewritten as

$$L_{3/2} = -\frac{1}{2}\bar{\phi}_\mu \gamma^{\mu\nu\rho}\,\phi_{\nu\rho}\ , \quad M = \pm\lambda\ . \tag{2.14}$$

2.3 Massive spin-3/2 field

In the Stueckelberg gauge-invariant formulation, we describe a massive spin-3/2 field of mass parameter $\omega = \sqrt{\sigma\lambda^2 + m^2}$ with a Majorana vector-valued spinor ψ_μ and a Majorana spinor Stueckelberg field χ with Lagrangian

$$L_{3/2\mathrm{M}} = -\frac{1}{2}\bar{\psi}_\mu \gamma^{\mu\nu\rho}\nabla_\nu \psi_\rho + \frac{\omega}{2}\,\bar{\psi}_\mu \gamma^{\mu\nu}\psi_\nu - \frac{3}{4}\bar{\chi}\,\gamma^\mu \nabla_\mu \chi - \frac{3\omega}{2}\,\bar{\chi}\,\chi - \frac{3m}{2}\bar{\psi}_\mu \gamma^\mu \chi\ , \tag{2.15}$$

which is invariant under the Stueckelberg gauge transformations

$$\begin{cases} \delta_0 \psi_\mu &= \nabla_\mu \theta + \frac{\omega}{2}\,\gamma_\mu\,\theta\ , \\ \delta_0 \chi &= m\,\theta\ . \end{cases} \tag{2.16}$$

Note that, when the mass m parameter is non-zero, the massive spin-3/2 field can be defined with the two signs of the cosmological constant $\sigma = \pm 1$, thus in both AdS$_4$ and dS$_4$. In the dS$_4$ case, the massless limit $m \to 0$ renders ω imaginary, in clash with the Majorana reality condition. This is consistent with the fact that Majorana massless spin-3/2 field cannot be defined in dS$_4$, as recalled earlier. Defining the gauge invariant field

$$\Psi_\mu = \psi_\mu - \frac{1}{m}\nabla_\mu \chi - \frac{\omega}{2m}\gamma_\mu\,\chi\ , \tag{2.17}$$

the Lagrangian for the massive spin-3/2 field can be rewritten as

$$L_{3/2\mathrm{M}} = -\frac{1}{2}\bar{\Psi}_\mu\gamma^{\mu\nu\rho}\nabla_\nu\Psi_\rho + \frac{\omega}{2}\,\bar{\Psi}_\mu\gamma^{\mu\nu}\Psi_\nu\,. \tag{2.18}$$

It is possible to set a gauge, named unitary gauge, where χ is set to zero using the form of its gauge transformation with parameter θ. Once this is done, the gauge invariant field Ψ_μ coincides with the field ψ_μ.

In the unitary gauge, the equations of motion obtained by taking the Euler-Lagrange derivative of the Lagrangian (2.18) are $\gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho - \omega\gamma^{\mu\nu}\psi_\nu = 0$. Using gamma matrices identities and commutators of covariant derivatives of the (A)dS$_4$ background, one can verify that this implies:

$$(\Box - \omega^2 + 4\sigma\lambda^2)\psi_\mu = 0\,, \qquad \omega = \sqrt{\sigma\lambda^2 + m^2}\,. \tag{2.19}$$

In AdS$_4$, when setting the mass parameter m to zero, we recover the gauge-fixed equation of motion of a massless spin-3/2 field $(\Box + 3\lambda^2)\phi_\mu = 0$. This equation can be obtained from the massless spin-3/2 Lagrangian (2.11) in the same way, after a gauge fixing. In AdS$_4$ spacetime, note that the parametrisation $\omega = \sqrt{\sigma\lambda^2 + m^2}$ does not allow to set ω to zero with a real value of the mass parameter m. This expresses the fact that the massive spin-3/2 with $\omega = 0$ is non-unitary in AdS. Indeed, in Anti-de Sitter background, the unitarity bound for the squared mass appearing in the D'Alembert-like gauge fixed equation $\Box\phi_\mu = M^2\phi_\mu$ for a Rarita-Schwinger potential is $M^2 \geqslant -3\lambda^2$. Setting ω to zero gives $\Box\phi_\mu = -4\lambda^2\phi_\mu$, which is below the unitarity bound.

In more general terms, for a spin-s field propagating in AdS$_4$, the mass-like term M^2 appearing in the D'Alembert-like equation $(\Box - M^2\lambda^2)\phi_s = 0$ is a quadratic function of the conformal dimension $\Delta = E_0$, the minimal energy of the corresponding $so(2,3)$ module. For a bosonic spin-s field, one has $M^2_{\Delta,s} = \Delta(\Delta-3) - s$, while for a fermionic spin-s field, one has $M^2_{\Delta,s} = \Delta(\Delta-3) - s - \frac{1}{4}$. Massless, propagating spin-s fields in AdS$_4$ have $\Delta = s+1$, be them bosonic or fermionic [53–55]. These are the propagating Fronsdal and Fang-Fronsdal fields. The unitarity bound on the conformal dimension is $\Delta \geqslant s+1$ for a propagating spin-s field. It is saturated by the massless fields.[2] Note that, for $s = 0$, the unitary conformally-coupled scalar field with $M^2 = -2$ admits not only $\Delta = s+1 = 1$, but also $\Delta = 2$, corresponding to two different boundary conditions [57, 58]. The mass-squared M^2 is bounded from below by the value taken by the massless field.

In this work, we will need a description of the non-unitary massive spin-3/2 field with $\omega = 0$ in AdS$_4$, as it is part of the shortest partially massless supermultiplet [33] that will be studied in the following. We therefore introduce a new parametrisation $\tilde{\omega} = \sqrt{\sigma(\lambda^2 - m^2)}$. In AdS$_4$, this parametrisation permits to reach only the non-unitary region between $\Box\phi_\mu = -3\lambda^2\phi_\mu$ and $\Box\phi_\mu = -4\lambda^2\phi_\mu$. If $m^2 > \lambda^2$, $\tilde{\omega}$ becomes imaginary. In de Sitter spacetime, $\tilde{\omega}$ matches with ω, and therefore also describes a unitary field, where m can take arbitrary values larger than or equal to λ^2. The Lagrangian consistent with $\tilde{\omega} = \sqrt{\sigma(\lambda^2 - m^2)}$ is

$$L_{3/2\tilde{\mathrm{M}}} = -\frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho + \frac{\tilde{\omega}}{2}\,\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu + \frac{3\sigma}{4}\bar{\chi}\,\gamma^\mu\nabla_\mu\chi + \frac{3\tilde{\omega}\sigma}{2}\,\bar{\chi}\,\chi + \frac{3m\sigma}{2}\bar{\psi}_\mu\gamma^\mu\chi\,. \tag{2.20}$$

[2] The spin-1/2 and spin-0 singletons, called Di and Rac, respectively [56], are unitary irreducible $so(2,3)$ modules with conformal weights $\Delta_{Di} = 1$ and $\Delta_{Rac} = 1/2$, respectively, that correspond to fields that propagate on the conformal boundary of AdS$_4$.

It is unvariant (up to total derivatives) under the gauge transformations

$$\begin{cases} \delta_0 \psi_\mu &= \nabla_\mu \theta + \frac{\tilde{\omega}}{2} \gamma_\mu \theta \,, \\ \delta_0 \chi &= m\, \theta \,. \end{cases} \tag{2.21}$$

The gauge-invariant Stueckelberg field-strength is

$$\Psi_\mu = \psi_\mu - \frac{1}{m} \nabla_\mu \chi - \frac{\tilde{\omega}}{2m} \gamma_\mu \chi \,. \tag{2.22}$$

2.4 Massless spin-1 field

We describe a massless spin-1 field by a vector field A_μ with curvature $G_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$. We recall the expression of the Maxwell Lagrangian,

$$L = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} \,, \tag{2.23}$$

which is invariant under the gauge transformation

$$\delta A_\mu = \nabla_\mu \alpha \,. \tag{2.24}$$

3 Cubic deformations using the BRST-BV-Stueckelberg formalism

In this section, we briefly summarize the BRST-BV-Stueckelberg method proposed in [44], further developed in [32], in the context where the set of fields contains a massive spin-3/2 field and a partially massless spin-2 field, among others. More specifically, we consider the fields that build up the shortest partially massless supermultiplet of [33] and investigate the possible couplings among those fields.

3.1 Set-up

We follow the notation of [33] as much as possible and recall the action of the shortest supermultiplet in AdS_4 with two extra parameters σ and σ', as compared to what was given in [33]. The first parameter was already introduced above and allows choosing the sign of the cosmological constant, while the parameter σ' gives us the freedom to change the sign of the kinetic terms for the massless spin-3/2 ϕ_μ and the vector gauge field A_μ. The supermultiplet of [33] is only defined in AdS_4 because there is no massless spin-3/2 fields in dS_4 obeying the Majorana reality condition [51], as we recalled above. Explicitly, the complete free action in the Stueckelberg formulation takes the form $S_0 = \int d^4x \sqrt{-\bar{g}}\; L_0$ with

$$\begin{aligned} L_0 = &-\frac{1}{2} \nabla_\rho h_{\mu\nu} \nabla^\rho h^{\mu\nu} + \nabla_\rho h^{\mu\nu} \nabla_\mu h^\rho{}_\nu - \nabla_\mu h \nabla_\nu h^{\mu\nu} + \frac{1}{2} \nabla_\mu h \nabla^\mu h + \frac{\sigma}{4} F_{\mu\nu} F^{\mu\nu} \\ &- 2\sigma\lambda^2 h_{\mu\nu} h^{\mu\nu} + \frac{1}{2} \sigma \lambda^2\, h^2 + 2\lambda \left[h \nabla_\mu B^\mu - h^{\mu\nu} \nabla_\mu B_\nu \right] + 3\lambda^2 B^\mu B_\mu \\ &- \frac{1}{2} \bar{\psi}_\mu \gamma^{\mu\nu\rho} \nabla_\nu \psi_\rho + \frac{\tilde{\omega}}{2} \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu + \sigma \frac{3\tilde{\omega}}{2} \bar{\chi}\chi + \sigma \frac{3}{4} \bar{\chi} \gamma^\mu \nabla_\mu \chi + \sigma \frac{3m}{2} \bar{\psi}_\mu \gamma^\mu \chi \\ &+ \frac{\sigma'}{2} \bar{\phi}_\mu \gamma^{\mu\nu\rho} \nabla_\nu \phi_\rho - \frac{\sigma'}{2} \lambda\, \bar{\phi}_\mu \gamma^{\mu\rho} \phi_\rho + \frac{\sigma'}{4} G^{\mu\nu} G_{\mu\nu} \,. \end{aligned} \tag{3.1}$$

One recovers the action presented in [33] by taking $\sigma' = 1 = \sigma$, $\tilde{\omega} = 0$, and setting all the Stueckelberg fields to zero. We recall the gauge transformations under which the free action is invariant, up to boundary terms that we neglect in this work:

$$\begin{cases} \delta_0 h_{\mu\nu} &= 2\,\nabla_{(\mu}\varepsilon_{\nu)} + \lambda\,\bar{g}_{\mu\nu}\,\pi \\ \delta_0 B_\mu &= \nabla_\mu \pi + 2\,\sigma\lambda\,\varepsilon_\mu \end{cases} , \tag{3.2}$$

$$\begin{cases} \delta_0 \psi_\mu &= \nabla_\mu \theta + \frac{\tilde{\omega}}{2}\,\gamma_\mu\,\theta \\ \delta_0 \chi &= m\,\theta \end{cases} , \qquad \begin{cases} \delta_0 \phi_\mu &= \nabla_\mu \rho + \frac{\lambda}{2}\gamma_\mu \rho \\ \delta_0 A_\mu &= \nabla_\mu \alpha \end{cases} . \tag{3.3}$$

For each gauge parameter, we introduce a ghost field with opposite Grassmann parity:

- For ε_ν, we introduce the Grassmann-odd vector field ξ_ν, and for π, we introduce the Grassmann odd scalar field C ;
- For θ we introduce the Grassmann-even Majorana spinor ζ ;
- For ρ we introduce the Grassmann-even Majorana spinor τ ;
- For α we introduce the Grassmann-odd scalar field ϵ .

The set of all the fields and ghosts will be denoted by $\{\Phi^I\}$. With each field or ghost Φ^I, one introduces a corresponding BV antifield $\widetilde{\Phi}^\star_I$ canonically conjugated to Φ^I through the BV antibracket

$$(A, B) = \frac{\delta^R A}{\delta \Phi^I}\frac{\delta^L B}{\delta \widetilde{\Phi}^*_I} - \frac{\delta^R A}{\delta \widetilde{\Phi}^*_I}\frac{\delta^L B}{\delta \Phi^I} , \tag{3.4}$$

for any local functionals A and B, and where we use De Witt's condensed notations for summations over repeated indices that imply integration over spacetime.

The BV functional for the free theory is given by

$$\begin{aligned} W_0[\Phi, \Phi^\star] = S_0 + \int d^4x\,\sqrt{-\bar{g}}\,\Big(& h^{\star\mu\nu}(2\,\nabla_{(\mu}\xi_{\nu)} + \lambda \bar{g}_{\mu\nu} C) + B^{\star\mu}(\nabla_\mu C + 2\,\sigma\,\lambda\,\xi_\mu) \\ & + \bar{\psi}^{\star\mu}\left(\nabla_\mu \zeta + \frac{\tilde{\omega}}{2}\gamma_\mu \zeta\right) + m\,\bar{\chi}^\star\,\zeta + \bar{\phi}^{\star\mu}\left(\nabla_\mu \tau + \frac{1}{2}\lambda\,\gamma_\mu\,\tau\right) + A^{\star\mu}\,\nabla_\mu \epsilon\Big). \end{aligned} \tag{3.5}$$

Note that, in our conventions for W_0 , we considered the antifields Φ^*_I as tensors and not as tensorial densities $\widetilde{\Phi}^*_I$ as is understood implicitly in (3.4). Therefore, by defining the BRST differential s for the free theory through the BV bracket $s\,\bullet := (W_0, \bullet)$, we have $s\,\Phi^I(x) = -\frac{\delta^R W_0}{\delta \widetilde{\Phi}^*_I(x)} = -\frac{1}{\sqrt{-\bar{g}(x)}}\frac{\delta^R W_0}{\delta \Phi^*_I(x)}$ and $s\,\widetilde{\Phi}^*_I(x) = \frac{\delta^R W_0}{\delta \Phi^I(x)} = \sqrt{-\bar{g}(x)}\,(s\,\Phi^*_I(x))$, hence $s\,\Phi^*_I(x) = \frac{1}{\sqrt{-\bar{g}(x)}}\frac{\delta^R W_0}{\delta \Phi^I(x)}$.

In the case of a free theory, the BRST differential can be written as the sum of two differentials $s = \gamma + \delta$, where δ is the Koszul differential that implements the field equations into the BRST cohomology, see e.g. the book [59] for a complete presentation. The action of these two differentials on the complete BV spectrum is given in table 1, where we also list the Grassmann parity, ghost number, antifield number and pureghost number of all the fields. The

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problem of perturbative deformation of a free theory, in the antifield reformulation of [46, 47], consists in solving the BV master equation $(W, W) = 0$ perturbatively around the free solution W_0, for the deformed BV-functional $W = W_0 + g\, W_1 + \mathcal{O}(g^2)$. The descent equations issued from the equation $sW_1 = 0$ at first order in the formal deformation parameters g, with $W_1 = \int d^4x\sqrt{-\bar{g}}\ (a_0 + a_1 + a_2)$ and antifield$(a_i) = i$, take the form

$$\delta a_1 + \gamma a_0 = \nabla_\mu j_0^\mu \ , \tag{3.6}$$

$$\delta a_2 + \gamma a_1 = \nabla_\mu j_1^\mu \ , \tag{3.7}$$

$$\gamma a_2 = 0 \ . \tag{3.8}$$

Importantly, the cochains a_0, a_1, a_2, as well as j_0^μ and j_1^μ, are all assumed to be local. All the cohomologies $H^*(s|d)$ that we will be computing are in the space of local forms, with representatives that depend on the fields and their derivatives up to a finite, but arbitrary, order. We refer to section 2 of [49] for a detailed presentation of the deformation procedure that we follow. The term a_2 in antifield number two encodes the deformations of the Abelian gauge algebra of the free theory, the term a_1 in antifield number one encodes the deformations of the gauge transformations, and the antifield number zero part a_0 encodes the deformation of the quadratic Lagrangian, i.e., the first-order vertices. We will make the abuse of terminology of designating by a "total derivative" a term $\nabla_\mu j^\mu$, it being understood the multiplication with the density $\sqrt{-\bar{g}}$ will effectively transform $\nabla_\mu j^\mu$ into a total derivative.

3.2 Cohomology of γ

The usual procedure is to start computing the cohomology $H^*(\gamma)$ of the differential γ, the differential along the gauge orbits. One finds

$$H^*(\gamma) = \Big\{ f\big([\Phi_I^\star], [\mathcal{K}_{\mu\nu\rho}], [\Psi_\mu], [\phi_{\mu\nu}], [G_{\mu\nu}], C, \nabla_\mu C, \xi_\mu, \nabla_{(\mu}\xi_{\nu)}, \tau, \epsilon\big) \Big\} \ , \tag{3.9}$$

where the notation $[\Phi]$ means the field Φ and all its derivatives up to some finite, but arbitrary, order, so as to ensure the locality of the function f. The quantities in the argument of the function f in the above expression are not all independent. First of all, the members of each pair $(\nabla_{(\mu}\xi_{\nu)}, C)$ and $(\nabla_\mu C, \xi_\mu)$ belong to the same cohomology class, as we discuss below, and the differential Bianchi identities give relations among the derivatives $[\mathcal{K}_{\mu\nu\rho}]$, $[\Psi_\mu]$, $[\phi_{\mu\nu}]$, and $[G_{\mu\nu}]$.

Details on the calculation of $H^*(\gamma)$ in the sector of the massless spin-1 and spin-3/2 fields around AdS_4 can be found in [45], for example. In the following, we detail the contribution of the massive spin-3/2 field and of the partially massless spin-2 field, both in the Stueckelberg formulation. It is direct to see that the gauge invariant objects constructed out of the fields $h_{\mu\nu}, \psi_\mu$ and their Stueckelberg companions are $\mathcal{K}_{\mu\nu\rho}$ and Ψ_μ, respectively defined in equations (2.9) and (2.17). Therefore, functions of these gauge invariant objects and their derivatives are part of the cohomology of γ: $f([\mathcal{K}_{\mu\nu\rho}], [\Psi_\mu]) \in H^*(\gamma)$.

The ghost fields require further discussion. In the sector of the partially massless field $h_{\mu\nu}$, there are two ghost fields. One can see in table 1 that both C and ξ_μ are elements of the cohomology group. Indeed, they are γ-closed and there is no way to express them as γ-exact objects. Let us focus on the derivatives of C. Because of the relation $\nabla_\mu C = \gamma B_\mu - 2\sigma\lambda\xi_\mu$,

	$\lvert\cdot\rvert$	gh	antfld	puregh	γ	δ
$h_{\mu\nu}$	0	0	0	0	$2\nabla_{(\mu}\xi_{\nu)} + \lambda\,\bar{g}_{\mu\nu}\,C$	0
ψ_μ	1	0	0	0	$-\nabla_\mu\zeta - \frac{1}{2}\tilde{\omega}\,\gamma_\mu\,\zeta$	0
ϕ_μ	1	0	0	0	$-\nabla_\mu\tau - \frac{1}{2}\lambda\,\gamma_\mu\,\tau$	0
A_μ	0	0	0	0	$\nabla_\mu\epsilon$	0
B_μ	0	0	0	0	$\nabla_\mu C + 2\sigma\lambda\,\xi_\mu$	0
χ	1	0	0	0	$-m\,\zeta$	0
ξ_μ	1	1	0	1	0	0
C	1	1	0	1	0	0
ζ	0	1	0	1	0	0
τ	0	1	0	1	0	0
ϵ	1	1	0	1	0	0
$h^{\star\mu\nu}$	1	-1	1	0	0	$\nabla_\rho K^{\rho(\mu\nu)} - \bar{g}^{\mu\nu}\nabla_\rho K^\rho + \nabla^{(\mu}K^{\nu)}$
$\bar{\psi}^{\star\mu}$	0	-1	1	0	0	$-\nabla_\nu\bar{\Psi}_\lambda\gamma^{\nu\lambda\mu} - \tilde{\omega}\bar{\Psi}_\lambda\gamma^{\mu\lambda}$
$\bar{\phi}^{\star\mu}$	0	-1	1	0	0	$\sigma'\bar{\phi}_{\nu\lambda}\gamma^{\mu\nu\lambda}$
$A^{\star\mu}$	1	-1	1	0	0	$-\sigma'\nabla_\nu G^{\nu\mu}$
$B^{\star\mu}$	1	-1	1	0	0	$-2\lambda\,K^\mu$
$\bar{\chi}^\star$	0	-1	1	0	0	$\frac{3\sigma m}{2}\,\bar{\Psi}_\mu\gamma^\mu$
$\xi^{\star\mu}$	0	-2	2	0	0	$-2\nabla_\alpha h^{\star\mu\alpha} + 2\sigma\lambda\,B^{\star\mu}$
$C^\star$	0	-2	2	0	0	$\lambda\,h^\star - \nabla_\mu B^{\star\mu}$
$\bar{\zeta}^\star$	1	-2	2	0	0	$-\nabla_\alpha\bar{\psi}^{\star\alpha} + \frac{1}{2}\tilde{\omega}\,\bar{\psi}^\star_\alpha\,\gamma^\alpha + m\,\bar{\chi}^\star$
$\bar{\tau}^\star$	1	-2	2	0	0	$-\nabla_\mu\bar{\phi}^{\star\mu} + \frac{1}{2}\lambda\,\bar{\phi}^{\star\mu}\gamma_\mu$
$\epsilon^\star$	0	-2	2	0	0	$-\nabla_\mu A^{\star\mu}$

Table 1. Properties and BRST differentials of every field and antifield.

we see that $\nabla_\mu C$ is an element of the cohomology in the same class as ξ_μ. At higher orders in derivatives we have the relation $\nabla_\alpha\nabla_\mu C = \gamma\nabla_{(\alpha}B_{\mu)} - \sigma\lambda\gamma h_{\alpha\mu} + \sigma\lambda^2\bar{g}_{\alpha\mu}C$ indicating that $\nabla_\alpha\nabla_\mu C$ is in the same cohomology class as C. Let us pursue with the derivatives of the ghost ξ_μ. When taking the antisymmetric part of the relation $\nabla_\alpha\nabla_\mu C = \gamma\nabla_\alpha B_\mu - 2\sigma\lambda\nabla_\alpha\xi_\mu$, one obtains $0 = \gamma\nabla_{[\alpha}B_{\mu]} - 2\sigma\lambda\nabla_{[\alpha}\xi_{\mu]}$. This shows that the antisymmetrized covariant derivative of ξ_μ is γ-exact, as well as all the derivatives thereof. For the symmetric part, the relation $\gamma h_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)} + \lambda\,\bar{g}_{\mu\nu}C$ indicates that $\nabla_{(\mu}\xi_{\nu)}$ is in the same cohomology class as C. Taking the covariant derivative of $\gamma h_{\mu\nu}$, one can show that $\nabla_\alpha\nabla_{(\mu}\xi_{\nu)}$ is equivalent to ξ_μ up to γ-exact terms. As a conclusion, in the PM spin-2 sector we have two cohomology classes given by $(\nabla_{(\mu}\xi_{\nu)}, C)$ and $(\nabla_\mu C, \xi_\mu)$, respectively, as announced above. It will be convenient to take one or another representative of each class, depending on the context.

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In the sector of the massive spin-3/2 field ψ_μ, one can see in table 1 that ζ is γ-exact, and thus it is not an element of the cohomology group, as it should for a Stueckelberg field. Moreover, because of the relation $\nabla_\mu\zeta = -\gamma\psi_\mu - \frac{1}{2}\tilde{\omega}\gamma_\mu\zeta$, ζ and $\nabla_\mu\zeta$ are related by a γ-exact term. Thus, neither ζ nor its derivatives belong to the cohomology of γ. This is the typical situation of a massive field theory, as detailed in [44]. The contributions of the ghost fields to the cohomology of γ therefore only come from the sectors of the PM spin-2 field $h_{\mu\nu}$, of the massless spin-3/2 field ϕ_μ, and of the vector gauge field A_μ.

Ambiguity relations. In a massive theory, the Stueckelberg fields transform under the differential γ into the ghost fields. We have an example here with the massive spin-3/2 field whose Stueckelberg field transforms as $\gamma\chi = -m\zeta$. As a consequence, the ghost ζ is not an element of the cohomology group and this gives an ambiguity in the expression of $\nabla_\mu\zeta$ as a γ-exact object:

$$\nabla_\mu\zeta = \gamma\left(-\frac{1}{m}\nabla_\mu\chi\right)\ , \quad \nabla_\mu\zeta = \gamma\left(-\psi_\mu + \frac{\tilde{\omega}}{2m}\gamma_\mu\chi\right)\ , \tag{3.10}$$

as a direct consequence of the gauge invariance of the quantity Ψ_μ defined in (2.17). Note that, while the second equality of (3.10) is similar to the case of a massless spin-3/2 field, the first equality above exists only because of the presence of the Stueckelberg field that transforms into the ghost field ζ.

While we work here in the Stueckelberg formulation, recall that the partially massless spin-2 field has a gauge symmetry (2.8) in the unitary gauge. Consequently, the ghost fields ξ_μ , C, and the derivatives $\nabla_{(\mu}\xi_{\nu)}$ and $\nabla_\mu C$, are in the cohomology of γ. Next, we note that one can build an ambiguity relation for $\nabla_\rho\nabla_{[\mu}\xi_{\nu]}$ as a γ-exact quantity, from the very fact that the quantity $\mathcal{K}_{\mu\nu\rho}$ defined in (2.9) is gauge invariant:

$$\nabla_\rho\nabla_{[\mu}\xi_{\nu]} = \gamma\left(\frac{1}{4\sigma\lambda}\nabla_\rho F_{\mu\nu}\right)\ , \quad \nabla_\rho\nabla_{[\mu}\xi_{\nu]} = \gamma(\nabla_{[\mu}h_{\nu]\rho} + \lambda\, g_{\rho[\mu}B_{\nu]})\ . \tag{3.11}$$

This ambiguity (3.11) has the same structure as (3.10), one expression only in terms of the Stueckelberg field and one expression in terms of the field and its Stueckelberg partner.

We make use of these ambiguity relations in the derivation of the cubic vertices, so as to ensure that they have a non-singular massless limit $m \to 0$ in the unitary gauge. The general strategy in the derivation of vertices involving massive fields in the Stueckelerg formulation is detailed in [44]. It amounts to building cubic vertices that retain the information of the cubic vertices of the massless theory.

3.3 Cubic deformations of the gauge algebra

In this section, we select the cubic deformations of the gauge algebra that are relevant for a localisation of the rigid $\mathcal{N}_4 = 1$ supersymmetry of [33]. First, let us describe a similar theory with which the comparison will be useful.

Pure $\mathcal{N}_4 = 2$ supergravity expanded around AdS$_4$ is a theory that, in the BRST spectrum, contains a massless graviton with its diffeomorphism Grassmann-odd ghost parameter ξ^μ, two massless gravitini with Grassmann-even ghost fields τ^Δ $(\Delta = 1, 2)$ associated with the Grassmann-odd gauge parameters ρ^Δ for local supersymmetry, and one vector gauge field A_μ

with ghost ϵ. The gauge transformations on the fields of the spectrum close, on-shell, to form the $\mathcal{N}_4 = 2$ anti-de Sitter superalgebra. This theory has been studied using the BRST-BV deformation method in [45]. Starting from the free action, it was shown in [45] that the cubic deformation of the Abelian algebra leading to $\mathcal{N}_4 = 2$ pure supergravity theory in AdS$_4$ is encoded, in the BRST-BV formalism, in the following expression at antifield number two,

$$\begin{aligned} a_2^{\text{sugra}} = \alpha_{3/2} \left(\frac{1}{4} k_{\Delta\Omega} \xi^{\star\mu} \bar{\tau}^{\Delta} \gamma_\mu \tau^{\Omega} + k_{\Delta\Omega} \bar{\tau}^{\star\Delta} \gamma^{\mu\nu} \tau^{\Omega} \nabla_{[\mu}\xi_{\nu]} - 2\lambda k_{\Delta\Omega} \bar{\tau}^{\star\Delta} \gamma^{\mu} \tau^{\Omega} \xi_\mu \right) \\ + y \left(t_{\Delta\Omega} \epsilon^\star \bar{\tau}^{[\Delta} \tau^{\Omega]} - 2\lambda t_{\Delta\Omega} \tau^{\star\Delta} \tau^{\Omega} \epsilon \right), \quad t_{\Delta\Omega} = -t_{\Omega\Delta}, \quad k_{\Sigma\Omega} = k_{\Omega\Sigma} \,. \end{aligned} \tag{3.12}$$

By comparing with the PM supermultiplet of [33], one can remark that, apart from the fact that the graviton is partially massless and that one of the gravitini is massive, there is still one spin-2, two spin-3/2 and one spin-1 field. This permits to take advantage of the analysis done in [45] in the selection of the deformations of the gauge algebra we will consider. Indeed, in the PM supermultiplet, there are also two fermionic gauge parameters that we shall denote by $\{\rho, \theta\}$: the parameter ρ for the massless spin-3/2 field ϕ_μ, while θ is associated, in the Stueckelberg formulation, with the massive spin-3/2 field ψ_μ. The analysis presented in the present section consists in the search for deformations of the Abelian gauge symmetries, that would correspond to a localisation of the $\mathcal{N}_4 = 1$ rigid supersymmetry of the model. Although there are two fermionic gauge parameters, θ is a Stueckelberg gauge parameter that represents an artificial gauge symmetry that vanishes in the unitary gauge. Therefore, it is expected that, if a localisation of the $\mathcal{N}_4 = 1$ rigid supersymmetry is possible, the gauge parameter can only be ρ, the gauge parameter of the massless spin-3/2 field ϕ_μ.

To make the analysis as complete as possible, we use as starting point the same deformations of the gauge algebra as in (3.12) for the analysis of the $\mathcal{N}_4 = 2$ supergravity. In doing so, the two gauge parameters ρ and θ with corresponding Grassmann-even ghosts τ and ζ are treated equally, which allows for a clarification of their roles in the possible gauging. The differences with (3.12) is that we do not assume the symmetry properties $t_{\Delta\Omega} = -t_{\Omega\Delta}$, $k_{\Sigma\Omega} = k_{\Omega\Sigma}$ provided by the analysis of [45], and we allow the presence of a γ_5 matrix as it is present in the global supersymmetry transformations of [33]. We consider the following list of terms:

$$\begin{aligned} a_2^{\text{Total}} = k_1\, a_2^{(1)} + k_2\, a_2^{(2)} + k_3\, a_2^{(3)} + k_4\, a_2^{(4)} + k_5\, a_2^{(5)} + k_6\, a_2^{(6)} + k_7\, a_2^{(7)} + k_8\, a_2^{(8)} + k_9\, a_2^{(9)} \\ + k_{10}\, a_2^{(10)} + k_{11}\, a_2^{(11)} + k_{12}\, a_2^{(12)} + k_{12'}\, a_2^{(12')} + k_{13}\, a_2^{(13)} + k_{14}\, a_2^{(14)} + k_{15}\, a_2^{(15)} \,, \end{aligned} \tag{3.13}$$

with

$$\begin{aligned} &a_2^{(1)} = \xi^{\star\mu} \bar{\zeta} \gamma_\mu \zeta \,, && a_2^{(5)} = \bar{\zeta}^\star \gamma^{\mu\nu} \tau \nabla_{[\mu}\xi_{\nu]} \,, && a_2^{(9)} = \bar{\zeta}^\star \gamma^{\mu} \tau \xi_\mu \,, && a_2^{(12')} = \epsilon^\star \bar{\zeta} (i\gamma_5) \tau \,, \\ &a_2^{(2)} = \xi^{\star\mu} \bar{\zeta} \gamma_\mu \tau \,, && a_2^{(6)} = \bar{\tau}^\star \gamma^{\mu\nu} \zeta \nabla_{[\mu}\xi_{\nu]} \,, && a_2^{(10)} = \bar{\tau}^\star \gamma^{\mu} \zeta \xi_\mu \,, && a_2^{(13)} = \bar{\zeta}^\star \zeta \epsilon \,, \\ &a_2^{(3)} = \xi^{\star\mu} \bar{\tau} \gamma_\mu \tau \,, && a_2^{(7)} = \bar{\tau}^\star \gamma^{\mu\nu} \tau \nabla_{[\mu}\xi_{\nu]} \,, && a_2^{(11)} = \bar{\tau}^\star \gamma^{\mu} \tau \xi_\mu \,, && a_2^{(14)} = \bar{\zeta}^\star \tau \epsilon \,, \\ &a_2^{(4)} = \bar{\zeta}^\star \gamma^{\mu\nu} \zeta \nabla_{[\mu}\xi_{\nu]} \,, && a_2^{(8)} = \bar{\zeta}^\star \gamma^{\mu} \zeta \xi_\mu \,, && a_2^{(12)} = \epsilon^\star \bar{\zeta} \tau \,, && a_2^{(15)} = \bar{\tau}^\star \zeta \epsilon \,. \end{aligned} \tag{3.14}$$

In this list, the deformations $a_2^{(1)}$, $a_2^{(4)}$, and $a_2^{(8)}$ are γ-exact, hence, cannot deform the gauge algebra. These were the deformations studied in [32], that led to two cubic vertices coupling the PM spin-2 field to the massive spin-3/2 field. Note that there are only four candidates in the above list that are in the cohomology of γ: $a_2^{(3)}$, $a_2^{(9)}$, $a_2^{(11)}$, and $a_2^{(14)}$. All the other ones are γ-exact terms, and thus can be canceled out by field dependent redefinitions of the gauge parameters. They do not deform the gauge algebra. Nevertheless, in accordance with the strategy explained in [44], which allows to take the massless limit for the vertices in the unitary gauge, the γ-exact candidates are considered.

3.4 Deformations of the gauge transformations a_1

In this section, we present our results for the solving of the descent equation (3.7) using the deformations a_2 of the gauge algebra as sources. Two cases are possible. The first case is when the a_2 is part of the cohomology of γ. In that case, either we obtain a solution a_1 of the descent equation, or either we obtain an obstruction and the deformation computation stops. An obstruction is manifest if δa_2 gives an element of the cohomology of γ. The second case is when $a_2 = \gamma A$. In that case there always exists a trivial solution $a_1^t = \delta A$ with $a_0^t = 0$. Such a solution is not interesting because the theory is not really deformed, a_2 and a_1^t can always be canceled out by field dependent redefinitions of the gauge parameters and quadratic redefinitions of the fields in the quadratic Lagrangian.

However, since we work with massive and partially massless fields in the Stueckelberg formulation, it is in general possible to obtain an alternative solution a_1 that differs from a_1^t by a γ-exact term γc_1 plus a term $\bar{a}_1$ in the cohomology of γ:

$$a_1 - a_1^t = \gamma c_1 + \bar{a}_1 \,, \qquad \bar{a}_1 \in H(\gamma) \,. \tag{3.15}$$

These non-trivial solutions a_1 are obtained by making use of the right-hand sides of the ambiguity relations (3.11) and (3.10), as well as a certain number of integrations by parts. The term $\bar{a}_1$ is a non-trivial deformation of the gauge transformations, that cannot be removed by field-dependent redefinition of the gauge parameters, and that could lead to a non-trivial interaction vertex.

At this stage, we obtain that all the non-trivial deformations of the gauge algebra, $a_2^{(3)}$, $a_2^{(9)}$, $a_2^{(11)}$ and $a_2^{(14)}$, are obstructed. Apart from $a_2^{(3)}$, $a_2^{(9)}$, $a_2^{(11)}$ and $a_2^{(14)}$, all the other candidates that survive are γ-trivial, hence do not deform the gauge algebra. This means that, if we find vertices at the next stage, they will be Abelian. Therefore, we can already conclude that there is no way to localise the rigid supersymmetry carried by the shortest PM supermultiplet of [33], without adding any extra fields to the spectrum.

As an example, starting from the γ-exact deformation $a_2^{(12)} = \epsilon^\star\,\bar{\zeta}\,\tau = \gamma\left(-\frac{1}{m}\epsilon^\star\,\bar{\chi}\,\tau\right)$, we obtain the results

$$a_1^{(12)} = A^{\star\mu}\left(-\bar{\psi}_\mu\tau + \bar{\phi}_\mu\zeta - \left(\frac{\tilde{\omega}-\lambda}{2m}\right)\bar{\chi}\gamma_\mu\tau\right), \tag{3.16}$$

$$c_1^{(12)} = -\frac{1}{m}A^{\star\mu}\,\bar{\chi}\,\phi_\mu, \quad \bar{a}_1^{(12)} = A^{\star\mu}\,\bar{\tau}\,\Psi_\mu \,. \tag{3.17}$$

3.5 Non-trivial cubic interaction vertices a_0

In this section, we pursue the deformation procedure with the a_1 terms that appeared at the previous stage, coming form the surviving a_2 candidates. These a_1's are now used as sources for the a_0 in equation (3.6). At this level, we can add deformations of the gauge transformations $\bar{a}_1^{(I)}$ solutions of $\gamma\bar{a}_1^{(I)}=0$. Only those $\bar{a}_1^{(I)}$ that are nontrivial representatives of $H(\gamma)$ will be considered. Such deformations solve (3.7) with an $a_2=0$.

For each candidate a_1 obtained at the previous section, with $\bar{a}_1\neq 0$, the goal is to find a combination $\alpha^{(A)}\bar{a}_1^{(A)}+\alpha^{(B)}\bar{a}_1^{(B)}+\alpha^{(C)}\bar{a}_1^{(C)}+\dots$ of elements $\bar{a}_1^{(I)}$ in the cohomology of γ with coefficients $\alpha^{(I)}$ such that

$$\begin{aligned}\delta(\bar{a}_1^{Tot})+\gamma a_0&=\text{t.d.}\,,\\ \bar{a}_1^{Tot}&=\bar{a}_1+\alpha^{(A)}\bar{a}_1^{(A)}+\alpha^{(B)}\bar{a}_1^{(B)}+\alpha^{(C)}\bar{a}_1^{(C)}+\dots\,.\end{aligned}\tag{3.18}$$

Then, to obtain the final result, we define a_1^{Tot}, which is just a_1 but with $\bar{a}_1$ substituted with $\bar{a}_1^{Tot}$,

$$a_1^{Tot}=a_1^t+\gamma c_1+\bar{a}_1^{Tot}\,.\tag{3.19}$$

Injecting a_1^{Tot} in the descent equation (3.6), we obtain the complete vertex

$$a_0^{Tot}=a_0+\delta c_1\,.\tag{3.20}$$

This solution a_0^{Tot} is a non-trivial deformation provided that $a_0\neq\delta b$. Indeed, if $a_0=\delta b$ for some local function b, this means that $\bar{a}_1^{Tot}=\gamma c$. Although $\bar{a}_1^{Tot}$ is a sum of elements, each of which non-trivial in the cohomology of γ, it may happen that the sum is trivial in the cohomology, because of some cancellations. Therefore, in that case where $a_0=\delta b$, the deformation is trivial and can be removed by field redefinitions.

In the following, we present our results for the only candidates a_1 that lead to interaction vertices a_0. We found three vertices denoted $a_0^{(12)}$, $a_0^{(12')}$ and $a_0^{(30)}$.

3.5.1 Interaction vertex $a_0^{(12)}$

Starting with the trivial deformation of the gauge algebra $a_2^{(12)}=\epsilon^\star\,\bar{\zeta}\,\tau$, we find a deformation $a_1^{Tot(12)}$ of the gauge transformations that gives rise to a vertex $a_0^{(12)}$ provided that $\tilde{\omega}=\sqrt{\sigma(\lambda^2-m^2)}$ vanishes. Explicitly, we have

$$\begin{aligned}a_1^{Tot(12)}=&\,A^{\star\mu}\Big(-\bar{\psi}_\mu\,\tau+\bar{\phi}_\mu\,\zeta+\frac{\lambda}{2m}\,\bar{\chi}\,\gamma_\mu\,\tau\Big)\\ &+\bar{\psi}^{\star\mu}\Big(\frac{\sigma'}{4}\,\gamma_{\mu\rho\sigma}\,\tau\,G^{\rho\sigma}-\frac{\sigma'}{2}\,\gamma^\beta\,\tau\,G_{\mu\beta}-\frac{\sigma'}{6\lambda}\,\nabla_\mu(\gamma^{\rho\sigma}\,\tau\,G_{\rho\sigma})\Big)\,,\end{aligned}\tag{3.21}$$

and

$$a_0^{(12)}=\sigma'\bar{\Psi}_\beta\Big(G^{\alpha\beta}-i\gamma^5(*G)^{\alpha\beta}\Big)\phi_\alpha+\frac{\sigma'}{m}\nabla_\nu G^{\nu\mu}\,\bar{\chi}\,\phi_\mu\,,\tag{3.22}$$

where we have used that $\gamma^{\mu\nu\rho\sigma}=-i\gamma^5\epsilon^{\mu\nu\rho\sigma}$ and $(*G)^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}$. This is a new nontrivial vertex involving a massless spin-3/2 field, a massive spin-3/2 field of mass parameter $\tilde{\omega}=0$ and a massless spin-1 field. The gauge transformations are deformed, but the gauge algebra is not. This reproduces a vertex of the $\mathcal{N}_4=2$ supergravity theory in AdS$_4$, except that here one of the two spin-3/2 fields is massive — see equation (3.48) of [45].

3.5.2 Interaction vertex $a_0^{(12')}$

Starting with the candidate $a_2^{(12')} = \epsilon^\star \bar{\zeta}(i\gamma_5)\tau$, we obtain a consistent vertex, provided that the mass parameter $\tilde{\omega} = \sqrt{\sigma(\lambda^2 - m^2)}$ vanishes. Explicitly,

$$
\begin{aligned}
a_1^{Tot(12')} = A^{\star\mu}\Big(-\bar{\psi}_\mu(i\gamma_5)\tau + \bar{\phi}_\mu(i\gamma_5)\zeta - \frac{\lambda}{2m}\bar{\chi}\gamma_\mu(i\gamma_5)\tau\Big) \\
- \frac{\sigma'}{2}\Big\{\bar{\psi}^{\star\rho}\Big(\gamma^\beta(i\gamma_5)\tau G_{\rho\beta} - \frac{1}{2}\gamma_\rho{}^{\alpha\beta}(i\gamma_5)\tau G_{\alpha\beta}\Big) \\
+ \bar{\chi}^\star\Big(\frac{\sigma\lambda}{3m}\gamma^{\alpha\beta}(i\gamma_5)\tau G_{\alpha\beta}\Big)\Big\} ,
\end{aligned}
\tag{3.23}
$$

and

$$
a_0^{Tot(12')} = \sigma'\bar{\Psi}_\sigma(i\gamma_5)\Big[G^{\rho\sigma} - i\gamma_5(*G)^{\rho\sigma}\Big]\phi_\rho + \frac{\sigma'}{m}\nabla_\nu G^{\nu\mu}\bar{\chi}(i\gamma_5)\phi_\mu \tag{3.24}
$$

This is a new non-trivial vertex involving a massless spin-3/2 field, a massive spin-3/2 field with mass parameter $\tilde{\omega} = 0$ and a massless spin-1 field. The gauge transformations are deformed, but the gauge algebra is not. The vertex is very similar to the vertex $a_0^{(12)}$ above in (3.22).

3.5.3 Interaction vertex $a_0^{(30)}$

In this section, we allow several massive spin-3/2 fields to couple to a gauge vector. Such states can generically appear in the supermultiplets of [35, 60], for example. We therefore consider the free Lagrangian for $N > 1$ massive, Majorana spin-3/2 fields ψ_μ^Ω $(\Omega = 1, \dots, N)$ with diagonal mass matrix $\omega_{\Delta\Omega}$, added to the Lagrangian for a gauge field A_μ with field strength $G_{\mu\nu}$:

$$
L_0 = -\frac{1}{2}\bar{\Psi}_\mu^\Delta\gamma^{\mu\nu\rho}\nabla_\nu\Psi_\rho^\Omega\delta_{\Delta\Omega} + \frac{1}{2}\omega_{\Delta\Omega}\bar{\Psi}_\mu^\Delta\gamma^{\mu\nu}\Psi_\nu^\Omega + \frac{\sigma'}{4}G^{\mu\nu}G_{\mu\nu} . \tag{3.25}
$$

We will not need to impose the mass parameters on the diagonal of $\omega_{\Delta\Omega}$ to vanish, therefore we use the parametrisation that describes the unitary region in AdS$_4$ spacetime: $\omega_{\Delta\Delta} = \sqrt{\sigma\lambda^2 + m_{\Delta\Delta}^2}$, where the index Δ is fixed. We consider the following trivial deformation of the gauge algebra, which is a generalization of $a_2^{(13)}$ in (3.14):

$$
a_2^{(30)} = \bar{\zeta}_\Delta^\star\,\zeta_\Omega\,\epsilon\,t^{\Delta\Omega} . \tag{3.26}
$$

This deformation $a_2^{(30)}$ can be lifted to a deformation of the gauge transformations, that in turn can be integrated to a consistent vertex provided $\omega_{\Delta\Omega} = \omega\delta_{\Delta\Omega}$ and $t^{\Delta\Omega} = t^{[\Delta\Omega]}$. The resulting deformation of the gauge transformations and the corresponding cubic vertex read

$$
a_1^{(30)} = \bar{\psi}_\Delta^{\star\alpha}\left(\psi_{\Omega\alpha}\,\epsilon\,t^{\Delta\Omega} - \zeta_\Omega\,A_\alpha\,t^{\Delta\Omega}\right) + \bar{\chi}_\Delta^\star\,\chi_\Omega\,\epsilon\,t^{\Delta\Omega} , \tag{3.27}
$$

$$
a_0^{(30)} = -\frac{1}{m}A_\alpha\bar{\Psi}_{\Delta\mu\nu}\gamma^{\alpha\mu\nu}\chi_\Omega\,t^{\Delta\Omega} - \frac{1}{2}A_\nu\bar{\Psi}_{\Delta\lambda}\gamma^{\alpha\nu\lambda}\Psi_{\Omega\alpha}t^{\Delta\Omega} . \tag{3.28}
$$

where we recall that $\Psi_\mu^\Delta = \psi_\mu^\Delta - \frac{1}{m}\nabla_\mu\chi^\Delta - \frac{\omega}{2m}\gamma_\mu\,\chi^\Delta$, and $\Psi_{\mu\nu}^\Delta = \nabla_{[\mu}\Psi_{\nu]}^\Delta + \frac{\omega}{2}\gamma_{[\mu}\Psi_{\nu]}^\Delta$.

This is, in the Stueckelberg formalism, the well-known minimal coupling of massive spin-3/2 fields to electromagnetism. The simplest case is $N = 2$, where the pair of Majorana

Rarita-Schwinger field forms a U(1) doublet, a complex Rarita-Schwinger field, and where one can take the matrix $t^{\Delta\Omega} = \epsilon^{\Delta\Omega}$, the $so(2)$-invariant symbol.

This problem has an old story, of course [61, 62]. Let us only mention the analyses [63, 64] that went beyond formal consistency, looking at causality of the models, in particular. There, general Pauli-type non-minimal couplings of the type (3.22) are added, on top of the minimal couplings, to ensure the correct number of propagating degrees of freedom. It is of course not a surprise that, among the 5-parameter family of non-minimal vertices considered in [63, 64], precisely the ones corresponding to (3.22) (but for equal-mass spin-3/2 fields) emerge on the account of counting of degrees of freedom. In the Stueckelberg formalism, we see that these non-minimal couplings appear automatically. It is one of the advantages of the Stueckelberg formalism, that it naturally account for the correct propagation of degrees of freedom. Note that in flat space, this coupling was also studied in [65].

3.6 Final results

In this section, we present our final results outside the BRST-BV formalism and in the unitary gauge, where all the Stueckelberg fields have been eliminated. The procedure to reach the unitary gauge at first order in deformation is detailed in [32].

3.6.1 Vertices $a_0^{(12)}$ and $a_0^{(12')}$

Our two results, applying to the free supermultiplet of [33], and that exist only when $\tilde{\omega} = 0$ are the following. Consider a model composed of one massive spin-3/2 field with mass parameter $\tilde{\omega} = 0$, one massless spin-3/2 field and one massless spin-1 field, in AdS$_4$ spacetime. The Lagrangian takes the form

$$L_0 = -\frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho + \frac{\sigma'}{2}\bar{\phi}_\mu\gamma^{\mu\nu\rho}\nabla_\nu\phi_\rho - \frac{\sigma'}{2}\lambda\ \bar{\phi}_\mu\gamma^{\mu\rho}\phi_\rho + \frac{\sigma'}{4}G^{\mu\nu}G_{\mu\nu}\,. \tag{3.29}$$

The action is invariant under the gauge transformations

$$\delta_0\phi_\mu = \nabla_\mu\rho + \frac{\lambda}{2}\gamma_\mu\rho\,,\quad \delta_0 A_\mu = \nabla_\mu\alpha\,,\quad \delta_0\psi_\mu = 0\,. \tag{3.30}$$

Our result is that the deformed action with Lagrangian $L_0 + L_1$,

$$L_1 = g^{(12)}\sigma'\bar{\psi}_\beta\Big[G^{\alpha\beta} - i\gamma_5(*G)^{\alpha\beta}\Big]\phi_\alpha + g^{(12')}\sigma'\bar{\psi}_\sigma(i\gamma_5)\Big[G^{\rho\sigma} - i\gamma_5(*G)^{\rho\sigma}\Big]\phi_\rho\,, \tag{3.31}$$

is invariant under the deformed gauge transformations $\delta_0 + \delta_1$, with

$$\delta_1 A_\mu = -g^{(12)}\bar{\psi}_\mu\rho - g^{(12')}\bar{\psi}_\mu(i\gamma_5)\rho\,, \tag{3.32}$$

$$\begin{aligned}\delta_1\psi_\mu = {}& \sigma' g^{(12)}\left[\frac{1}{4}\gamma_{\mu\nu\rho}\,\rho\,G^{\nu\rho} - \frac{1}{2}\gamma^\nu\rho\,G_{\mu\nu} - \frac{1}{6\lambda}\nabla_\mu\Big(\gamma^{\rho\sigma}\rho\,G_{\rho\sigma}\Big)\right]\\ &+\sigma' g^{(12')}\left[\frac{1}{4}\gamma_\mu{}^{\alpha\beta}(i\gamma_5)\rho\,G_{\alpha\beta} - \frac{1}{2}\gamma^\nu(i\gamma_5)\rho\,G_{\mu\nu} + \frac{\sigma}{6\lambda}\nabla_\mu\Big(\gamma^{\alpha\beta}(i\gamma_5)\,\rho\,G_{\alpha\beta}\Big)\right],\end{aligned} \tag{3.33}$$

$$\delta_1\phi_\mu = 0\,. \tag{3.34}$$

The gauge algebra remains Abelian.

3.6.2 Vertex $a_0^{(30)}$

Consider a model composed of an even number N of massive spin-3/2 fields ψ_μ^Ω, $\Omega = 1, \ldots, N$ of same mass ω and a massless spin-1 field A_μ. This model is defined in (A)dS$_4$. The free Lagrangian takes the form

$$L_0 = -\frac{1}{2}\bar{\psi}_\mu^\Omega \gamma^{\mu\nu\rho}\nabla_\nu \psi_\rho^\Delta \delta_{\Omega\Delta} + \frac{\omega}{2}\bar{\psi}_\mu^\Omega \gamma^{\mu\nu}\psi_\nu^\Delta \delta_{\Omega\Delta} + \frac{\sigma'}{4}G^{\mu\nu}G_{\mu\nu}\,. \tag{3.35}$$

The action is invariant under the free gauge transformations

$$\delta_0 A_\mu = \nabla_\mu \alpha\,. \tag{3.36}$$

Our result is that the action with deformed Lagrangian $L_0 + L_1$,

$$L_1 = -g^{(30)}\frac{1}{2}A_\nu \bar{\psi}_{\Delta\lambda}\gamma^{\alpha\nu\lambda}\psi_{\Omega\alpha}t^{\Delta\Omega}\,, \quad t^{\Delta\Omega} = t^{[\Delta\Omega]}\,, \tag{3.37}$$

is invariant under the deformed gauge transformations $\delta_0 + \delta_1$ with

$$\delta_1\psi_\mu^\Delta = g^{(30)}\psi_{\Omega\mu}\alpha t^{\Delta\Omega}\,, \qquad \delta_1 A_\mu = 0\,. \tag{3.38}$$

The gauge algebra remains Abelian. We recover of course the well known minimal coupling between a doublet of Majorana massive spin-3/2 fields and a massless spin-1 field [63–65].

4 Partially massless supergravity vertex

In this section, we push further the analogy with $\mathcal{N} = 2$ pure supergravity around AdS$_4$ by adding at least one extra massless gravitino to the spectrum. We will classify all the possible deformations of the gauge algebra and obtain a non-Abelian vertex that bears striking resemblance with the gravitational minimal coupling in $\mathcal{N} = 2$ sugra, except that the role of the graviton is taken over by the PM spin-2 field.

4.1 Set-up

We consider a free theory consisting of a PM spin-2 field $h_{\mu\nu}$ in the unitary gauge and an unspecified number of massless spin-3/2 fields ϕ_μ^Δ, $\Delta = 1, \ldots, N$, with $N > 1$ so that at least two massless gravitini are present, which ensures that one can extract the spectrum of $\mathcal{N} = 2$ pure supergravity from the total spectrum, recalling that a partially massless spin-2 field decomposes into a massless spin-2 and a massless spin-1 fields in the flat spacetime limit. The total, free Lagrangian, takes the form

$$L_0 = -\frac{1}{4}K_{\mu\nu\rho}K^{\mu\nu\rho} + \frac{1}{2}K^\mu K_\mu - \frac{1}{2}\bar{\phi}_\mu^\Delta \gamma^{\mu\nu\rho}\nabla_\nu \phi_\rho^\Sigma \delta_{\Delta\Sigma} + \frac{\lambda}{2}\bar{\phi}_\mu^\Delta \gamma^{\mu\rho}\phi_\rho^\Sigma M_{\Delta\Sigma}\,, \tag{4.1}$$

where $K_{\mu\nu\rho} = 2\,\nabla_{[\mu}h_{\nu]\rho}$ and $K_\mu = \bar{g}^{\nu\rho}K_{\mu\nu\rho}$, with gauge transformations

$$\delta_0 h_{\mu\nu} = \nabla_\mu\nabla_\nu\pi - \sigma\lambda^2\bar{g}_{\mu\nu}\pi\,, \tag{4.2}$$

$$\delta_0\phi_\mu^\Delta = \nabla_\mu\rho^\Delta + \frac{\lambda}{2}\gamma_\mu\rho^\Sigma M_\Sigma{}^\Delta\,. \tag{4.3}$$

The mass matrix $M_{\Delta\Sigma} = \text{diag}(\pm 1, \pm 1, \ldots, \pm 1)$, $M_{\Delta\Gamma}M^{\Gamma}{}_{\Sigma} = \delta_{\Delta\Sigma}$, allows for massless spin-3/2 fields with opposite mass-like terms. We note that, in order to have an invariance under the R-symmetry group $O(N)$, one needs $M_{\Delta\Sigma}$ to correspond to an invariant tensor of $O(N)$. The only possibility is $M_{\Delta\Sigma} = \pm\delta_{\Delta\Sigma}$. Therefore, in the case $M_{\Delta\Sigma} \neq \pm\delta_{\Delta\Sigma}$, the R-symmetry $O(N)$ is explicitly broken from the start. Note also that for $N > 2$, say $N = 4$, we can construct an $M_{\Delta\Sigma} \neq \pm\delta_{\Delta\Sigma}$ breaking $O(4)$, but that is a direct sum of invariant tensors of $O(2)$: $M_{\Delta\Sigma} = \pm\text{diag}(-1, -1, +1, +1)$.

As in the previous section, we introduce a ghost field τ^Δ associated to each gauge parameter ρ^Δ and a ghost field C associated with the PM gauge parameter π. The set of all fields and ghost fields is denoted Φ^I. To each field Φ^I is associated an antifield $\tilde{\Phi}^\star_I$ canonically conjugated to Φ^I by the BV antibraket defined at (3.4). The complete list of fields and antifields as well as their BRST differentials are summarized in table 2. The BV functional of the free theory takes the form

$$W_0[\Phi, \Phi^\star] = S_0 + \int d^4x\, \sqrt{-\bar{g}} \left(h^{\star\mu\nu}(\nabla_\mu\nabla_\nu C - \sigma\lambda^2 \bar{g}_{\mu\nu} C) + \bar{\phi}^{\star\mu}_\Delta \left(\nabla_\mu \tau^\Delta + \frac{\lambda}{2}\gamma_\mu \tau^\Sigma M_\Sigma{}^\Delta \right) \right). \tag{4.4}$$

The cohomology group of the differential along the gauge orbits γ is

$$H^*(\gamma) = \left\{ f\left([\Phi^\star_I], [K_{\mu\nu\rho}], [\phi^\Delta_{\mu\nu}], C, \nabla_\mu C, \tau^\Delta\right) \right\} . \tag{4.5}$$

As already discussed in subsection 3.1, we search for perturbative deformations of the BV functional at first order, $W = W_0 + gW_1 + \mathcal{O}(g^2)$, $W_1 = \int d^4x \sqrt{-\bar{g}}\ (a_0 + a_1 + a_2)$, by solving classical master equation $(W, W) = 0$ to first order that take form of the descent of equations

$$\delta a_1 + \gamma a_0 = \nabla_\mu j_0^\mu \ , \tag{4.6}$$

$$\delta a_2 + \gamma a_1 = \nabla_\mu j_1^\mu \ , \tag{4.7}$$

$$\gamma a_2 = 0 \ . \tag{4.8}$$

We recall that the terms of antifield number 2, a_2, are deformations of the gauge algebra, terms of antifield number 1, a_1, deformations of the gauge transformations, and terms of antifield number 0, a_0, deformations of the free action.

4.2 Cubic deformations of the gauge algebra

As a starting point, we consider the list of all candidate deformations of the gauge algebra in the cohomology of γ at pureghost number 2, $H^2(\gamma)$, mixing the PM spin-2 parameter with the spin-3/2 parameters. The only restriction is that we exclude candidates involving a γ_5 matrix that would break parity. There are only three independent terms:

$$\begin{aligned} a_2^{(1)} &= k^{(1)}_{\Delta\Sigma}\, \bar{\tau}^{\star\Delta}\gamma^\mu\tau^\Sigma\nabla_\mu C \ , \\ a_2^{(2)} &= k^{(2)}_{\Delta\Sigma}\, \bar{\tau}^{\star\Delta}\tau^\Sigma C \ , \\ a_2^{(3)} &= k^{(3)}_{[\Delta\Sigma]}\, C^\star\bar{\tau}^\Delta\tau^\Sigma \ . \end{aligned} \tag{4.9}$$

	$\vert.\vert$	gh	afld	puregh	γ	δ
$h_{\mu\nu}$	0	0	0	0	$\nabla_\mu\nabla_\nu C - \sigma\lambda^2\bar{g}_{\mu\nu}C$	0
ϕ_μ^Δ	1	0	0	0	$-\nabla_\mu\tau^\Delta - \frac{\lambda}{2}\gamma_\mu\tau^\Omega M_\Omega{}^\Delta$	0
C	1	1	0	1	0	0
τ^Δ	0	1	0	1	0	0
$h^{\star\mu\nu}$	1	-1	1	0	0	$\nabla_\rho K^{\rho(\mu\nu)} - \bar{g}^{\mu\nu}\nabla_\rho K^\rho + \nabla^{(\mu}K^{\nu)}$
$\bar{\phi}^{\star\mu}_\Delta$	0	-1	1	0	0	$-\bar{\phi}_{\Delta\nu\lambda}\gamma^{\mu\nu\lambda}$
$C^\star$	0	-2	2	0	0	$\nabla_\mu\nabla_\nu h^{\star\mu\nu} - \sigma\lambda^2 h^\star$
$\bar{\tau}^\star_\Delta$	1	-2	2	0	0	$-\nabla_\mu\bar{\phi}^{\star\mu}_\Delta + \frac{\lambda}{2}\bar{\phi}^{\star\mu}_\Omega\gamma_\mu M^\Omega{}_\Delta$

Table 2. Properties and BRST differentials of every field and antifield.

4.3 Deformations of the gauge transformations

The next step is to use the candidates (4.9) as source for the equation (4.7). We obtain a solution

$$\begin{aligned} a_1^{(1-2-3)} = {} & k^{(3)}_{\Delta\Sigma}\Big(2\nabla_\mu h^{\star\mu\nu}\bar{\tau}^\Delta\phi^\Sigma_\nu + \lambda h^{\star\mu\nu}\bar{\phi}^\Delta_\mu\gamma_\nu\tau^\Omega M_\Omega{}^\Sigma + \lambda h^{\star\mu\nu}\bar{\tau}^\Delta\gamma_\mu\phi^\Omega_\nu M_\Omega{}^\Sigma\Big) \\ & + k^{(1)}_{\Delta\Sigma}\Big(\bar{\phi}^{\star\Delta}_\mu\gamma^\nu\phi^{\Sigma\mu}\nabla_\nu C - \bar{\phi}^{\star\Delta}_\mu\gamma_\nu\tau^\Sigma h^{\mu\nu} + \lambda M_\Omega{}^\Sigma\,\bar{\phi}^{\star\Delta}_\mu\phi^{\Omega\mu}C\Big) \end{aligned} \tag{4.10}$$

under the constraints

$$\begin{aligned} \{k^{(1)}, M\} &= 0\,, \\ \{k^{(3)}, M\} &= 0\,, \\ k^{(2)}_{\Delta\Sigma} &= \lambda\, k^{(1)}_{\Delta\Omega}\, M_\Sigma{}^\Omega\,. \end{aligned} \tag{4.11}$$

This imposes that the matrix M should take the form

$$M = \pm\mathrm{diag}(-1,\dots,-1,1,\dots,1)\,, \tag{4.12}$$

with an equal number of -1's and $+1$'s eigenvalues, with $\Delta = 1,\dots,2n$, providing the matrices k^1 and k^3 are anti-diagonal. Already, after solving this first equation, we see that the case $M_{\Delta\Sigma} = \pm\delta_{\Delta\Sigma}$ is excluded. In particular, in the case $n = 1$ ($\Delta = 1, 2$), this means the R-symmetry is explicitly broken. A consequence of these constraints is also that the minimal number of spin-3/2 fields is two, thereby excluding any non-Abelian coupling involving one PM spin-2 and one Majorana massless spin-3/2 field. This result is consistent with the result obtained in the previous section; it is not possible to render the supersymmetric transformations of [33] local without introducing additional fields.

4.4 Cubic vertex

We pursue by using the result (4.10) as source for the next equation in the descent (4.6). In solving (4.6), the solution a_1 to the previous equation (4.7) can always be combined with deformations $\bar{a}_1$ solutions to $\gamma\bar{a}_1 = 0$ that do not contribute to the gauge algebra. We find a solution

$$\delta\Big(a_1^{(1-2-3)} + \bar{a}_1\Big) + \gamma a_0^{\text{PM sugra}} = \text{t.d.}\,, \tag{4.13}$$

with

$$\bar{a}_1 = b_{\Delta\Omega}\bar{\phi}^{\star\Delta\rho}\gamma^{\mu\nu}\tau^{\Omega}K_{\mu\nu\rho} + d_{\Delta\Omega}\bar{\phi}^{\star\Delta\nu}\phi^{\Omega}_{\mu\nu}\nabla^{\mu}C\,, \tag{4.14}$$

under the constraints

$$\begin{aligned} b_{\Delta\Omega} &= b_{(\Delta\Omega)}\,, \qquad d_{\Delta\Omega} = -8b_{\Delta\Omega}\,,\\ k^{(1)}_{\Delta\Sigma} &= k^{(1)}_{[\Delta\Sigma]} = -2\lambda b_{(\Delta\Omega)}M_{\Sigma}{}^{\Omega}\,,\\ b_{(\Delta\Omega)} &= \lambda k^{(3)}_{[\Delta\Sigma]}M_{\Omega}{}^{\Sigma}\,. \end{aligned} \tag{4.15}$$

The resulting interaction vertex is

$$\begin{aligned} a_0^{\text{PM sugra}} = b_{\Delta\Omega}\Big\{ &-\frac{1}{2}\bar{\phi}^{\Delta}_{\mu}\gamma^{\mu\nu\rho}\gamma^{\alpha\beta}\phi^{\Omega}_{\rho}K_{\alpha\beta\nu}\\ &-2\bar{\phi}^{\Delta}_{\mu}\gamma^{\nu\delta\rho}\phi^{\Omega}_{\delta\rho}h^{\mu}{}_{\nu} - 4\bar{\phi}^{\Delta}_{\mu}\gamma^{\mu\nu\rho}\phi^{\Omega}_{\delta\rho}h^{\nu\delta} + 2\bar{\phi}^{\Delta}_{\mu}\gamma^{\mu\nu\rho}\phi^{\Omega}_{\nu\rho}h\Big\}\,. \end{aligned} \tag{4.16}$$

Using the symmetry properties of the matrices $b_{\Delta\Omega}$, this result can be rewritten as

$$\begin{aligned} a_0^{\text{PM sugra}} = b_{\Delta\Omega}\Big\{ &-\frac{1}{2}\bar{\phi}^{\Delta}_{\mu}\gamma^{\mu\nu\rho}\gamma^{\alpha\beta}\phi^{\Omega}_{\rho}K_{\alpha\beta\nu}\\ &-2\bar{\phi}^{\Delta}_{\mu}\gamma^{\nu\delta\rho}\nabla_{\delta}\phi_{\rho}{}^{\Omega}\,h^{\mu}{}_{\nu} - 4\bar{\phi}^{\Delta}_{\mu}\gamma^{\mu\nu\rho}\nabla_{[\delta}\phi_{\rho]}{}^{\Omega}\,h^{\nu\delta} + 2\bar{\phi}^{\Delta}_{\mu}\gamma^{\mu\nu\rho}\nabla_{\nu}\phi_{\rho}{}^{\Omega}\,h\Big\}\,. \end{aligned} \tag{4.17}$$

The final deformation of the gauge transformations is

$$\begin{aligned} a_1^{\text{PM sugra}} = b_{\Delta\Gamma}\Big\{ & h^{\star\mu\nu}\Big(\nabla_{\mu}\Big(-\frac{2}{\lambda}\bar{\tau}^{\Delta}\phi^{\Sigma}_{\nu}M_{\Sigma}{}^{\Gamma}\Big) + 2\bar{\phi}^{\Delta}_{\nu}\gamma_{\mu}\tau^{\Gamma}\Big)\\ &+\bar{\phi}^{\star\Delta}_{\nu}\Big(-2\lambda\gamma^{\mu}\phi^{\Sigma\nu}\nabla_{\mu}CM_{\Sigma}{}^{\Gamma} - 2\lambda^2\phi^{\Gamma\nu}C + 8\phi^{\Gamma\nu\mu}\nabla_{\mu}C\\ &\qquad + 2\lambda\gamma^{\mu}\tau^{\Sigma}h^{\nu}{}_{\mu}M_{\Sigma}{}^{\Gamma} + \gamma^{\mu\delta}\tau^{\Gamma}K_{\mu\delta}{}^{\nu}\Big)\Big\}\,, \end{aligned} \tag{4.18}$$

and the final deformation of the gauge algebra is

$$a_2^{\text{PM sugra}} = -2\lambda b_{\Delta\Omega}M_{\Sigma}{}^{\Omega}\bar{\tau}^{\star\Delta}\gamma^{\mu}\tau^{\Sigma}\nabla_{\mu}C - 2\lambda^2 b_{\Delta\Sigma}\bar{\tau}^{\star\Delta}\tau^{\Sigma}C + \frac{1}{\lambda}b_{\Delta\Omega}M_{\Sigma}{}^{\Omega}C^{\star}\bar{\tau}^{\Delta}\tau^{\Sigma}\,. \tag{4.19}$$

Comparison with $\mathcal{N}_4 = 2$ supergravity Let us compare the PM supergravity cubic interacting model obtained in this section to the $\mathcal{N}_4 = 2$ supergravity model studied in [45] that shares the same spectrum of helicities $(\pm 2, \pm 3/2, \pm 3/2, \pm 1)$. In subsection 2.1, we have presented the Stueckelberg formulation of the PM spin-2 field allowing to introduce a Stueckelberg gauge parameter ε_{μ} playing the role of linearised diffeomorphisms. In the unitary gauge where the Stueckelberg fields are set to zero, ε_{μ} is fixed to $\varepsilon_{\mu} = -\frac{\sigma}{2\lambda}\nabla_{\mu}\pi$, where π is the gauge parameter of the PM spin-2 field. In the following, we are going to compare the deformations of the gauge algebra, gauge transformations, and finally the interaction vertex of $\mathcal{N}_4 = 2$ supergravity obtained in [45] with the results of the present paper. In light of the Stueckelberg formulation of the PM spin-2 field, we identify $\nabla_\mu C$ in the partially massless case as a diffeomorphism ghost ξ_μ in $\mathcal{N}_4 = 2$ supergravity. The ghost field C in the partially massless case is now replaced by the U(1) ghost ϵ for the massless vector gauge field in $\mathcal{N}_4 = 2$ supergravity.

We begin by the deformation of the gauge algebra (4.19) that we compare to the corresponding gauge algebra of $\mathcal{N}_4 = 2$ supergravity given at equation (3.12). The three algebra terms of the PM supergravity algebra (4.19) are present in (3.12), but a_2^{sugra} has two supplementary terms. In one term the diffeomorphism BV-antifield $\xi^\star_\mu$ appears, and in the other the diffeomorphism ghost appears through $\nabla_{[\mu}\xi_{\nu]}$. The first represents the commutator of two gauge supersymmetry transformations that gives a local diffeomorphism, and the second represents the commutator of a supersymmetry transformation and a local Lorentz transformation. The first term is absent because there is no way to represent $\xi^\star_\mu$ within the PM supergravity framework. The second term is absent because $\nabla_{[\mu}\xi_{\nu]}$ vanishes identically when ξ_μ is interpreted as $\nabla_\mu C$. Therefore, we conclude that the PM supergravity gauge algebra (4.19) is as close as possible to (3.12).

Let us rewrite the cubic results obtained in [45] for the gauge transformations of the $\mathcal{N}_4 = 2$ supergravity in AdS$_4$ with the same method:

$$
\begin{aligned}
a_1^{\text{sugra}} = \alpha_{3/2}\, k_{\Delta\Omega}\Big[&- h^{\star\mu\nu}\bar{\phi}^{\Delta}_{\mu}\gamma_\nu\tau^\Omega + \bar{\phi}^{\star\Delta\rho}\gamma_{\mu\nu}\phi^{\Omega}_{\rho}\nabla^{[\mu}\xi^{\nu]} - \bar{\phi}^{\star\Delta\rho}\gamma^{\mu\nu}\tau^\Omega\nabla_{[\mu}h_{\nu]\rho} \\
&+ \lambda\,\bar{\phi}^{\star\Delta\mu}\gamma^\nu\tau^\Omega h_{\mu\nu} - 2\lambda\bar{\phi}^{\star\Delta\mu}\gamma_\nu\phi^\Omega_\mu\xi^\nu - 8\bar{\phi}^{\star\Delta\mu}\phi^\Omega_{\mu\nu}\xi^\nu\Big] \\
+\, y\, t_{\Delta\Omega}\Big[&- 2A^{\star\mu}\bar{\phi}^\Delta_\mu\tau^\Omega - 2\lambda\bar{\phi}^{\star\Delta\mu}(\phi^\Omega_\mu\epsilon - \tau^\Omega A_\mu) - \frac{1}{2}\bar{\phi}^{\star\Delta}_\mu F_{\rho\sigma}\gamma^{\rho\sigma}\gamma^\mu\tau^\Omega\Big]\,.
\end{aligned}
\tag{4.20}
$$

Here, the two spin-3/2 fields denoted ϕ^Δ_μ have a trivial mass matrix $M_{\Delta\Omega} = \pm\text{diag}(1,1)$, $h_{\mu\nu}$ stands for a massless spin-2 field with diffeomorphism ghost ξ_μ, and A_μ stands for a massless vector field with ghost ϵ. In light of the Stueckelberg formulation of the PM spin-2 field, we recall that we identify a ξ_μ in supergravity as a $\nabla_\mu C$ in the partially massless case. All terms in the first and second lines of (4.20) are present in (4.18), except the term in $\nabla^{[\mu}\xi^{\nu]}$ that is identically zero in the partially massless case where ξ_μ is identified a $\nabla_\mu C$. In the last line of (4.20), the only term in which the massless spin-1 is not involved is present in (4.18). In fact, the only term present in the PM supergravity model and not in (4.20) is the first term on the first line of (4.18). Notice that this term has the form $\nabla_{(\mu}\tilde{\xi}_{\nu)}$ of a particular linearised diffeomorphism transformation of parameter $\tilde{\xi}_\mu = -\frac{2}{\lambda}b_{\Delta\Gamma}\bar{\tau}^\Delta\phi^\Sigma_\mu M_\Sigma{}^\Gamma$.

We pursue by comparing the PM supergravity interaction vertex (4.17) with the interaction vertex of $\mathcal{N}_4 = 2$ supergravity obtained in [45]. The interaction vertices are very similar, but there are three main differences. First, in (4.17), the helicity-1 modes are enclosed in the PM spin-2 field. Then, the PM sugra vertex in (4.17) does not contain any term proportional to λ. Finally, the symmetric matrix that contracts the Δ indices of the spin-3/2 fields is different. In the case of $\mathcal{N}_4 = 2$ supergravity in AdS$_4$, this matrix is proportional to the Kronecker delta $\delta_{\Delta\Omega}$, while in (4.17) the matrix $b_{\Delta\Omega}$ is symmetric and anti-diagonal. This means that the partially massless supergravity vertex (4.17) contains only cross terms of the type "$\phi^1\phi^2 h$", while the supergravity vertex contains only diagonal terms of the type "$\phi^1\phi^1 h + \phi^2\phi^2 h$".

4.5 Final result

In this subsection, we rewrite and comment the obtained results without reference to the BRST-BV formalism. The Lagrangian at first order in deformation reads

$$L^{\text{PM sugra}} = L_0 + L_1 \,, \tag{4.21}$$

with

$$L_0 = -\frac{1}{4}K_{\mu\nu\rho}K^{\mu\nu\rho} + \frac{1}{2}K^{\mu}K_{\mu} - \frac{1}{2}\bar{\phi}^{\Delta}_{\mu}\gamma^{\mu\nu\rho}\nabla_{\nu}\phi^{\Sigma}_{\rho}\delta_{\Delta\Sigma} + \frac{\lambda}{2}\bar{\phi}^{\Delta}_{\mu}\gamma^{\mu\rho}\phi^{\Sigma}_{\rho}M_{\Delta\Sigma} \,, \tag{4.22}$$

and

$$\begin{aligned} L_1 = b_{\Delta\Omega}\Big\{ &-\frac{1}{2}\bar{\phi}^{\Delta}_{\mu}\gamma^{\mu\nu\rho}\gamma^{\alpha\beta}\phi^{\Omega}_{\rho}K_{\alpha\beta\nu} \\ &- 2\bar{\phi}^{\Delta}_{\mu}\gamma^{\gamma\beta\rho}\nabla_{\beta}\phi_{\rho}{}^{\Omega}h^{\mu}{}_{\gamma} - 4\bar{\phi}^{\Delta}_{\beta}\gamma^{\beta\gamma\rho}\nabla_{[\delta}\phi_{\rho]}{}^{\Omega}h^{\delta}{}_{\gamma} + 2\bar{\phi}^{\Delta}_{\beta}\gamma^{\beta\gamma\rho}\nabla_{\gamma}\phi_{\rho}{}^{\Omega}h\Big\} \,. \end{aligned} \tag{4.23}$$

We have shown that this interaction vertex is equivalent to the one obtained at equation (74) of [48]. At this order, this interacting model is invariant under gauge transformations

$$\begin{aligned} \delta h_{\alpha\beta} &= \nabla_{\alpha}\nabla_{\beta}\pi - \sigma\lambda^2\bar{g}_{\alpha\beta}\pi \\ &\quad + b_{(\Delta\Gamma)}\left(\nabla_{(\alpha}\left(-\frac{2}{\lambda}\bar{\phi}^{\Delta}_{\beta)}\rho^{\Sigma}M_{\Sigma}{}^{\Gamma}\right) + 2\bar{\phi}^{\Delta}_{(\alpha}\gamma_{\beta)}\rho^{\Gamma}\right) , \end{aligned} \tag{4.24}$$

$$\begin{aligned} \delta\phi^{\Delta\beta} &= \nabla^{\beta}\rho^{\Delta} + \frac{\lambda}{2}\gamma^{\beta}\rho^{\Sigma}M_{\Sigma}{}^{\Delta} \\ &\quad + b^{\Delta}{}_{\Gamma}\Big(- 2\lambda\gamma^{\alpha}\phi^{\Sigma\beta}\nabla_{\alpha}\pi M_{\Sigma}{}^{\Gamma} - 2\lambda^2\phi^{\Gamma\beta}\pi + 8\phi^{\Gamma\beta\alpha}\nabla_{\alpha}\pi \\ &\qquad + 2\lambda\gamma^{\alpha}\rho^{\Sigma}h^{\beta}{}_{\alpha}M_{\Sigma}{}^{\Gamma} + \gamma^{\alpha\gamma}\rho^{\Gamma}K_{\alpha\gamma}{}^{\beta}\Big) \,. \end{aligned} \tag{4.25}$$

The gauge algebra takes the form

$$[\delta_{\vec{\rho}}, \delta_{\vec{\zeta}}]h_{\alpha\beta} = \nabla_{\alpha}\nabla_{\beta}\tilde{\pi} - \lambda^2\bar{g}_{\alpha\beta}\tilde{\pi} + \mathcal{O}(b^2) \,, \tag{4.26}$$

$$[\delta_{\pi}, \delta_{\vec{\rho}}]\phi^{\Delta}_{\alpha} = \nabla_{\alpha}\tilde{\rho}^{\Delta} + \frac{\lambda}{2}\gamma_{\alpha}\tilde{\rho}^{\Sigma}M_{\Sigma}{}^{\Delta} + \mathcal{O}(b^2) \,, \tag{4.27}$$

$$[\delta_{\pi}, \delta_{\vec{\rho}}]h_{\alpha\beta} = \mathcal{O}(b^2) \,, \quad [\delta_{\vec{\rho}}, \delta_{\vec{\zeta}}]\phi^{\Delta}_{\alpha} = \mathcal{O}(b^2) \,, \tag{4.28}$$

with

$$\tilde{\pi} = \frac{2}{\lambda}\bar{\zeta}^{\Delta}\rho^{\Sigma}b_{\Delta\Gamma}M_{\Sigma}{}^{\Gamma} \,, \tag{4.29}$$

$$\tilde{\rho}^{\Delta} = b^{\Delta}{}_{\Gamma}(2\lambda\gamma^{\alpha}\rho^{\Sigma}\nabla_{\alpha}\pi M_{\Sigma}{}^{\Gamma} + 2\lambda^2\rho^{\Gamma}\pi) \,. \tag{4.30}$$

Rather suggestively, after a rescaling $b_{\Delta\Omega} \to -\frac{1}{8}b_{\Delta\Omega}$, the action with quadratic and cubic terms can be rewritten as the truncation up to (and including) the cubic order of

$$\begin{aligned} S^{\text{PM sugra}} &= \int d^4x\,\bar{e}\left(-\frac{1}{4}K_{\mu\nu\rho}K^{\mu\nu\rho} + \frac{1}{2}K^{\mu}K_{\mu}\right) \\ &\quad + \int d^4x\,E_{\Delta}{}^{\Omega}\left(-\frac{1}{2}\bar{\phi}^{\Delta}_{\mu}\gamma^{abc}\tilde{\phi}_{\nu\rho\Pi}\right)(E^{\mu}_{a})_{\Omega}{}^{\Theta}(E^{\nu}_{b})_{\Theta}{}^{\Gamma}(E^{\rho}_{c})_{\Gamma}{}^{\Pi} \,, \end{aligned} \tag{4.31}$$

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with

$$
\begin{aligned}
(G_{\mu\nu})_\Omega{}^\Delta &\equiv (E^a{}_\mu)_\Omega{}^\Theta (E_{a\nu})_\Theta{}^\Delta = \bar{g}_{\mu\nu}\delta_\Omega{}^\Delta + h_{\mu\nu} b_\Omega{}^\Delta \,, \\
(E^a{}_\mu)_\Omega{}^\Delta &= \bar{e}^a{}_\mu \delta_\Omega{}^\Delta + \frac{1}{2}\bar{e}^{a\nu} h_{\nu\mu} b_\Omega{}^\Delta \,, \\
(E_a{}^\mu)_\Omega{}^\Delta &= \bar{e}_a{}^\mu \delta_\Omega{}^\Delta - \frac{1}{2}\bar{e}_{a\nu} h^{\nu\mu} b_\Omega{}^\Delta \,, \\
E_\Omega{}^\Delta &= \bar{e}\delta_\Omega{}^\Delta + \frac{1}{2}\bar{e} h b_\Omega{}^\Delta \,, \\
\tilde{\phi}^\Delta_{\mu\nu} &= \tilde{\nabla}_{[\mu}\phi_{\nu]}{}^\Delta + \frac{\lambda}{2}\gamma_c (E^c{}_{[\mu})_\Gamma{}^\Omega\, \phi^\Gamma_{\nu]} M_\Omega{}^\Delta \,, \\
\tilde{\nabla}_\mu \phi^\Delta_\nu &= \nabla_\mu \phi_\nu{}^\Delta - \frac{1}{4}\nabla_\alpha h_{\beta\mu}\gamma^{\alpha\beta}\phi^\Omega_\nu b_\Omega{}^\Delta \,.
\end{aligned}
\tag{4.32}
$$

This first order interacting model has the form of a gravitational minimal coupling, but where the role of the graviton is taken over by the partially massless spin-2 field. Compared to conventional minimal coupling in General Relativity, the partially massless spin-2 field is accompanied by the symmetric anti-diagonal matrix $b_\Omega{}^\Delta$ such that all the first order interactions terms are cross terms. Remark that (4.31) contains all the corrections analogous to the gravitational minimal coupling, including corrections to the mass-like part of the spin-3/2 Lagrangian proportional to λ. However, the sum of all the first-order terms proportional to λ is identically zero using the symmetry properties of the matrices $b_\Omega{}^\Delta$ and $b_\Omega{}^\Gamma M_\Gamma{}^\Delta$, in agreement with (4.23).

4.6 Global symmetries of the free model

From the result of this first-order deformation computation, we can deduce global symmetries of the free Lagrangian. Consider global parameters $\breve{\pi}^\Delta$ and $\breve{\rho}$ solutions to $\delta_0\phi^\Delta_\mu = 0$ and $\delta_0 h_{\mu\nu} = 0$:

$$\nabla^\beta \breve{\rho}^\Delta + \frac{\lambda}{2}\gamma^\beta \breve{\rho}^\Sigma M_\Sigma{}^\Delta = 0 \,, \tag{4.33}$$

$$\nabla_\alpha \nabla_\beta \breve{\pi} - \lambda^2 \bar{g}_{\alpha\beta}\breve{\pi} = 0 \,. \tag{4.34}$$

Equation (4.33) defines Killing spinors of the AdS_4 background. Moreover, if $\breve{\pi}$ is a solution to (4.34), then $\breve{\xi}_\mu = \nabla_\mu \breve{\pi}$ satisfies the conformal Killing equation [1, 66, 67]

$$\nabla_\mu \breve{\xi}_\nu + \nabla_\nu \breve{\xi}_\mu = \frac{1}{2}\bar{g}_{\mu\nu}\nabla^\rho \breve{\xi}_\rho \,, \tag{4.35}$$

and defines $\breve{\xi}_\mu$ as a conformal Killing vector of the AdS_4 background. Evaluating the first order gauge invariance equation

$$\delta_0 L_1 + \delta_1 L_0 = 0 \,, \tag{4.36}$$

on global parameters $\breve{\pi}$ and $\breve{\rho}^\Delta$, we obtain $\breve{\delta}_1 L_0 = 0$, meaning that the first-order gauge transformations evaluated on global parameters define new symmetries of the free action with Lagrangian (4.22). First, we obtain global fermionic transformations that mix the bosonic and fermionic sectors:

$$\breve{\delta}_{\breve{\rho}} h_{\alpha\beta} = b_{\Delta\Gamma}\Big(-\frac{2}{\lambda}\nabla_{(\alpha}(\bar{\phi}^\Delta_{\beta)}\rho^\Sigma M_\Sigma{}^\Gamma) + 2\bar{\phi}^\Delta_{(\alpha}\gamma_{\beta)}\rho^\Gamma\Big) \,, \tag{4.37}$$

$$\breve{\delta}_{\breve{\rho}} \phi^{\Delta\beta} = b^\Delta{}_\Gamma\Big(+ 2\lambda\gamma^\alpha \breve{\rho}^\Sigma h^\beta{}_\alpha M_\Sigma{}^\Gamma + \gamma^{\alpha\gamma}\breve{\rho}^\Gamma K_{\alpha\gamma}{}^\beta\Big) \,. \tag{4.38}$$

As expected, these transformations are very similar to the $\mathcal{N}_4 = 1$ supersymmetry transformations obtained in [33] at equation (5.18) with a different spectrum. Second, we obtain a symmetry that mixes the spin-3/2 sector of (4.22) along a conformal Killing vector $\check{\xi}_\mu = \nabla_\mu \check{\pi}$. Up to a free gauge transformation $\tilde{\delta}_0 \phi^{\Delta\beta}$ of free gauge parameter $\tilde{\rho}^\Delta = b^\Delta{}_\Gamma \phi^{\Gamma\alpha} \check{\xi}_\alpha$, this symmetry takes the form

$$\check{\delta}_{\check{\pi}} \phi^\Delta_\mu = b^\Delta{}_\Gamma \Big(-\check{\xi}^\nu \nabla_\nu \phi^\Gamma_\mu - \lambda\, \check{\xi}^\nu \gamma_\nu \phi^\Sigma_\mu M_\Sigma{}^\Gamma + \lambda\, \gamma_\mu \check{\xi}^\nu \phi^\Sigma_\nu M_\Sigma{}^\Gamma - \frac{3}{2}\, \lambda^2\, \phi^\Gamma_\mu \check{\pi} \Big) + \tilde{\delta}_0 \phi^\Delta_\mu \,. \tag{4.39}$$

This is very close to the recently derived conformal-like symmetry of the massless Dirac spin-3/2 field in dS_4 spacetime, see equation (6.21) of [68].[3] We leave the study of the commutators of these global symmetries for future work.

5 Obstruction to second order, a road to conformal supergravity

At this stage, we have obtained a consistent cubic deformation $W_1^{\text{PM sugra}}$ of the BV functional $W[\Phi^I, \Phi^*_I] = W_0 + W_1 + W_2 + \dots$ up to first order in the coupling constants. The local functional we obtained, $W_1^{\text{PM sugra}} = \int d^4x \sqrt{\bar{g}}\,(a_0 + a_1 + a_2)$, satisfies $(W_0, W_1) \equiv sW_1 = 0$ and is not s-exact in the space of local functionals.

To second order in the deformation, the BRST-BV master equation $(W, W) = 0$ takes the form

$$sW_2 = -\frac{1}{2}(W_1, W_1)\,. \tag{5.1}$$

Once W_2 is expanded in antifield number, $W_2 = \int d^4x \sqrt{\bar{g}}\,(b_0 + b_1 + b_2)$, one obtains the descent of equations

$$\delta b_1 + \gamma b_0 = -(a_1, a_0) + \nabla_\mu t_0^\mu \,, \tag{5.2}$$

$$\delta b_2 + \gamma b_1 = -\frac{1}{2}(a_1, a_1) - (a_2, a_1) + \nabla_\mu t_1^\mu \,, \tag{5.3}$$

$$\gamma b_2 = -\frac{1}{2}(a_2, a_2) + \nabla_\mu t_2^\mu \,. \tag{5.4}$$

To solve this descent of equations for the triplet (b_0, b_1, b_2), one starts with (5.4) to be solved for b_2, using as a source the quantity a_2 that was obtained at previous order. In the particular case of the final a_2 of this section, see (4.19), the computation of the antibracket (a_2, a_2) leads to obstructions $\mathcal{O}_i$ to the existence of b_2, in the form

$$\mathcal{O}_1 = 8\lambda^2 b_{\Delta\Omega} M_K{}^\Omega b_\Gamma{}^K M_\Theta{}^\Gamma \bar{\tau}^{\star\Delta} \gamma^{\mu\nu} \tau^\Theta \nabla_\mu C \nabla_\nu C \,, \tag{5.5}$$

$$\mathcal{O}_2 = 16\lambda^3 b_{\Delta\Omega} M_\Pi{}^\Omega b_\Sigma{}^\Pi \bar{\tau}^{\star\Delta} \gamma^\mu \tau^\Sigma \nabla_\mu C C \,, \tag{5.6}$$

$$\mathcal{O}_3 = -8 b_{\Delta\Omega} M_\Pi{}^\Omega b_\Gamma{}^\Pi M_\Sigma{}^\Gamma C^\star \bar{\tau}^{(\Delta} \gamma^\mu \tau^{\Sigma)} \nabla_\mu C \,, \tag{5.7}$$

$$\mathcal{O}_4 = -4\lambda b_{\Delta\Sigma} b_{\Gamma\Omega} M_\Theta{}^\Omega \bar{\tau}^{\star\Delta} \tau^\Sigma \bar{\tau}^\Gamma \tau^\Theta \,, \tag{5.8}$$

$$\mathcal{O}_5 = 4\lambda b_{\Gamma\Theta} b_{\Delta\Omega} M_\Sigma{}^\Omega \bar{\tau}^{\star\Delta} \gamma^\mu \tau^\Sigma \bar{\tau}^\Gamma \gamma_\mu \tau^\Theta \,. \tag{5.9}$$

Unless one trivially sets the deformation structures $b_{\Delta\Omega}$ to zero, thereby setting W_1 to zero, there is no way to make these obstructions vanish. This implies that the first-order

[3] The authors thank Vasileios Letsios for fruitful discussions on this subject.

deformation cannot be pushed to the next order in interactions. These obstructions signal a failure to the Jacobi identity for the gauge algebra to be satisfied.

The model considered so far contains the same spectrum of fields as the one of $\mathcal{N}_4 = 2$ pure supergravity in AdS$_4$, a theory that has been studied in [45] with the same cohomological techniques as the ones we have been following here. We recall that the on-shell spectrum of helicity modes of this model is $\{\pm 2, \pm 3/2, \pm 3/2, \pm 1\}$. This is a remarkable point indeed, that motivated our investigations of the couplings between a PM spin-2 field with two massless gravitini around AdS$_4$. We recall that the PM spin-2 field contains, in his decomposition in helicity modes in the flat limit $\lambda \to 0$, a massless spin-2 field as well as a massless spin-1 field, accounting for the helicity modes $(\pm 2, \pm 1)$.

A noticeable difference between $\mathcal{N}_4 = 2$ pure sugra and the PM model we found up to cubic order is that, in the latter, the coupling between the spin-1 and the spin-3/2 modes is fixed by the coupling of the spin-2 modes with the spin-3/2 modes, while in the perturbative reconstruction of $\mathcal{N}_4 = 2$ pure sugra around the free model around AdS$_4$, at cubic order one still has the freedom in the coupling constants between the gauge vector and the rest of the spectrum. In other words, all the coefficients in the vertex (4.23) are already fixed at cubic-order, which means that the spin-1 modes are fixed to interact with the other modes, in a way that is not free to choose. By contrast, in $\mathcal{N}_4 = 2$ supergravity the coefficients in the vertices involving the spin-1 sector are fixed by the consistency relations imposed at second order.

A possible way out to cure the obstructions (5.5)–(5.9) we encountered in building a partially massless supergravity theory is the addition of a massless spin-2 field and a massless spin-1 field to the spectrum, that bring respectively the diffeomorphism ghost ξ^μ and the U(1) ghost ϵ into the BRST spectrum. The total deformation a_2 of the gauge algebra now reads

$$a_2 = a_2^{\text{PM Sugra}} + a_2^{\text{EH}} + a_2^{\text{Weyl}} + a_2^{\text{sugra}} + a_2^{\text{R}} \,, \tag{5.10}$$

where the algebra deformation $a_2^{\text{PM Sugra}}$ is given in (4.19) and with

$$a_2^{\text{EH}} = \kappa\, \xi^{\star\mu}\xi^\nu \nabla_{[\mu}\xi_{\nu]} \,, \tag{5.11}$$

$$a_2^{\text{Weyl}} = \alpha^{\text{Weyl}} \left(2\, C^\star \nabla^\mu C\, \xi_\mu - \frac{\lambda^2}{2}\xi^{\star\mu} C\, \nabla_\mu C\right) , \tag{5.12}$$

$$a_2^{\text{sugra}} = \frac{1}{4} k^1_{\Delta\Omega}\xi^{\star\mu}\bar{\tau}^{\Delta}\gamma_\mu\tau^{\Omega} + k^2_{\Delta\Omega}\bar{\tau}^{\star\Delta}\gamma^{\mu\nu}\tau^{\Omega}\nabla_{[\mu}\xi_{\nu]} - 2\lambda k^3_{\Delta\Omega}\bar{\tau}^{\star\Delta}\gamma^\mu\tau^\Omega\xi_\mu \,, \tag{5.13}$$

$$a_2^{\text{R}} = q^1_{\Delta\Omega}\epsilon^\star\bar{\tau}^\Delta i\gamma_5\tau^\Omega + 2\lambda q^2_{\Delta\Omega}\bar{\tau}^{\star\Delta} i\gamma_5\tau^\Omega\epsilon \,. \tag{5.14}$$

The term a_2^{EH} is the Einstein-Hilbert cubic deformation of the diffeomorphism algebra that uniquely leads to General Relativity [49]. The second deformation, a_2^{Weyl}, is the cubic deformation that mixes the gauge parameters of the massless and partially massless spin-2 fields. This deformation leads to conformal gravity at cubic order around the AdS$_4$ background. Indeed, the spectrum of conformal gravity linearised around (A)dS$_4$ spacetime is a massless spin-2 with a relatively ghostly partially massless spin-2 field [1]. In [31], it is proved that the relative sign and coefficients between a_2^{EH} and a_2^{Weyl} are fixed by the consistency conditions at second order.

The third deformation a_2^{sugra} is a supergravity gauge algebra deformation that leads to cubic interactions between the massless spin-2 field and the two massless gravitini. Finally, the deformation $a_2^{\rm R}$ is an algebra deformation that leads to cubic interactions between the massless spin-1 and spin-3/2 fields.

Considering that the two massless gravitini have opposite signs in front of their respective mass-like terms in the quadratic Lagrangian, we showed that the deformations $a_2^{\rm sugra}$ and $a_2^{\rm R}$ are consistently giving rise to linear (in the fields) gauge transformation laws, and to cubic vertices. The details of the complete analysis of the gauge algebra deformations (5.11)–(5.14) to cubic and quartic order will be presented elsewhere, together with the values of the constants $k^1_{\Delta\Omega}, k^2_{\Delta\Omega}$, $k^3_{\Delta\Omega}$, $q^1_{\Delta\Omega}$, and $q^2_{\Delta\Omega}$.

The structure of the candidate $a_2^{\rm sugra}$ is based on the general form of the gauge algebra deformation (3.12) leading to $\mathcal{N}_4 = 2$ pure supergravity [45]. One observes that algebra deformation candidate (5.10) contains the structures for all the commutators and anticommutators of the $\mathcal{N}_4 = 1$ superconformal algebra $su(2, 2|1)$, see for instance [34, 37]. We have shown, in the case $\Delta = 1, 2$, that they produce obstruction terms in (a_2, a_2) that combine with (5.5)–(5.9). In fact, the more than thirty obstruction terms in (a_2, a_2) remarkably cancel if a set of four independent quadratic equations on the four free constants $b_{\Delta\Omega}$, κ, $k^1_{\Delta\Omega}$ and $q^1_{\Delta\Omega}$ are satisfied. However, these four equations cannot all be satisfied at the same time, and a family of obstructions always survives. Therefore, the cubic PM supergravity model remains obstructed at quartic order.

This result contrasts with a consistent non-Abelian theory around AdS$_4$ that involves a PM spin-2 field and spin-3/2 fields that we expected to recover: $\mathcal{N}_4 = 1$ pure conformal supergravity [36, 37]. The spectrum of this theory expanded around AdS$_4$ is

- one massless spin-2 field,
- one partially massless spin-2 field,
- two massless, real spin-3/2 fields,
- one massive, real spin-3/2 field, and
- one massless vector.

Remarkably, this spectrum is the same as the one considered to cure the obstructions (5.5)–(5.9). The spectrum of conformal (Weyl) gravity around AdS$_4$ had already been found in [1] while the fermionic sector of $\mathcal{N}_4 = 1$ pure conformal supergravity can be found in [34, 35]. From section 4 and appendix C of [35] where $\mathcal{N} = 2$ conformal supergravity is considered, discarding the Yang-Mills, matter supermultiplets and extra $s_{\max} = 3/2$ multiplets that appear once coupled to gravity when $\mathcal{N} > 1$, see e.g. [69] section 6, one can retrieve[4] the spectrum and conformal dimensions of the various fields of pure $\mathcal{N}_4 = 1$ conformal supergravity expanded around AdS$_4$. The result is summarized in table 3 where we added in the last line the number of physical degrees of freedom of the corresponding fields.

[4]From the Lagrangian and tables 1 and 2 of [35], where the Einstein-Hilbert sector was added, one should formally take the limit $G_N \to \infty$ in their parameter $\delta = \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{L^2}{8\pi G_N(c_1 - c_2)}}$ so as to decouple the Einstein-Hilbert sector and only consider the higher-derivative terms (with coefficient $c_1 - c_2$ therein) of pure conformal supergravity.

s	2	2	3/2	3/2	3/2	1
Δ	3	2	5/2	5/2	3/2	2
$\#_{d.o.f.}$	2	4	2	2	4	2

Table 3. Set of fields of pure conformal supergravity expanded around AdS_4, with the conformal dimensions and number of physical degrees of freedom

Considering the gauge-fixed equation $(\Box - \lambda^2 m^2)\varphi_s = 0$ around AdS_4, where one has the mass formula $m^2_{\Delta,s} = \Delta(\Delta-3) - s$ for bosonic fields and $m^2_{\Delta,s} = \Delta(\Delta-3) - s - \frac{1}{4}$ for fermionic fields,[5] one can see that

- the field of spin-2 and conformal dimension $\Delta = 3$ in table 3 satisfies the linearised gauge-fixed field equation $(\Box + 2\lambda^2)\varphi_{\mu\nu} = 0$. This is the massless graviton with its two physical degrees of freedom;
- the field of spin-2 and conformal dimension $\Delta = 2$ in table 3 satisfies the linearised gauge-fixed field equation $(\Box + 4\,\lambda^2)h_{\mu\nu} = 0$. This is the partially massless spin-2 field with its four physical degrees of freedom;
- the two fields of spin-3/2 and conformal dimension $\Delta = 5/2$ in table 3 satisfy the linearised gauge-fixed field equation $(\Box + 3\,\lambda^2)\phi_\mu = 0$. These are two massless gravitini with their two physical degrees of freedom, where the field equation is recovered from (2.19) with $m^2 = 0$;
- the field of spin-3/2 and conformal dimension $\Delta = 3/2$ in table 3 satisfies the linearised field equation $(\Box + 4\,\lambda^2)\psi_\mu = 0$ which coincides with the equation (2.19) when $\omega = 0$, the massive fields ψ_μ that enter the vertex (3.31). This is a massive gravitino with its four physical degrees of freedom with helicities $(\pm 3/2, \pm 1/2)$ in the flat limit;
- finally, the field of spin-1 and conformal dimension $\Delta = 2$ in table 3 satisfies the linearised gauge-fixed field equation $(\Box + 3\,\lambda^2)A_\mu = 0$. This is a massless vector field with its two physical degrees of freedom.

It was found in [1] that the first two spin-2 fields in the list above make up the spectrum of Weyl (conformal) gravity around AdS_4. At the level of the linearised Lagrangian around AdS_4 background, suitable field redefinitions allow one to decompose the action of a Weyl-invariant spin-2 field into the sum of the Fierz-Pauli Lagrangian for a massless spin-2 field minus the Lagrangian for a PM spin-2 field, with the relative sign expressing the non-unitarity of the model [27, 28, 74, 75].

In the case of the Weyl invariant spin-3/2 field that is part of the spectrum of $\mathcal{N}_4 = 1$ conformal supergravity [34], it is not known whether it is possible to decompose the linearised Lagrangian around AdS_4 into a sum or difference of ordinary Rarita-Schwinger Lagrangians for the spin-3/2 helicity modes in table 3. This question has been addressed in flat spacetime

[5] A complete analysis in arbitrary dimension is given in [70–72], together with original references for the special case of $so(2,3)$ that we consider here. See also [73] for a review in both de Sitter and anti de Sitter.

in [74], but not in AdS background, to the best of our knowledge. We hope to return to this question in the near future.

We recall that the cubic PM supergravity vertex (4.23) is based on the sum of two Rarita-Schwinger Lagrangian densities with opposite mass-like terms (4.22). Therefore, this vertex can only be part of conformal supergravity if one assumes that the Lagrangian of a conformal spin-3/2 field can be decomposed into a sum or a difference of Rarita-Schwinger Lagrangians such that (4.22) describes the two massless spin-3/2 modes of table 3. However, whether this decomposition is possible or not around the AdS background has not yet been studied, to the best of our knowledge. If this decomposition is not possible, this would mean that our vertex (4.23) cannot be part of the conformal supergravity Lagrangian, in agreement with the inconsistency of our vertices at quartic order.

6 Conclusions

In this paper, we have made a detailed study of the possible interactions between a partially massless spin-2 field and spin-3/2 fields, massless and massive. The motivation behind our work is partly based on the paper [33] where a minimal partially massless supermultiplet around AdS_4 was found that contains the three types of fields cited above, plus a gauge vector. The findings of [33] stimulates the search for a partially massless supergravity model that would gauge the rigid supersymmetry carried by the supermultiplet. Our results show that the presence of the massless spin-3/2 field of the PM supermultiplet is not enough to localise the rigid supersymmetry: there is no way to make local the global supersymmetry algebra represented on the minimal PM supermultiplet of [33] without introducing extra fields. On the way to this result, we found two vertices that couple the massive and massless spin-3/2 fields to the gauge vector of the PM supermultiplet, in a way that deforms the Abelian gauge transformations of the free theory, but not the gauge algebra that remains Abelian.

In order to search for a non-Abelian model where the PM spin-2 field would not remain sterile, we took advantage of the observation that the spectrum of $\mathcal{N}=2$ pure supergravity around AdS_4 is identical to the set of fields $\{h_{\mu\nu}, \phi_\mu^\Delta\}$, $\Delta=1,2$, where $h_{\mu\nu}$ is the partially massless spin-2 field and $\{\phi_\mu^\Delta\}$, $\Delta=1,2$ a doublet of massless gravitini, and classified all the possible non-Abelian deformations of the gauge algebra which can lead to a deformation of the Lagrangian. We found the existence of a single non-Abelian vertex, that coincides with the vertex recently presented in [48], in a different formalism and using different field representations. In the representation we use where the two massless gravitini are Majorana spinors, we showed a very suggestive analogy between this vertex and the minimal coupling in $\mathcal{N}=2$ pure supergravity, except that now, the role of the graviton is taken over by the partially massless spin-2 field. This vertex is a candidate for a partially massless supergravity.

We pushed our analysis to second order in deformation, at the level of the Jacobi identity for the gauge algebra, and showed that the non-Abelian partially massless supergravity vertex (4.23) is obstructed. We then added a massless graviton to the spectrum with its corresponding diffeomorphism gauge parameter, and argued that this addition could potentially be sufficient to cure the obstruction to the Jacobi identity of the resulting gauge algebra, that now includes diffeomorphisms. The final spectrum of fields coincides with the spectrum of $\mathcal{N}=1$ pure conformal supergravity around AdS_4, therefore viewed as the only

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consistent (although non-unitary at the classical level) and non-Abelian theory for what one could call partially massless supergravity. However, the complete analysis at quartic order, that will be presented elsewhere, reveals that the model is still obstructed at quartic order with the free Lagrangian considered in section 4.1, where the two gravitini are decoupled. We are therefore unable to recover $\mathcal{N} = 1$ pure conformal supergravity starting from the free Lagrangian and mass-like matrix used in section 4.1. We intend to study the consistent interactions between a PM spin-2 and a conformal spin-3/2 field in the future to clarify this point and to uncover the form of the PM supergravity model enclosed in $\mathcal{N} = 1$ pure conformal supergravity.

In this work, as long as there was at least one massless Majorana spin-3/2 field in the spectrum, the background considered was AdS_4 with its negative cosmological constant. On the other hand, the results we obtained on the coupling of massive spin-3/2 fields with a vector gauge fields were valid in both AdS_4 and dS_4 backgrounds. The relations we made with conformal supergravity stimulates us to consider an expansion of that theory around dS_4, using the recent findings and spinor field representations of [68, 76, 77]. In fact, it is probably not a coincidence that we recuperated the rigid conformal transformation of one of these works, as our findings suggest that the closure of the transformations laws (4.39), very close to those found in [68], should lead to a representation of the superconformal algebra. We hope to pursue our investigations along those lines in the future, and investigate to which extend and with which field representation for the spinor fields one can expand $\mathcal{N} = 1$ conformal supergravity around dS_4.

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