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Principles and Applications of Model-free Extremum Seeking – A Tutorial Review

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ABSTRACT

This article aims to tutorial a few important extremum seeking control approaches that can be used for the model-free optimization of industrial processes in various fields. The application of several methods is illustrated with a simple case study related to the production of algal biomass in photobioreactors. Other methods and applications are briefly reviewed.

Keywords: Process Control, Optimization, Biosystems

INTRODUCTION

Extremum seeking (ES) has become a large family of perturbation-based methods whose origins can be traced back to the work of the French Engineer Leblanc in 1922, who sought to transmit electrical power to a train car contactlessly [1]. Since then, extremum seeking has gained significant attention, especially in the past three decades, notably thanks to the seminal work of Krstic and his co-workers, who provided conditions for convergence and stability analysis [2]. ES is a practical approach aimed at achieving optimal performance of a system by continuously seeking the extremum of an online-measured cost function. The method finds various applications in different fields, provided that the required measurement information is available, and an optimality principle can be formulated. While many simulation studies have confirmed the effectiveness of extremum seeking, relatively few experimental studies have been conducted to validate its real-world applications. This gap between theory and practice is a common challenge in real-time optimization and control.

This article will provide a tutorial introduction to the foundational principles and applications of extremum seeking control (ESC), using a simple case study of micro-algal growth in a photo-bioreactor (PBR) whose productivity can be adjusted by manipulating the dilution rate. More practical applications are briefly cited and available in previous review papers [3-5].

TUTORIAL EXAMPLE

A dynamic Droop model describes the growth of microalgae by the following differential equations:

$$\dot{X} = \mu(Q,I)X - D\,X \tag{1a}$$

$$\dot{S} = -\rho(S,Q)X - D\,(S - S_{in}) \tag{1b}$$

$$\dot{Q} = \rho(S,Q) - \mu(Q,I)\,Q \tag{1c}$$

where *X*, *S* and *Q* respectively stand for the biomass, nitrogen substrate source, and internal nitrogen quota, *I* is the light irradiance, S_{in} the substrate concentration in the feed, $D = F_{in}/V$ the dilution rate (assumed to be the sole input). In this latter expression, F_{in} is the feed flow rate, and *V* is the photobioreactor volume. μ and ρ are the growth and substrate uptake rates, respectively, which are nonlinear functions of the states and light. μ is modeled by Droop kinetics:

$$\mu(Q,I) = \mu_{max}\frac{I}{K_{I,s}+I+\frac{I^2}{K_{I,i}}}\left(1-\frac{Q_{min}}{Q}\right) \tag{2}$$

and $\rho(S)$ by Monod kinetics:

$$\rho(S,Q) = \rho_{max}\frac{S}{S+K_S}\left(1-\frac{Q}{Q_{max}}\right) \tag{3}$$

where μ_{max} and ρ_{max} are the maximum rate constants, Q_{min} and Q_{max} the minimum and maximum internal quota capacities, $K_{I,s}$ and K_S the half-saturation constants, and $K_{I,i}$ is the rate inhibition constant. Parameter values relative to the strain Isochrysis galbana are reported in [6]. A bifurcation analysis is also achieved in this study, providing stability conditions on the input steady-state value D_e as follows:

$$D_e < \mu_{max} \quad (4a)$$

$$D_e < \frac{\mu_{max}\,\rho_{max}}{\rho_{max} + \mu_{max}\,Q_{min}} \quad (4b)$$

Defining biomass productivity $y = D\,X$ as the cost function to optimize, a steady-state convex objective function can be obtained:

$$y(D_e) = \frac{D_e S_{in}(\mu_{max} - D_e)}{\mu_{max}\,Q_{min}} \quad (5)$$

and the corresponding optimal operating conditions:

$$D_e^* = \frac{1}{2}\mu_{max} \quad (6a)$$

$$y_e^* = \frac{S_{in}\,\mu_{max}}{4\,Q_{min}} \quad (6b)$$

The process model and optimal operating conditions are unknown in the following, but productivity can be measured online at a fast rate (every minute), and the objective of extremum seeking is to determine the optimal operating conditions quickly and reliably.

PRINCIPLES OF ESC

Modulation-Demodulation

The nowadays standard ES approach [2] consists of injecting a periodic perturbation signal (also called dither signal) in the system to extract information using a modulation-demodulation procedure, as represented in Figure 1. The perturbation can be a simple periodic signal, such as a sinusoid with adjustable amplitude and pulsation if only one parameter θ has to be estimated, or a combination of several periodic signals with different frequencies if several parameters have to be extracted. These parameters refer to a measurable static map

$$y = g(\theta) \quad (7)$$

of a dynamic system

$$\begin{aligned} \dot{x} &= f(x, y) \\ y &= h(x) \\ u &= q(x, \theta) \end{aligned} \quad (8)$$

If the perturbation signal changes slowly with respect to the system dynamics, the latter can be assumed in pseudo-steady state (the process output is then noted y_{ss}) and the following relations can be established for the modulated signal $y_{mod}(t)$, the high-pass filtered signal $y_{HP}(t)$, the demodulated signal $y_{demod}(t)$, and the low-pass filtered signal $y_{LP}(t)$

$$\begin{aligned} y_{mod}(t) &\approx y_{ss} + g' a \sin(\omega t) \\ y_{HP}(t) &\approx |G_{HP}(\omega)| g' a \sin(\omega t + \varphi_{HP}) \\ y_{demod}(t) &\approx |G_{HP}(\omega)| g' \frac{a^2}{2} (\cos(\varphi_{HP}) - \cos(2\omega t + \varphi_{HP})) \\ y_{LP}(t) &\approx |G_{LP}(\omega)| |G_{HP}(\omega)| g' \frac{a^2}{2} \cos(\varphi_{HP}) \end{aligned} \quad (9)$$

If the cutoff pulsations of the filters ω_{HP} and ω_{LP} are well-chosen (they must be smaller than the pulsation of the perturbation signal), then

$$y_{LP}(t) \approx g' \frac{a^2}{2} \qquad if \quad \omega > \omega_{HP}, \omega_{LP} \quad (10)$$

and the low-pass filtered signal contains information about the gradient of the static map $g(\theta)$. Modulation-demodulation thus provides a way to estimate the gradient of the cost function to optimize, and the integrator, which follows the low-pass filter in the block diagram of Figure 1, will act as a simple controller pushing to zero this gradient, i.e., seeking for an extremum of the static map.

The application of BOF-ESC (for bank of filters as the modulation-demodulation approach uses several filters) to the microalgal PBR is shown in Figures 2-3.

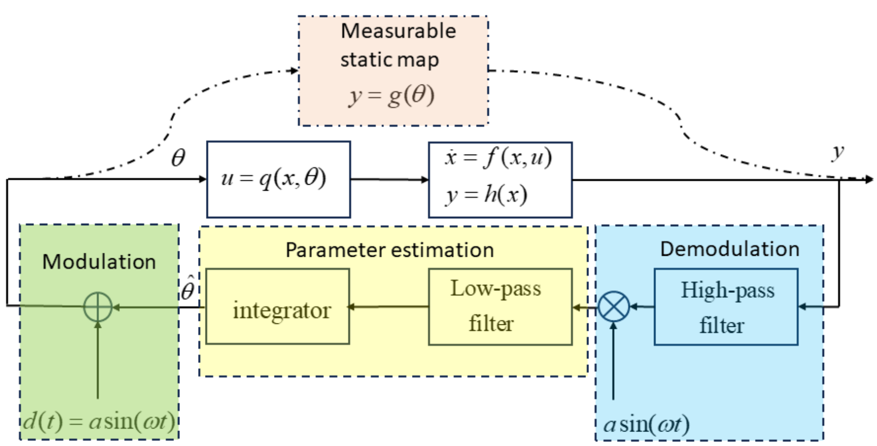

Figure 1: Extremum seeking control based on modulation with a dither signal, and demodulation and parameter estimation using demodulation and a bank of filters – BOF (high pass, low pass filters) + integrator.

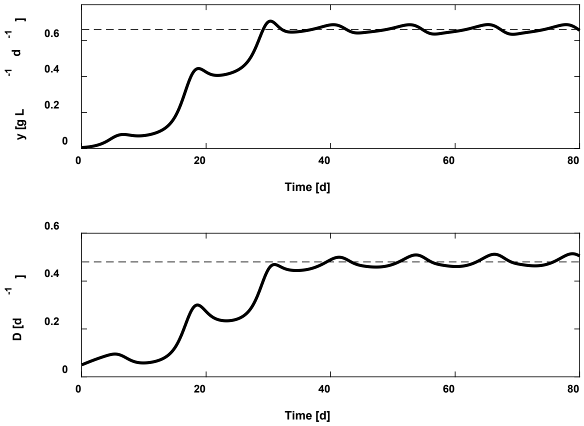

Figure 2. Productivity y and dilution D trajectories over time.

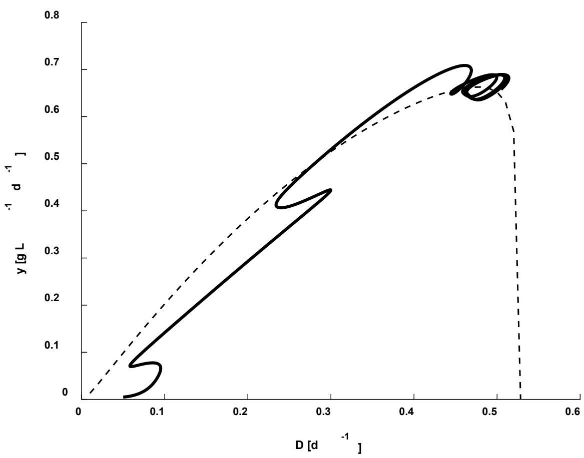

Figure 3. Productivity y vs dilution D diagram.

The perturbation signal is $d(t) = 0.02\,sin(\,0.5t)$, the high-pass filter pulsation $\omega_{HP} = 0.45$ rad/day, and the integrator gains 70. There is no low-pass filter in this design, as the integrator already plays an attenuation role. Figure 3 shows in the dashed line the static map and in the solid line the trajectory of the ESC, with sustained oscillation around the optimum due to the perturbation signal. Convergence is achieved in about 40 days, a pretty long time interval! This highlights the challenges of applying ESC to slow processes (such as those encountered in biotechnology) facing the limitation of the three-time scale separation:

- The process is considered as the fast dynamics;
- The perturbation signal is slow compared to the process dynamics but fast compared to the filter bandwidths;
- Parameter estimation is, therefore, the slowest.

BOF-ESC is a popular approach that has been applied in various fields (see the review paper [3] for a panel of applications) and has been successfully implemented for fast processes such as wind turbines or solar panels (see, for instance, [7] for an application to solar panels). For slower processes, it is critical to accelerate convergence, and various ways are explored in the following.

PI Extremum seeking

One intuitive way to speed up convergence is to use a PI controller instead of a plain integrator, as initially proposed and analyzed in [8]. The results of applying this strategy to our PBR example by including a proportional action with a gain of 0.5 (see Table 1 for a summary of the parameter values) are shown in Figures 4-6. Convergence is twice as fast (i.e., about 20 hours).

Block-Oriented Models and ESC

An alternative is to eliminate the filters and replace the modulation-demodulation approach with an estimator, e.g., recursive least-squares, as originally introduced in [9], or Kalman filters, as proposed in [10] where it is applied to the control of thermoacoustic instabilities (another fast system).

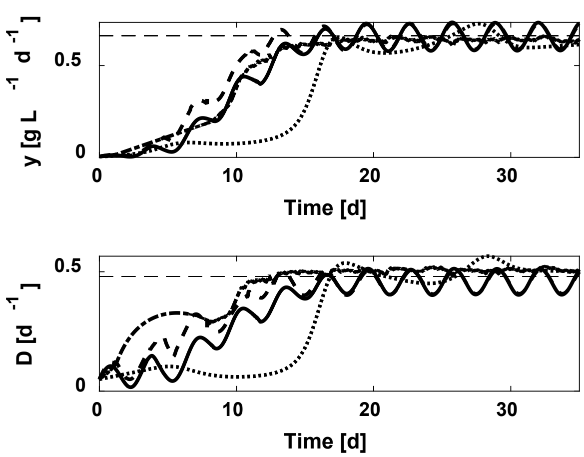

Figure 4. Productivity y and dilution D trajectories over time. Dotted line: PI-ESC; Continuous line: BOM-ESC; Dashed line: NB-ESC; Dashed-dotted line: STAB-ESC.

The structure of this ESC approach is displayed in Figure 7, where the controller can be a simple I control, a PI controller as in the previous subsection, or even a pole placement controller as in [11]. If we consider a recursive least square estimator it is also possible to describe the process in a slightly more detailed way to distinguish the static map from the dominant dynamics, as in Figure 8. The model of choice is a Hammerstein model, which combines a static block with a linear dynamic block (this type of model belongs to the family of block-oriented models – BOM in the following). The consideration of the process dynamics can improve parameter estimation and alleviate biased estimation. In our example, a quadratic map

Table 1. Parameter tuning of the reviewed extremum-seeking methods

Parameter	PI-ESC	BOM-ESC	NB-ESC	STAB-ESC
ω_1 (rad/d)	0.5	2	2	100
ω_2 (rad/d)	/	1	1	/
a_1 (d^{-1})	0.02	0.05	0.05	/
a_2 (d^{-1})	/	0.025	0.025	/
k_i	70	6	6	/
k_p	0.5	0	0	/
g	/	3	3	/
λ	/	0.99	0.99	/
ω_{HP} (rad/d)	0.45	/	/	/
α	/	/	/	0.0002
k	/	/	/	4000

with two parameters, M_1 and M_2, is considered in combination with a first-order transfer function. The estimation of 4 parameters requires a sufficiently rich excitation signal and here $d(t) = a_1 \sin\omega_1 t + a_2 \sin\omega_2 t$. The quadratic map is a local approximation of the local convex optimum. The selected parameters are given in Table 1 and the results displayed in Figures 4-6.

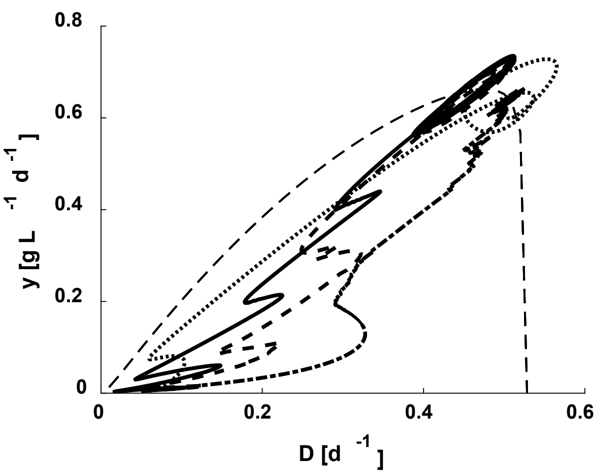

Figure 5. Productivity *y* vs dilution *D* diagram trajectories. Dotted line: PI-ESC; Continuous line: BOM-ESC; Dashed line: NB-ESC; Dashed-dotted line: STAB-ESC.

$$
\begin{aligned}
&e = y - \Phi^T \hat{\xi} \\
&\dot{\hat{\xi}} = K R^{-1} \Phi^T e \\
&\dot{R} = K(\Phi^T \Phi - \lambda R) \\
&\dot{\hat{\theta}} = k_i \, \hat{\xi} \\
&\theta = \hat{\theta} + d, \qquad (11)
\end{aligned}
$$

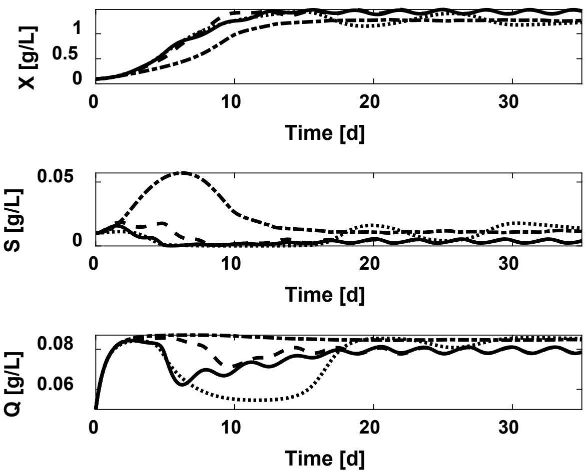

Figure 6. State trajectories over time. Dotted line: PI-ESC; Continuous line: BOM-ESC; Dashed line: NB-ESC; Dashed-dotted line: STAB-ESC.

Newton seeking

It is also possible to improve ESC's line search by considering second-order information in addition to the gradient information. This allows scaling the gradient by the Hessian, as in the classical Gauss-Newton optimization algorithm. This approach was originally introduced in [12]. The application of this technique to our tutorial example is illustrated in Table 1 and Figures 4-6 (NB-ESC). When used in combination with a Hammerstein model, the Hessian is readily available in parameter M_1, which makes the application of the method particularly straightforward. Newton seeking is, however, mostly useful in multiple input problems with different relative gains, where the Hessian scaling will be particularly beneficial.

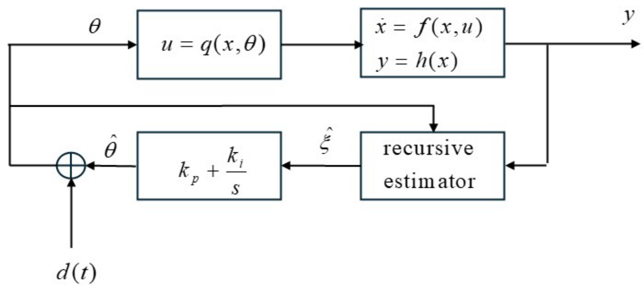

Figure 7. ESC using a parameter estimator (recursive least-squares or Kalman filtering) and a PI controller.

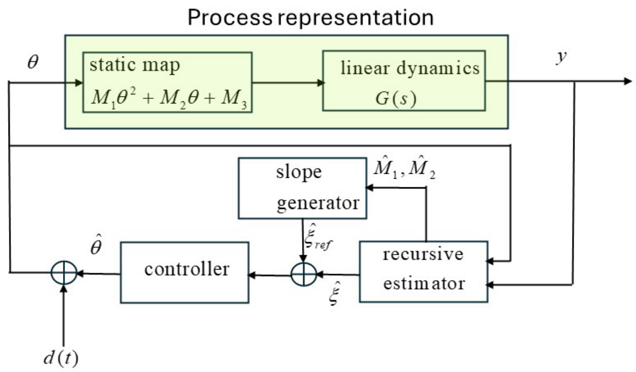

Figure 8. ESC using a parameter estimator (recursive least-squares or Kalman filtering), a block-oriented model (BOM) of the process, a slope generator and a generic controller.

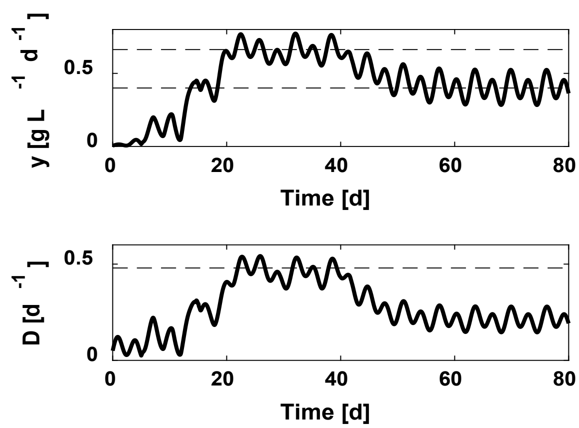

Figure 9. Productivity *y* and dilution *D* trajectories over time with BOM-ESC until 40 days ($y_{ref} = y^*$), followed by slope seeking ($y_{ref} = 0.4$ $g\ L^{-1}\ d^{-1}$) until 80 days.

Slope seeking

As illustrated in Figure 8, using ESC to achieve suboptimal operating points corresponding to non-zero gradients is also possible. Indeed, the controller can achieve any gradient setpoint. This can be used to alleviate the problem posed by a flat optimum, as might be the case in active flow control [13]. This can also be used in conjunction with a slope generator to achieve the desired level of productivity as suggested by [14].

Figure 9 shows a scenario where the extremum seeking is switched after 40 days to a specific slope setpoint, corresponding to a productivity reference $y_{ref} = 0.4$ $g\ L^{-1}\ d^{-1}$.

Stabilizing Extremum seeking

In the search for more robustness, particularly to measurement noise, a new ESC framework was proposed in [15].

The cost function J, assumed to be a cost Lyapunov function (CLF), is included in the argument of a bounded periodic signal

$$\dot{\theta} = \sqrt{\alpha\omega}\, cos(\omega t - kJ) \tag{12}$$

as shown in Figure 9.

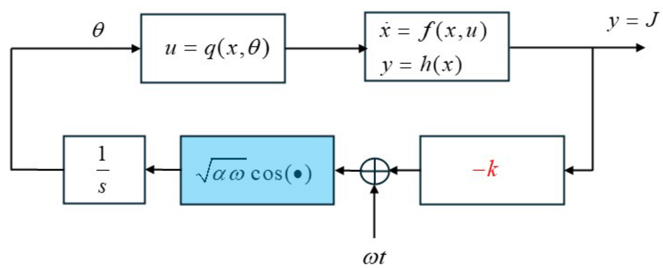

Figure 10. ESC using a parameter estimator (recursive least-squares or Kalman filtering) and a PI controller

As for the original proofs of convergence provided in [2], averaging techniques are required, with in addition, the concept of weak limits. This elaborate mathematical framework is detailed in [15] with the end result that

$$\dot{\bar{\theta}} = \frac{k\alpha}{2}\nabla J \tag{13}$$

The application of this technique is also shown in Table 1 and Figures 4-6.

CONCLUSIONS

Extremum-seeking control is a multifaceted, versatile approach that allows optimizing process operation without the need for the tedious development of a dynamic model. It usually yields very satisfactory results in fast applications (MPPT in solar or wind systems, flow control, etc), but is more delicate to design and tune in slower processes, as illustrated in our tutorial application. Fundamental multi-input aspects can be found in [18]. The interested reader can also find experimental applications of ESC to micro-algae systems in [14, 16] and an application of the recent stabilizing ESC approach [15] to bioprocesses in [17].

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