

Carrollian-Holographic Derivation of Gravitational Flux-Balance Laws

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(Received 9 May 2025; revised 14 July 2025; accepted 3 November 2025; published 31 December 2025)

We demonstrate that the Bondi-van der Burg-Metzner-Sachs (BMS) evolution equations for the mass and angular momentum aspects in asymptotically flat Einstein gravity follow from local Carroll, Weyl, and diffeomorphism invariance at the null conformal boundary, upon providing a minimalistic holographic dictionary as the sole input from the bulk. This result is a significant step in the quest for a flat-space holographic correspondence and offers a geometric implementation of the radiative degrees of freedom that source the boundary theory in the presence of bulk gravitational waves.

DOI: [10.1103/qv17-ks32](https://doi.org/10.1103/qv17-ks32)

Introduction—The holographic paradigm posits a duality between a gravitational theory and a lower-dimensional, nongravitational field theory, suggesting that bulk quantum gravity is encoded in the dynamics of codimension-one boundary degrees of freedom. Although the correspondence remains conjectural, it has yielded striking results in the presence of a negative cosmological constant, where spacetime asymptotically approaches anti-de Sitter geometry and the dual theory is a relativistic conformal field theory. This success has motivated significant efforts over the past decade to develop *flat-space holography*, i.e., an analogous framework for the more realistic setting of asymptotically flat spacetimes. The latter are ubiquitous gravitational models in modern physics as they provide the kinematical arena for a broad class of phenomena, from collider physics to gravitational-wave astronomy, involving localized sources of gravity such that the spacetime geometry asymptotically approaches that of Minkowski in the far, empty region.

Implementing this construction on the foundations of the AdS/CFT correspondence encounters two main challenges. First, the codimension-one conformal boundary of flat spacetime is null, implying that the boundary theory cannot

be relativistic but must instead obey the principles of *Carroll physics* [1,2]. The latter corresponds to the low-velocity limit of special relativity in which spacelike intervals dominate over timelike ones, effectively sending the speed of light to zero and collapsing the light-cone onto the time axis. In this context, the boundary theory is not a conformal field theory in the usual sense, but rather a Carrollian conformal field theory [3–15].

The second key distinction between AdS and flat-space dynamics lies in the fact that asymptotic inertial observers genuinely assess energy and momentum loss due to gravitational waves reaching null infinity. Asymptotically, the frame adapted to such observers is formed by $\mathbf{v}$, the congruence of generators of null infinity, and $\mathbf{e}_a$ ($a = 1, 2$), an orthonormal frame on the two-dimensional celestial sphere. On account of Einstein equations, one finds [16–21] (see also Refs. [22–26] for reviews)

$$\mathbf{v}(M) = \frac{1}{4} \mathcal{D}_a \mathcal{D}_b \mathcal{N}^{ab} - \frac{1}{8} \mathcal{N}_{ab} \mathcal{N}^{ab} + \frac{1}{8} \mathcal{D}_a \mathcal{D}^a \mathcal{R} \quad (1)$$

for the mass aspect M , representing the observed distribution of gravitational energy on the celestial sphere, and

$$\begin{aligned} \mathbf{v}(N_a) = & \mathcal{D}_a M + \frac{1}{4} \mathcal{C}_{ab} \mathcal{D}^b \mathcal{R} - \frac{1}{4} \mathcal{N}^{bc} \mathcal{D}_a \mathcal{C}_{bc} \\ & + \frac{1}{16} \mathcal{D}_a (\mathcal{C}_{bc} \mathcal{N}^{bc}) + \frac{1}{2} \mathcal{D}^b (\mathcal{D}_{[a} \mathcal{D}^c \mathcal{C}_{b]c} + \mathcal{C}^c_{[a} \mathcal{N}_{b]c}) \end{aligned} \quad (2)$$

for the angular momentum aspect N_a . In Eqs. (1) and (2), $\mathcal{D}$ stands for the boundary Weyl-covariant derivative, and $\mathcal{R}$ denotes the related Ricci scalar [27]. The right sides are sourced by $\mathcal{C}_{ab}$, the shear of outgoing null geodesics, and $\mathcal{N}_{ab}$ the so-called Bondi news tensor, whose precise

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relation to $\mathcal{C}_{ab}$ will be disclosed in the main content. It encodes the strain of gravitational radiation: when non-vanishing, gravitational dynamics does not yield conservation equations on the boundary but rather *flux-balance laws* for the asymptotic charges [28–31]. This, once again, stands in contrast to the AdS intuition, where Einstein’s equations reduce to the covariant conservation of a boundary energy–momentum tensor. From the holographic point of view, this means that gravitational radiation would act as *external sources* [8,9,32–34] coupled to the boundary conformal Carrollian field theory that are responsible for breaking the global symmetries on the boundary. This is, of course, a matter of viewpoint, and the present analysis aims to show that the boundary symmetries are, in fact, preserved upon introducing the appropriate geometric tools to describe the boundary theory. In this Letter, we refer to Eqs. (1) and (2) as *Bondi–van der Burg–Metzner–Sachs (BMS) flux-balance laws* [17,18].

More than the null character of the conformal boundary, the nonconservation of gravitational charges presents a significant puzzle and has led to a bifurcation in the development of flat-space holography into two seemingly disconnected approaches. The first, focused solely on radiation, has given rise to the theory of massless *Carrollian amplitudes*, introduced in [8,9] and further explored in [35–43], notably via the asymptotically flat limit of AdS amplitudes [44]. The complementary sector has been investigated either through group-theoretical methods [45,46] or by considering the asymptotically flat limit of the AdS fluid-gravity correspondence [27,47–50], drawing on the substantial progress made in three-dimensional flat-space holography [51–63], where radiative sources are absent. However, aside from the heuristic attempt in Ref. [9], there has been no proper understanding of the coupling between these sources and the remaining Carrollian degrees of freedom; consequently, no intrinsic Carrollian derivation of the key Eqs. (1) and (2) has been achieved.

The aim of this Letter is to resolve this long-standing puzzle in flat-space holography providing such a derivation. To that end, we begin by reviewing the key geometric features of null infinity. We highlight that (part of) the boundary connection depends on extrinsic data [5,64–69], namely $\mathcal{C}_{ab}$ [70,71], motivating the introduction of *hypermomenta* [72–74], associated with variations of the effective action of these additional boundary data. This offers a natural geometric framework for implementing radiative sources and their coupling to the boundary Carrollian conformal field theory. Finally, we establish a holographic dictionary between Carrollian (hyper)momenta and bulk gravitational data [8,9,27] for which the Carrollian dynamical equations derived herein reproduce the BMS flux-balance laws.

Compared to recent analyses, the novelty of our approach is threefold. First, our derivation relies purely on boundary considerations, which reveals the genuinely Carrollian origin of the Bondi flux-balance laws without

employing limiting procedures in the bulk near null infinity. Second, it is developed directly in the conformally compactified spacetime, where null infinity is treated as just another null hypersurface. Finally, it introduces hypermomenta conjugate to the radiative degrees of freedom, which has the merit of clarifying and geometrically grounding their coupling to the putative holographic theory. The latter is of paramount importance for improving our understanding of the flat-space holographic correspondence.

Throughout the Letter, we make use of the transformation properties of various geometric objects under the relevant symmetries. For readability, we have gathered all of these technical expressions in the End Matter, specifically in Appendix A. The latter is supplemented by Appendix B, which provides a detailed derivation of the Letter’s main result, that is, Eq. (20).

Geometry of null infinity—The conformal boundary of asymptotically flat spacetime, denoted by $\mathcal{I}$, is a null hypersurface, so its normal vector is also tangent to it. It therefore induces an intrinsic vector field $\mathbf{v}$ on the boundary, dubbed the *asymptotic field of observers*, which generates the geodesic congruence of null generators of $\mathcal{I}$. Consequently, the pull-back $\mathbf{g}$ of the bulk metric is degenerate in this null direction,

$$\mathbf{g}(\mathbf{v}, \cdot) = 0. \quad (3)$$

Moreover, since the structure on $\mathcal{I}$ emerges from the conformal compactification procedure [75,76], it is endowed with a conformal structure on which Weyl rescalings by an arbitrary smooth nonvanishing function $\mathcal{B}$ are at work [77],

$$\mathbf{g} \mapsto \mathcal{B}^{-2}\mathbf{g}, \quad \mathbf{v} \mapsto \mathcal{B}\mathbf{v}, \quad (4)$$

mapping physically indistinguishable boundary Carroll structures. From (3) and (4), null infinity is endowed with a *conformal Carroll structure* [3–5], which provides a natural distinction between longitudinal and transverse quantities in the following sense. Introducing a clock form τ related to the field of observers $\mathbf{v}$ such that $\tau(\mathbf{v}) = 1$ [5,68], transverse vectors at any point $P \in \mathcal{I}$ are designed to belong to

$$H_P(\mathcal{I}) = \{\mathbf{V} \in T_P\mathcal{I} | \tau(\mathbf{V}) = 0\}. \quad (5)$$

To connect with the usual nomenclature, we refer to the two-dimensional manifold spanned by integral curves of transverse vectors at P as the *cut* of $\mathcal{I}$ at P and take it to be homeomorphic to two spheres. We denote by $\{\mathbf{e}_a\}$ for $a = 1, 2$ an orthonormal frame on $H_P(\mathcal{I})$, in the sense that the boundary metric expands as $\mathbf{g} = \delta_{ab}\theta^a \otimes \theta^b$ in the dual coframe $\{\theta^a\}$, i.e., such that $\theta^a(\mathbf{v}) = 0$ and $\theta^a(\mathbf{e}_b) = \delta^a_b$.

We refer to $\{\mathbf{e}_A\} = \{\mathbf{v}, \mathbf{e}_1, \mathbf{e}_2\}$ as a local *Carroll–Cartan frame* at the point P of the boundary ($A = 0, 1, 2$) and by definition its dual coframe is given by $\{\theta^A\} = \{\tau, \theta^1, \theta^2\}$.

In spite of the metric being degenerate, the boundary volume form is well defined and reads as $\mu = \tau \wedge \theta^1 \wedge \theta^2$ in terms of it. There are several local transformations at work on the defined boundary local frame. First, one can rotate the transverse basis independently of the longitudinal components:

$$\delta_r \mathbf{v} = \mathbf{0} = \delta_r \tau, \quad \delta_r \mathbf{e}_a = r_a^b \mathbf{e}_b, \quad \delta_r \theta^a = r_a^b \theta^b, \quad (6)$$

where $r_{ab} \in \mathfrak{so}(2)$ and $r_{(ab)} = 0$. Second, the clock form is genuinely defined up to a transverse one-form $\lambda_a \theta^a$, which implies the existence of the following transformation:

$$\delta_\lambda \mathbf{v} = \mathbf{0} = \delta_\lambda \theta^a, \quad \delta_\lambda \mathbf{e}_a = \lambda_a \mathbf{v}, \quad \delta_\lambda \tau = -\lambda_a \theta^a. \quad (7)$$

The transformations (6) and (7) correspond to the action of the homogeneous Carroll algebra $\mathfrak{car}(3) \simeq \mathfrak{iso}(2)$ on the frame. In particular, λ_a parameterizes a local Carroll boost. The existence of Weyl transformations (4) on the boundary also affects the local Carroll–Cartan basis infinitesimally as

$$\delta_B \mathbf{e}_A = B \mathbf{e}_A, \quad \delta_B \theta^A = -B \theta^A. \quad (8)$$

We discuss the implications of these local transformations, seen as variational symmetries of the boundary theory, in the next section.

Remark. Contrary to their timelike and spacelike counterparts, the normal vector to a null hypersurface cannot be canonically normalized. Therefore, as the normal vector is also tangent in the null case, this unavoidable ambiguity induces another local transformation:

$$\delta_\eta \mathbf{v} = \eta \mathbf{v}, \quad \delta_\eta \tau = -\eta \tau, \quad \delta_\eta \mathbf{e}_a = \mathbf{0} = \delta_\eta \theta^a. \quad (9)$$

However, because of its extrinsic status compared to the previous local transformations, it is not considered as a symmetry of the boundary theory in what follows.

The boundary basis obeys the following structure equations:

$$[\mathbf{v}, \mathbf{e}_a] \equiv \varphi_a \mathbf{v} - c_a^b \mathbf{e}_b, \quad [\mathbf{e}_a, \mathbf{e}_b] \equiv 2\varpi_{ab} \mathbf{v} + c_{ab}^c \mathbf{e}_c, \quad (10)$$

where ϖ_{ab} measures the nonintegrability of the distribution of cuts (5) on the flow of $\mathbf{v}$ (when $\varpi_{ab} \neq 0$, the cuts do not define a spacelike foliation of $\mathcal{S}$). The evolution of the transverse geometry under the geodesic flow generated by $\mathbf{v}$ is encoded into the rank-two tensor,

$$c_{ab} = \frac{1}{2} \theta \delta_{ab} + \xi_{ab} + c_{[ab]}, \quad (11)$$

where θ measures the expansion of the null congruence $\mathbf{v}$ and the transverse trace-free tensor ξ_{ab} is its shear. On account of Einstein’s equations [78], the latter vanishes identically on the boundary [28,70,79]. Therefore, the bulk Levi–Civita connection naturally induces an affine

connection ∇ on $\mathcal{S}$ which is torsion-free and obeys [70,71]

$$\nabla \mathbf{g} - 2\alpha \otimes \mathbf{g} = \mathbf{0}, \quad \nabla \mathbf{v} + \alpha \otimes \mathbf{v} = \mathbf{0}. \quad (12)$$

The boundary connection is therefore both metric and field-of-observer compatible *up to Weyl rescalings* [80–82]. For the definition (12) to be compatible with the boundary Weyl structure, the boundary one-form α transforms as $\alpha \mapsto \alpha - d \ln \mathcal{B}$ under Weyl rescaling (4). The boundary connection ∇ is therefore naturally a *Weyl-affine Carroll connection* [48]. Einstein equations further impose that $\alpha_0 = \frac{1}{2} \theta$ while the spacelike Weyl connection α_a is encoded into purely bulk metric data and thus constitutes another boundary background field that contributes to the definition of ∇ . In the following, we assume that α is closed for simplicity.

Now comes a crucial observation: if ω^A_B denote the connection one-forms of ∇ in the Carroll–Cartan frame, the vanishing of the torsion and the conditions (12) are insufficient to constrain completely $\beta_{ab} \equiv \omega^0_b(\mathbf{e}_a)$, as they leave $\beta_{(ab)}$ arbitrary. In Carroll geometry, this property is understood as the statement that torsion-free compatible Carroll connections constitute an affine space modeled on transverse symmetric rank-two tensors [5,64–69]. The transverse part of ω^0_a is therefore *not* entirely fixed from purely boundary considerations. However, relating the boundary conformal manifold $\mathcal{S}$ to a bulk geometry allows for resolving this apparent ambiguity and give a physical interpretation of it. Indeed, it can be proven that β_{ab} coincides with the boundary value of the deviation tensor of the congruence of outgoing null geodesics generated by a vector field $\mathbf{k}$, such that the pull-back of the associated one-form to $\mathcal{S}$ is equal to (minus) the clock form τ . This observation derives from an explicit computation of the induced connection on $\mathcal{S}$ from the bulk Weyl–Levi–Civita connection on the unphysical conformally compactified spacetime using the so-called *rigging method* introduced in [83] (see also Refs. [84–87]), with the rigging vector being $\mathbf{k}$. In particular, its traceless part, denoted as $\beta_{(ab)}$, is the *Bondi asymptotic shear* and encodes the two degrees of freedom of bulk gravitational radiation [70]. This concludes our presentation of the relevant kinematical aspects of the boundary geometry.

Carrollian momenta and dynamics—We now turn to the derivation of the dynamical equations that the dual conformal Carrollian field theory is committed to obey. Let S be an action for this theory coupled to the background geometry. In the perspective of [27,47–50,59,60,88], we are agnostic of the microscopic “matter” field content of this theory, which we denote collectively by Φ . However, we demand that it is simultaneously invariant under local Carroll transformations (6) and (7), Weyl transformations (8) and diffeomorphisms to extract general features on the boundary theory.

Taking an arbitrary variation of S yields

$$\delta S = \frac{1}{16\pi G} \int_{\mathcal{S}} \mu (\mathcal{E}_\Phi \delta \Phi + \delta \theta^A (\mathbf{T}_A) + \delta \omega^A_B (\boldsymbol{\Omega}_A^B)) \quad (13)$$

where G is the Newton–Cavendish constant in four dimensions. The first term in Eq. (13) represents the contribution of the “matter” fields and $\mathcal{E}_\Phi \approx 0$ are their equations of motion. From now on, we denote by $\approx$ equalities that hold on-shell for them. Next, variations with respect to the boundary background geometry, which is fully encoded in the Carroll–Cartan coframe, define the *Carrollian momenta* [47,48,88,89] that we expand as

$$\mathbf{T}_0 = \Pi \mathbf{v} + \Pi^a \mathbf{e}_a, \quad \mathbf{T}_a = P_a \mathbf{v} + \Pi^b_a \mathbf{e}_b. \quad (14)$$

They respectively stand for the energy density (Π), the energy flux (Π^a), the momentum density (P_a), and the stress tensor (Π^a_b). Finally, we profit from the main lesson of the previous section, which was that (part of) the boundary connection as an independent field with respect to the geometry. This is why we introduce the vector fields $\boldsymbol{\Omega}_A^B$ as the response to fluctuations of the boundary connection one-forms ω^A_B , and we refer to them as the *hypermomentum* [72–74]. For S to be Weyl invariant, we assume that

$$\mathbf{T}_A \mapsto \mathcal{B}^4 \mathbf{T}_A, \quad \boldsymbol{\Omega}_A^B \mapsto \mathcal{B}^3 \boldsymbol{\Omega}_A^B, \quad (15)$$

i.e., that the Carrollian momenta and hypermomenta transform as weight-four and three Weyl-covariant objects. At this stage, it is important to emphasize that the dynamical fields and the geometry play different roles in the variational principle, Eq. (13). Specifically, both the frame θ^A and the connection ω^A_B are treated as background fields since, unlike Φ , they are not subject to equations of motion. Nonetheless, their variations are retained in the variational principle as they are required to define the associated momenta and derive the constraints dictated by local symmetries (*Noether identities*) [90].

This is precisely what we now set out to do, i.e., we study the constraints imposed on the Carrollian momenta by the local symmetries. Let us start with local Carroll boost invariance. Contracting Eq. (13) with the transformations (7) for the frame and (A4) for the connection and performing an integration by parts to isolate the boost parameter, we obtain

$$\delta_\lambda S = \mathcal{N}_\lambda[\mathcal{E}_\Phi] - \frac{1}{16\pi G} \int_{\mathcal{S}} \mu \lambda_a (\Pi^a + \mathcal{D} \cdot \boldsymbol{\Omega}_0^a), \quad (16)$$

where $\mathcal{N}_\lambda[\mathcal{E}_\Phi]$ is a linear functional in the equation of motion for the fields Φ and $\mathcal{D}$ stands for the Weyl-covariant derivative on the boundary. It acts as

$$\mathcal{D} = \nabla + w\alpha \quad (17)$$

on weight- w quantities and $\mathcal{D} \cdot \mathbf{V} \equiv \mathcal{D}_v V^0 + \mathcal{D}_a V^a$ denotes the divergence of any $\mathbf{V} \in T\mathcal{S}$. Because λ_a is an arbitrary function on $\mathcal{S}$, requiring local Carroll boost invariance yields

$$\delta_\lambda S = 0 \Rightarrow \Pi^a \approx -\mathcal{D} \cdot \boldsymbol{\Omega}_0^a, \quad (18)$$

when the fields Φ are on-shell, and therefore, the functional $\mathcal{N}_\lambda$ vanishes identically. Crucially, the fact that $\Pi^a \neq 0$ is *not* in contradiction with local Carroll boost invariance, which is often thought of as synonymic of the absence of energy flux [91–93]. Our analysis shows that the latter statement is generally incomplete and should be replaced by “Carroll-boost symmetry implies that energy flux is of geometric nature only.” This subtle feature, already derived in [27,48,50] from direct computations, is rooted in the fact that Carroll connections possess degrees of freedom that cannot be fixed by the background geometry, namely $\beta_{(ab)}$, as we stressed earlier.

We can apply similar techniques to derive the remaining Noether identities, namely for rotation, Weyl, and diffeomorphism invariance. First, rotation and Weyl symmetries further respectively fix the skew-symmetric and trace-full parts of the stress tensor as

$$\Pi_{[ab]} \approx \mathcal{D} \cdot \boldsymbol{\Omega}_{[ab]}, \quad \Pi + \Pi^a_a \approx -\mathcal{D} \cdot (\boldsymbol{\Omega}_0^0 + \boldsymbol{\Omega}_a^a), \quad (19)$$

where the spacelike indices are lowered in with δ_{ab} . Furthermore, diffeomorphism invariance, i.e., $\mathcal{L}_\xi S = 0$, gives rise to the sought-after dynamical equations [47,48,88], which take the form of flux-balance equations for the energy–momentum densities as

$$\mathcal{D} \cdot \mathbf{T}_0 \approx \mathcal{R}^A_B(\mathbf{v}, \boldsymbol{\Omega}_A^B), \quad \mathcal{D} \cdot \mathbf{T}_a \approx \mathcal{R}^B_C(\mathbf{e}_a, \boldsymbol{\Omega}_B^C). \quad (20)$$

Further details about their derivation are provided in End Matter (Appendix B). As the latter involves explicitly the constraints (18) and (19), these equations are genuinely Carroll and Weyl covariant. The right sides display the curvature two-form $\mathcal{R}^A_B(\cdot, \cdot)$ associated with the boundary Weyl connection [94]. Already at this stage, it is worth noticing that, since the boundary connection is not completely determined in terms of intrinsic data, the flux terms cannot be completely recast as a divergence [96]. In other words, there exists no local modification of the Carrollian momenta that would transform (20) into covariant conservation equations. This feature was observed long ago in [97] in the study of Galilean fluid dynamics. Upon identifying the Carrollian (hyper)momenta with the appropriate gravitational data, the dynamical equations (20) are nothing but the celebrated BMS evolution equations at null infinity. We prove this statement in the next section by deriving the precise form of the fluxes in the right sides within a concrete boundary gauge fixing. Therefore, the Eq. (20) should be understood as the flat-space avatar of the

conservation of the holographic energy–momentum tensor in the AdS/CFT framework.

Remark. Among the connection one-forms, ω^a_0 and $\omega_{(ab)}$ transform homogeneously under local Carroll and Weyl transformations and can therefore be safely set to zero without breaking any of these symmetries. Consequently, these variables can always be ignored in the variational principle (13), with the related hypermomenta, Ω_a^0 and $\Omega_{(ab)}$, set to zero, which we assume in what follows.

Gravitational flux-balance laws—For definiteness, we opt for relating the Carrollian dynamical equations (20) to the BMS flux-balance laws in the *BMS frame* for which the clock form is exact so that both φ_a and ϖ_{ab} identically vanish, making the distribution of cuts integrable on $\mathcal{I}$. So, there exists a smooth function u on $\mathcal{I}$ such that $\tau = du$, which defines a coordinate along the null direction. Boundary coordinates (u, x^i) are fixed upon providing a given coordinate system (x^i) on the cuts. Furthermore, we require that the clock form is lifted as $e^{-b}du$ (where the function b vanishes at $\mathcal{I}$) in the bulk, which corresponds to choosing null Gaussian normal coordinates (r, u, x^i) in the vicinity of future null infinity. By construction, r is a parameter along the null geodesics generated by $\mathbf{k}$, and making a choice for r would complete the gauge fixing, which we do not discuss here [98].

For these conditions to emerge from the bulk gauge fixing, boundary diffeomorphisms ξ act on the boundary in such a way to preserve the BMS frame. This can be achieved by compensating the change induced by the Lie derivative by appropriate local boost, rotation, and Weyl transformations to put the frame back to its original value [101]. The key point is that the fitting local and Weyl transformations explicitly depend on the diffeomorphism at hand, as we show below. The *gauge-fixed variation* under boundary diffeomorphisms δ_ξ is therefore defined as the appropriate combination of diffeomorphisms and compensating local gauge transformations that fix the (co)frame, i.e.,

$$\delta_\xi \equiv \mathcal{L}_\xi + \delta_{\lambda(\xi)} + \delta_{r(\xi)} + \delta_{B(\xi)}: \quad \delta_\xi \tau = 0 = \delta_\xi \theta^a. \quad (21)$$

Decomposing the diffeomorphism generator as $\xi = f\mathbf{v} + Y^a \mathbf{e}_a$ in the Carroll–Cartan frame, we get that, in order for the gauge-fixed variation to indeed define a gauge-fixing, the local parameters must be determined in terms of the components of the diffeomorphism generator as

$$\lambda_a(\xi) = \mathbf{e}_a(f), \quad B(\xi) = \mathbf{v}(f), \quad r_{ab}(\xi) = \delta_{c[b} \theta^c(\mathcal{L}_{\mathbf{e}_{a]}} \xi) \quad (22)$$

and that the diffeomorphism generator must also satisfy the following constraints:

$$\theta^a(\mathcal{L}_{\mathbf{v}} \xi) = 0, \quad \delta_{c(a} \theta^c(\mathcal{L}_{\mathbf{e}_{b)}} \xi) = \delta_{ab} B(\xi). \quad (23)$$

The first one implies that the spatial components of the diffeomorphism parameter are invariant along the longitudinal direction; ξ therefore generates a Carrollian diffeomorphism in the sense of [5,47]. The second one can be rewritten as $\mathcal{L}_\xi \mathbf{g} = 2B(\xi)\mathbf{g}$, implying that Y^a solves the conformal Killing equation on each cut of $\mathcal{I}$.

For simplicity and to align with the seminal Refs. [28,70,79] we further set $\theta = 0$ from now on at the price of restricting the Weyl freedom at the boundary to functions that are invariant along the null direction, $\mathbf{v}(B) = 0$. We also choose to set $c_{[ab]} = 0$, which restricts rotation transformations to $\mathbf{v}(r_{ab}) = 0$. Both conditions are compatible with the previous equations defining the gauge-fixed variation. The boundary connection one-forms, the solution of Eq. (12), expand as

$$\omega^0_0 = -\alpha_a \theta^a, \quad \omega^0_a = -\alpha_a \tau - \frac{1}{2} C_{ab} \theta^b, \quad (24a)$$

$$\omega^a_0 = 0, \quad \omega^a_b = \gamma^a_{cb} \theta^b, \quad (24b)$$

where C_{ab} is symmetric and taken trace-free by choice of a subleading Weyl rescaling [103] in the direction of $\mathbf{k}$; see, e.g., Refs. [99,104]. As mentioned above, it corresponds to the Bondi shear of the dual asymptotically flat spacetime. The purely horizontal connection coefficients,

$$\gamma^a_{bc} = \frac{1}{2} (c^a_{bc} + c^a_{cb} + c^a_{ca}) - \delta^a_b \alpha_c - \delta^a_c \alpha_b + \alpha^a \delta_{bc}, \quad (25)$$

are those of the Weyl–Levi–Civita connection on the cuts. Within the same set of hypotheses, the diffeomorphism parameters then obey

$$\mathbf{v}(f) = \frac{1}{2} \mathcal{D}_a Y^a, \quad \mathbf{v}(Y^a) = 0, \quad 2\mathcal{D}_{(a} Y_{b)} = \mathcal{D}_c Y^c \delta_{ab}, \quad (26)$$

which define an element of $\mathfrak{bm}\mathfrak{g}_4$, the BMS algebra in four dimensions [17–19]. The gauge-fixed variation (21) then induces a representation of this algebra on the boundary and acts therefore as expected from the bulk analysis [21,105] on dynamical variables as, e.g., the asymptotic shear [106]:

$$\delta_\xi \mathcal{C}_{ab} = f\mathbf{v}(\mathcal{C}_{ab}) + (\mathcal{L}_Y \mathcal{C})_{ab} - \mathbf{v}(f)\mathcal{C}_{ab} - 2\mathcal{D}_{(a} \mathcal{D}_{b)} f. \quad (27)$$

Importantly, the inhomogeneous piece above betrays that the transverse tensor $\mathcal{C}_{ab}$ appears in the spacelike connection under Carroll boosts.

The Carroll and Weyl-covariant flux-balance laws (20) now read explicitly in the BMS frame as

$$\mathbf{v}(\Pi) = \mathcal{D}_a \mathcal{D}_b \Omega_0^{ba} + \mathbf{v}(\mathcal{D}_a \Omega_0^{0a}) - \frac{1}{2} \mathcal{N}_{ab} \Omega_0^{ab}, \quad (28a)$$

$$\begin{aligned} \mathbf{v}(P_a) = & \frac{1}{2} \mathcal{D}_a \Pi - \mathcal{D}^b \Pi_{[ab]} + \frac{1}{2} \mathbf{v}(\mathcal{D}_a \Omega_0^{00} + \mathcal{D}_a \Omega_b^{0b}) \\ & + \frac{1}{2} \mathcal{D}_a \mathcal{D}_b (\Omega_0^{b0} + \Omega_c^{bc}) + \mathbf{v}(\mathcal{D}^b \Omega_{[a|0|b]}) + \mathcal{R} \Omega_{[a}{}^b{}_{b]} \\ & + \mathcal{D}^b \mathcal{D}^c \Omega_{[a|c|b]} + \frac{1}{2} \mathcal{N}_{ab} \Omega_0^{0b} - \frac{1}{2} \mathcal{D}_{[a} \mathcal{C}_{b]c} \Omega_0^{bc}, \end{aligned} \quad (28b)$$

where the constraints (18) and (19) have been extensively used and $\Omega_A^{CB} \equiv \theta^C(\Omega_A^B)$ denote the components of the hypermomentum. Note that the news tensor has been defined as $\mathcal{N}_{ab} \equiv -2\mathcal{R}_{ab}^0$ [27]. This definition coincides with the retarded-time-derivative of the shear, up to geometrical contributions ensuring its Weyl-covariance. A straightforward computation shows that, upon identifying the hypermomenta as

$$\begin{aligned} \Omega_0^0 &= 0, & \Omega_{ab} &= \Omega_{[ab]} = \mathcal{D}_{[a} \mathcal{C}_{b]} \mathbf{e}_c, \\ \Omega_0^a &= \left(\mathcal{N}^{ab} + \frac{1}{2} \mathcal{R} \delta^{ab} \right) \mathbf{e}_b, \end{aligned} \quad (29)$$

the Eqs. (28a) and (28b) reproduce the BMS flux-balance equations (1) and (2), provided that the Carrollian momenta are given in terms of the bulk gravitational data as

$$\Pi = 4M, \quad P_a = 2N_a + \frac{1}{16} \mathcal{D}_a (\mathcal{C}^{bc} \mathcal{C}_{bc}), \quad (30a)$$

$$\Pi^a = -\mathcal{D}_b \mathcal{N}^{ab} - \frac{1}{2} \mathcal{D}^a \mathcal{R}, \quad (30b)$$

$$\Pi_{ab} = \mathcal{D}^c \mathcal{D}_{[a} \mathcal{C}_{b]c} - \frac{1}{2} \mathcal{N}_{[a}{}^c \mathcal{C}_{b]c} - \frac{1}{4} \mathcal{R} \mathcal{C}_{ab} - 2M \delta_{ab}. \quad (30c)$$

Equations (29) and (30) constitute our proposal for the flat-space holographic dictionary for gravitational fields. In particular, the boundary momenta are identified, up to radiative decorations, with the bulk gravitational momenta M, N_a , which appear at third subleading order in the conformally rescaled metric, exactly as in the AdS case [107,108]. This completes our boundary-intrinsic and first-principles derivation of the BMS evolution equations for asymptotically flat gravitational fields, which is an important step forward in understanding holographic duality in this context.

Discussion—To conclude our analysis, some remarks are in order. First, the holographic dictionary (30) allows for recovering the BMS charges computed via covariant-phase-space methods from a purely boundary analysis. Indeed, taking the identifications (30) into account, the boundary Noether charges on any cut Σ of $\mathcal{I}$ are given by

$$\begin{aligned} \mathcal{Q}_{(f,Y)} &= \frac{1}{16\pi G} \int_{\Sigma} \mu_{\Sigma} (f\Pi + Y^a P_a) \\ &= \frac{1}{16\pi G} \int_{\Sigma} \mu_{\Sigma} \left(4fM + 2Y^a N_a + \frac{1}{16} Y^a \mathcal{D}_a (\mathcal{C}_{bc} \mathcal{C}^{bc}) \right), \end{aligned} \quad (31)$$

where $\mu_{\Sigma} \equiv \theta^1 \wedge \theta^2$ is the volume form on Σ , which corresponds exactly to the Barnich–Troessaert charges [31]. Whether these Carrollian momenta correspond to the Brown–York energy–momentum tensor of null infinity [84], as is the case in AdS [108,109], is a relevant question which deserves further investigation. Furthermore, the rationale behind the fact that our analysis lands on the “bare” BMS momenta of [31] and not their covariant avatars, conveniently encoded by the Newman–Penrose coefficients Ψ_1^0 and Ψ_2^0 and promoted into preferred BMS charge prescription in [24,27,110–113], is that we have also considered the “bare” momenta coming from the variation (13) without any improvement. The right sides in Eq. (20) contain some terms that can be recast as divergences, which are able to nourish the “bare” momenta with radiative contributions and transform them into their covariant counterpart.

In conclusion, our Letter sheds a new light on the properties of the putative dual field theory in flat-space holography. First, it shows that the asymptotic Einstein equations can be derived in a holographic manner with a very minimalistic holographic dictionary at disposal. In particular, the use of hypermomenta techniques in this setting offers a geometric perspective on the interplay between the dual theory and the radiative sources, further supporting the heuristic approach of Ref. [9]. Second, it allows for a genuinely Carrollian understanding of asymptotically flat gravity and its presumed holographic realization without relying on limits from AdS [27]. Finally, it paves the way toward grasping the microscopic structure of the dual theory, namely the subsector governing the sources: at this stage, it appears clear that the latter should encompass a theory for the connection itself, whose variation with respect to boundary geometric data is constrained by the form (29) of the hypermomenta. Unravelling this theory could be of relevance for Carrollian amplitudes.

Acknowledgments—We would like to thank Ismaël Ahlouche, Mathieu Beauvillain, Andrea Campoleoni, Marc Geiller, Daniel Grumiller, Damianos Iosifidis, Lionel Mason, Anastasios C. Petkou, Antoine Rignon-Bret, Romain Ruzzi, Konstantinos Siampos, and Céline Zwikel for useful discussions. A.F. acknowledges support from the Austrian Science Fund (FWF) [Grants DOI: 10.55776/P33789 and DOI: 10.55776/P36619] (Austria) during the early phases of this work (Projects No. P 33789 and No. P 36619). The work of S.P. was supported by the Fonds Friedmann run by the Fondation de

l'École polytechnique until October 31, 2024. Since November 1, 2024, S.P. has been supported by the European Research Council (ERC) Project 101076737—CeleBH. S.P. is also partially supported by INFN Iniziativa Specifica ST&FI. The work of M.V. is supported by the F.R.S.-FNRS (Belgium) under the Grant No. T.0047.24. M.V. is grateful to the Centre de Physique Théorique de l'École polytechnique, and in particular Jean-René Chazottes, for financial support during summer 2024. Finally, we would like to acknowledge support from the Partenariat Hubert Curien France/Grèce, project entitled “Symétries asymptotiques en gravitation, théories des champs conformes et holographie.” We are also grateful to the Laboratory of Theoretical Physics of the Aristotle University of Thessaloniki and the Mathematical Institute of the University of Oxford for the organization of brainstorming meetings, where parts of this work have been carried out and its fundamental ideas have been discussed.

Views and opinions expressed are however those of the author only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

Data availability—No data were created or analyzed in this study.

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End Matter

Appendix A: Transformation laws—We collect here the transformation laws of the structure functions and the connection one-forms under infinitesimal local transformations and diffeomorphisms on the boundary. We keep denoting by λ_a and r^a_b the infinitesimal parameters of local Carroll rotations and boosts, which act respectively as (6) and (7) on the Carroll–Cartan basis.

As our definition of the structure functions is still given by Eq. (10), we can check that

$$\delta_\lambda \varphi_a = \lambda_c c^c_a + \mathbf{v}(\lambda_a), \quad (\text{A1a})$$

$$\delta_\lambda \varpi_{ab} = \mathbf{e}_{[a}(\lambda_{b]}) + \lambda_{[a} \varphi_{b]} - \frac{1}{2} \lambda_c c^c_{ab}, \quad (\text{A1b})$$

$$\delta_\lambda c^a_b = 0, \quad (\text{A1c})$$

$$\delta_\lambda c^c_{ab} = 2c^c_{[a} \lambda_{b]} \quad (\text{A1d})$$

under local Carroll boosts;

$$\delta_r \varphi_a = r^b_a \varphi_b, \quad (\text{A2a})$$

$$\delta_r \varpi_{ab} = r^c_a \varpi_{cb} + r^c_b \varpi_{ac}, \quad (\text{A2b})$$

$$\delta_r c^a_b = r^a_c c^c_b + r^c_b c^a_c + \mathbf{v}(r^a_b), \quad (\text{A2c})$$

$$\delta_r c^a_{bc} = r^a_d c^d_{bc} + r^d_b c^a_{dc} + r^d_c c^a_{bd} - 2\mathbf{e}_{[b}(r^a_{c]}) \quad (\text{A2d})$$

under local rotations; and finally

$$\delta_B \varphi_a = B \varphi_a - \mathbf{e}_a(B), \quad (\text{A3a})$$

$$\delta_B c^a_b = B c^a_b - \delta^a_b \mathbf{v}(B), \quad (\text{A3b})$$

$$\delta_B \varpi_{ab} = B \varpi_{ab}, \quad (\text{A3c})$$

$$\delta_B c^a_{bc} = B c^a_{bc} - 2\delta^a_{[b} \mathbf{e}_{c]}(B) \quad (\text{A3d})$$

under Weyl rescalings. From Eq. (A3d), it can be observed that the gauge choice $\theta = c^a_a = 0$ implies the reduction

of Weyl transformations to time-invariant parameters, $\mathbf{v}(B) = 0$. The additional choice $c_{[ab]} = 0$ reduces the local symmetry group to time-invariant rotations, $\mathbf{v}(r_{ab}) = 0$, as indicated by Eq. (A2c). Assuming this, the transverse tensor c_{ab} identically vanishes on account of asymptotic Einstein equations, as explained in the main text.

Let us now consider a general connection ∇ on $T\mathcal{J}$. Looking at the transformation laws of the related connection one-forms ω^A_B , we find

$$\delta_\lambda \omega^0_0 = -\lambda_a \omega^a_0, \quad \delta_\lambda \omega^a_0 = \mathbf{0}, \quad (\text{A4a})$$

$$\delta_\lambda \omega^0_a = \nabla \lambda_a + \lambda_a \omega^0_0, \quad \delta_\lambda \omega^a_b = \lambda_b \omega^a_0 \quad (\text{A4b})$$

under local Carroll boosts;

$$\delta_r \omega^0_0 = \mathbf{0}, \quad \delta_r \omega^a_0 = r^a_b \omega^b_0, \quad (\text{A5a})$$

$$\delta_r \omega^0_a = r^b_a \omega^0_b, \quad \delta_r \omega^a_b = \nabla r^a_b \quad (\text{A5b})$$

under local rotations; and

$$\delta_B \omega^0_0 = dB, \quad \delta_B \omega^a_0 = \mathbf{0}, \quad (\text{A6a})$$

$$\delta_B \omega^0_a = \mathbf{0}, \quad \delta_B \omega^a_b = \delta^a_b dB \quad (\text{A6b})$$

under Weyl rescalings. In Eqs. (A4b) and (A5b), the bold symbol $\nabla(\cdot) = \tau \nabla_\nu(\cdot) + \theta^a \nabla_a(\cdot)$ represents the covariant exterior derivative. The connection one-forms ω^0_0 (together with ω^a_a), ω^0_a , and $\omega_{[ab]}$ are respectively recognized as Weyl, boost, and rotation connections. Furthermore, ω^a_0 transforms homogeneously under all the local transformations. The latter can always be set to zero without any restriction as it can be shown to form the connection for Galilean boosts,

$$\delta_w \mathbf{v} = w^a \mathbf{e}_a, \quad \delta_w \mathbf{e}_a = \mathbf{0}, \quad \delta_w \boldsymbol{\tau} = \mathbf{0}, \quad \delta_w \boldsymbol{\theta}^a = -w^a \boldsymbol{\tau}, \quad (\text{A7})$$

that mirror the operation of (7) and are absent in Carroll relativity. In the same vein, we observe that $\omega_{\langle ab \rangle}$ also

transforms homogeneously under all the local Carroll and Weyl transformations and can always be safely set to zero.

Appendix B: Derivation of Carroll flux-balance laws—In this appendix, we provide some details about the derivation of Eq. (20). We start with the general variation (13) of the boundary action; we evaluate it on a boundary diffeomorphism ξ and then demand S to be invariant under its action when the fields Φ are on-shell:

$$\mathcal{L}_\xi S \approx \frac{1}{16\pi G} \int_{\mathcal{J}} \mu \left(\mathcal{L}_\xi \theta^A(\mathbf{T}_A) + \mathcal{L}_\xi \omega^A_B(\Omega_A^B) \right) \approx 0. \quad (\text{B1})$$

The Lie derivatives that appear in the integrand are computed using Cartan's magic formula as

$$\begin{aligned} \mathcal{L}_\xi \theta^A &= (\mathbf{e}_B(\xi^A) + C^A_{BC} \xi^C) \theta^B, \\ \mathcal{L}_\xi \omega^A_B &= \left(\xi^C \mathbf{e}_C(\omega^A_{DB}) + \omega^A_{CB} \mathbf{e}_D(\xi^C) + \omega^A_{CB} C^C_{DE} \xi^E \right) \theta^D, \end{aligned} \quad (\text{B2})$$

where $[\mathbf{e}_A, \mathbf{e}_B] = C^C_{AB} \mathbf{e}_C$ and $\omega^A_B = \omega^A_{CB} \theta^C$. At the moment, ω^A_B are the one-forms of a general connection on the boundary. To isolate the diffeomorphism parameter, we need an inverse Leibniz rule on the differential operator $\mathbf{e}_A$, which is again derived using Cartan's magic formula and reads as

$$\mu \mathbf{e}_A(g) = d(\dots) - \mu(g \mathbf{e}_A(f) - f g C^B_{AB}) \quad (\text{B3})$$

for all functions f, g on $\mathcal{J}$. Since the boundary metric admits no inverse, the computation should be done separately for $A = 0$ and $A = a$. The final result can be recast covariantly with respect to the Carroll–Cartan frame as in Eq. (B3).

Next, we keep in mind that, after taking the variation, the result needs to be evaluated for the boundary connection ∇ which is torsion-free and obeys (12). These conditions are solved by

$$\omega^0_0 = -\alpha, \quad \omega^a_0 = 0, \quad \omega_{(ab)} = -\alpha \delta_{ab}, \quad (\text{B4})$$

and the other connection coefficients are determined using the fact that the torsion vanishes, which, by virtue of first Cartan structure equation, implies

$$d\theta^A + \omega^A_B \wedge \theta^B = 0 \Rightarrow \omega^A_{[BC]} = \frac{1}{2} C^A_{BC}. \quad (\text{B5})$$

We then find

$$\omega^0_{0a} = \varphi_a - \alpha_a, \quad \omega^0_{ab} = \beta_{(ab)} + \varpi_{ab}, \quad (\text{B6a})$$

$$\omega^a_{0b} = -c^a_b, \quad \omega^a_{bc} = \gamma^a_{bc} = (25), \quad (\text{B6b})$$

where $\beta_{(ab)}$ are utterly free data, as we explained in the main text. Note that the identities $\omega^B_{AB} = -3\alpha_A$ and $\omega_{(a|C|b)} = -\alpha_C \delta_{ab}$ hold, which is verifiable by direct computation. The condition (B1) can then be worked out to get

$$\int_{\mathcal{J}} \mu \xi^A \left(\mathcal{D} \cdot \mathbf{T}_A - \mathcal{R}^C_{BAD} \Omega_C^{DB} + \omega^C_{AB} \Delta^B_C \right) \approx 0, \quad (\text{B7})$$

where the boundary term has been discarded and ξ^A is a general function. The last term involves the combination $\Delta^B_C \equiv T^B_C + \mathcal{D} \cdot \Omega_C^B$ and can thus be shown to vanish if one requires local Carroll and Weyl invariance. Indeed,

$$\begin{aligned} \omega^C_{AB} \Delta^B_C &= -\alpha_A \Delta^0_0 + \omega^0_{Ab} \Delta^b_0 + \omega^c_{Ab} \Delta^b_c \\ &= \omega_{cAb} \Delta^{[bc]} - \alpha_A (\Delta^0_0 + \Delta^a_a) = 0. \end{aligned} \quad (\text{B8})$$

The first equality above holds by virtue of the second condition in Eq. (B4). Thanks to Carroll-boost invariance, the second term cancels out, since condition (18) can be rewritten as $\Delta^a_0 = 0$. One then obtains the second equality in (B8) by decomposing the spacelike indices (b, c) into their symmetric and antisymmetric parts and using the third condition in Eq. (B4). The result vanishes by virtue of the constraints (19) imposed by local rotation and Weyl invariance, i.e., $\Delta_{[ab]} = 0$ and $\Delta^A_A = 0$. Therefore, Eq. (B7) implies the general flux-balance laws displayed in Eq. (20).