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Asymmetric Smith-Purcell radiation via coupled high-index gratings

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***Abstract* – In this work, we present a structure consisting of a double photonic crystal with a high-refractive-index substrate. By shifting the two crystals, we induce an asymmetry in the Smith-Purcell radiation between the upper and lower directions. This asymmetry is explained using a semi-analytical model based on Coupled Mode Theory.**

I. INTRODUCTION

Smith-Purcell Radiation (SPR) results from the interaction of a relativistic electron beam with a periodic surface. Since its discovery in 1953, this emission has gained interest for applications in compact terahertz sources, high-resolution spectroscopy, and imaging. The broad spectral coherence is valuable for non-destructive diagnostics and wakefield accelerators. Recent studies have optimized SPR emission using various geometries and materials. For example, dielectric photonic crystals improve dispersion and directionality control, while Bound States in the Continuum (BICs) enhance emission.

This study employs numerical simulations and Coupled Mode Theory (CMT) to report and explain the asymmetric emission when two photonic crystals are in a staggered configuration. Unlike prior work, we focus on emission into a high-index substrate (n = 3.91). Analyzing the resonances via CMT reveals the interference and coupling effects, aiding the optimization of these devices. This work thus supports the development of tunable and broadband photonic crystal sources.

II. GEOMETRY AND DISPERSION

Fig. 1a illustrates our basic unit cell, which consists of a silicon tooth (n = 3.91) with a height $L = 200\ nm$ and a width $w = 100\ nm$ (50% fill factor). This grating is placed on a variable-thickness SiO_2 layer (n = 1.4). The entire structure is supported by a silicon substrate (n = 3.91) and is surrounded by air (n = 1). The color codes in Fig. 1a-b represent the different materials. An electron beam is positioned at a distance d_e from the top of the grating.

In the second structure (Fig. 1b), we duplicate the grating with respect to the x-axis, resulting in a grating on either side of the electron beam. To generate asymmetric radiation between the top and bottom of the structure, we shift the gratings by a distance of $\alpha_x w$. A value $\alpha_x = 0$ corresponds to perfectly aligned gratings, while $\alpha_x = 1$ represents gratings in perfect anti-phase. The electrons have an energy of 15 keV, allowing for the generation of a single SPR order (order -1), as indicated in Fig. 1c.

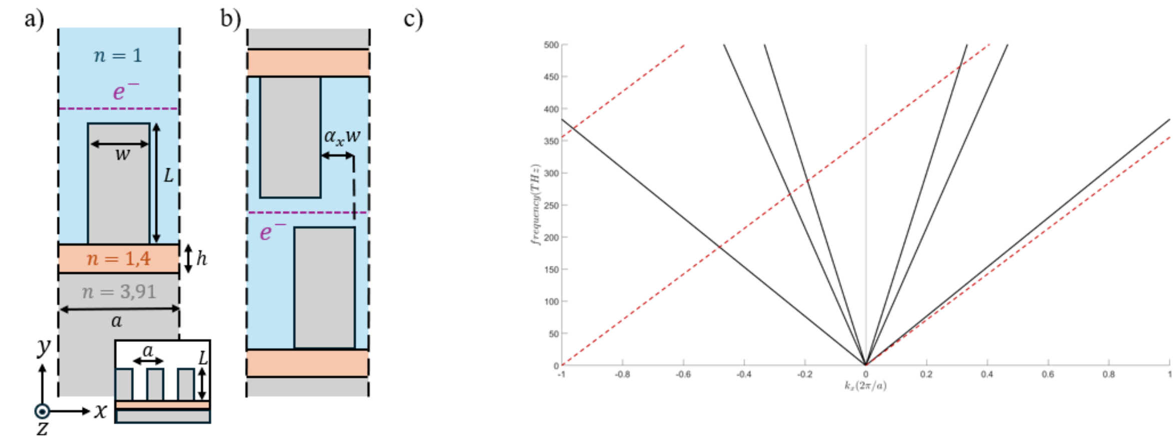

Fig. 1. a) The structure with a single grating. b) The structure with two gratings. In both cases, the electron is positioned at a distance d_e from the grating. c) Dispersion of the light cones for the different materials, along with the SPR orders for an electron energy of 15 keV.

III. Single Grating Results

We begin by exciting the structure with a single grating. The resulting spectrum is shown in Fig. 2c, where a peak appears around 250 THz for a zero SiO_2 thickness. The position and intensity of this peak vary depending on the SiO_2 thickness.

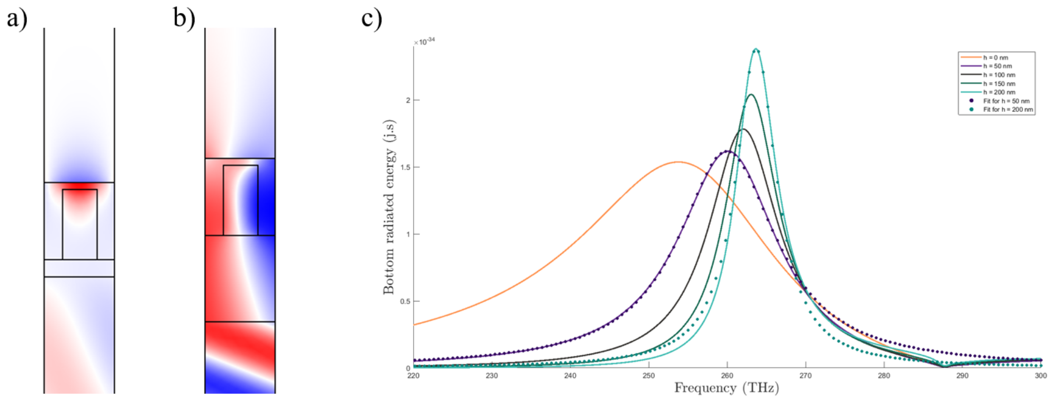

Fig. 2. a) and b) Out-of-plane (z-direction) magnetic field profile for the resonance peak at a) $h = 50\,nm$ and b) $h = 250\,nm$. c) Spectrum obtained via electron excitation (solid lines) with the fitted model (dots).

We can then fit our model for a single grating using the following equation, where a represents the mode amplitude, ω_0 the resonance angular frequency, γ the losses, and s the source amplitude.

$$\frac{da}{dt} = (j\omega_0 - \gamma)a + s \qquad a = a_0 e^{j\omega t} \qquad s = s_0 e^{j\omega t} \tag{1}$$

By adjusting γ for different values of h, we obtain the dots in Fig. 2c, demonstrating an excellent fit between the model and the spectra obtained from electron excitation.

IV. Double Grating Results

We examine the results obtained with the double structure from Fig. 1b. Our theoretical model now includes coupling between the mode in the upper and lower gratings:

$$\frac{da_1}{dt} = (j\omega_0 - \gamma)a_1 + j\kappa a_2 + s_1 \qquad a_1 = a_{1_0} e^{j\omega t} \qquad s_1 = s_{1_0} e^{j\omega t} \tag{2}$$

$$\frac{da_2}{dt} = (j\omega_0 - \gamma)a_2 + j\kappa a_1 + s_2 \qquad a_2 = a_{2_0} e^{j\omega t} \qquad s_2 = s_1 e^{j\phi} \tag{3}$$

Here, κ represents the coupling constant between the two modes, s_1 is the source amplitude of mode 1, s_2 is the source amplitude of mode 2, and ϕ is the phase shift between the two sources.

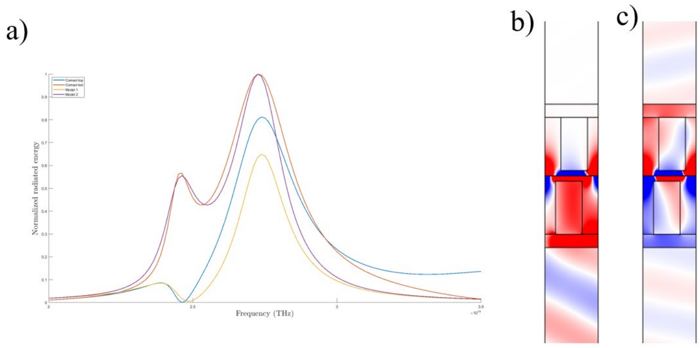

Fig. 3. a) Fit between the COMSOL simulation and the model for $\alpha_x = 0.2$. Out-of-plane (z-direction) magnetic field profile for b) f = 246 THz and c) f = 272 THz.

By adjusting the parameters, we obtain the results displayed in Fig. 3a, where a qualitatively satisfying fit can be observed for $\alpha_x = 0.2$, corresponding to $\phi = 0.2$. Fig. 3b and 3c show the out-of-plane magnetic field profile for two different frequencies, with Fig. 3b showing a strong asymmetry between the top and bottom radiation, while in Fig. 3c the radiation is nearly identical on the two sides of the double array.

The advantage of this model is that once the parameters are fitted for a few specific cases, it enables the exploration of a wide range of possible configurations due to its analytical nature. This allows for a qualitative understanding of phenomena observed in the structure. Another benefit of the model is the variety of parameters that can be explored. A lower γ simulates reduced losses, leading to higher quality resonances, which can be used to model different SiO_2 thicknesses. The κ parameter affects the coupling between the two modes, allowing the simulation of varying distances between the gratings and the electron beam.

VI. Conclusion

In this work, we demonstrate that by duplicating an array and inducing a symmetry breaking between the top and bottom of the structure, it is possible to generate asymmetric radiation through the excitation of the photonic crystal with an electron beam. We then fit the radiation obtained via COMSOL to a semi-analytical model based on Coupled Mode Theory, thereby enabling rapid exploration of a large number of parameters.

References

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