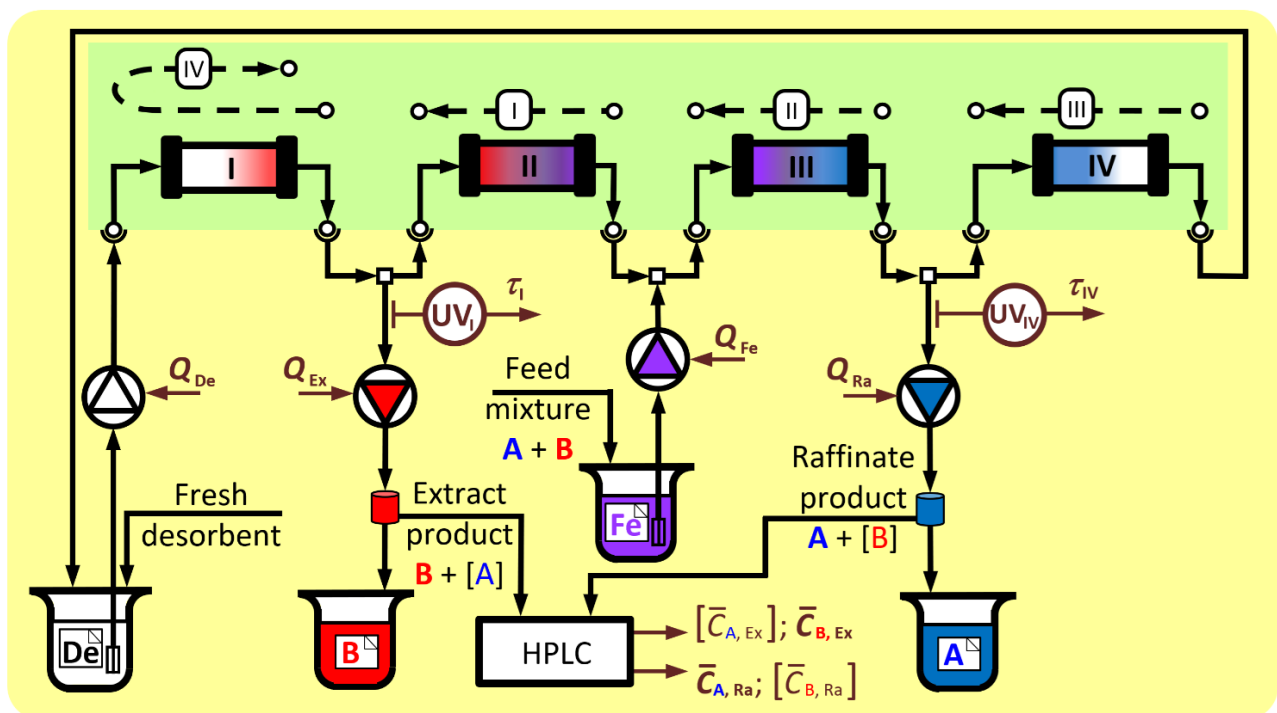


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Moving Horizon Estimation and Control of Simulated Moving Bed Chromatographic Processes

Doctoral thesis



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Moving Horizon Estimation and Control of Simulated Moving Bed Chromatographic Processes

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Abstract

Chromatography is a powerful separation technique mainly used for difficult separations of temperature-sensitive substances or substances with similar properties that are difficult to separate by other methods. Among the various chromatographic processes, the simulated moving bed (SMB) process, which is the continuous implementation of the basic chromatographic process, has found great practical application.

The SMB process allows for significantly higher productivity than the chromatographic batch process while simultaneously reducing solvent consumption, thereby lowering the cost per unit of production. Its main disadvantage is its complexity. The process is inherently nonlinear, both due to the chromatographic system involved, consisting of the solutes, the adsorbent and the desorbent, and the nature of the interaction between them. Furthermore, at their economic optimal operating point, SMB processes are highly sensitive to disturbances. In practice, SMB processes are most often operated in an open-loop mode, with the selected operating point not at the optimum but at a suboptimal one located in an operating region with higher robustness to disturbances but reduced productivity. That is why the use of automatic control in SMB operations is particularly attractive.

From a control perspective, SMB processes are very challenging – they are characterised by complex hybrid dynamics including cyclic operation, long settling times, interconnected control channels, and limited measurement information. That requires a deep understanding of the process. Mathematical modelling and computer simulations are indispensable tools for the design, optimisation, and control of such processes, which are also used in this work.

This thesis builds upon the research on the control of SMB processes first carried out by Marco Fütterer (Fütterer, 2010) and then further continued by Paul Suvarov (Suvarov, 2016) in Magdeburg and Mons. They proposed a simple yet powerful control strategy, which was successfully validated experimentally by Suvarov, for SMB processes operating with adsorption isotherms in the linear region of the isotherms. At the same time, this strategy showed some weaknesses when operating in the nonlinear region of the isotherms, which is the main focus of the current thesis, particularly for chromatographic systems with Langmuir isotherms.

The pilot plant considered has four controlled variables – the two product purities and two dimensionless retention times to control the total regeneration in the outer zones. The manipulated variables are the flowrates in each of the zones of the SMB plant. The original control concept of Suvarov is first further extended by adding two decouplers to reduce cross-coupling in the product control channels, thus providing a decentralised control scheme. Moreover, to achieve a faster response, the PI controllers are replaced with PID ones. The results showed that this control strategy performs very well around the operating point at which the controllers and decouplers are tuned. However, performance deteriorates when not operating close to it. Moreover, the capability

of handling input constraints is limited.

Therefore, a different control strategy based on online optimising control was developed. In particular, a novel semi-centralised control scheme is proposed in which the two product purities are controlled by a nonlinear model predictive controller, whereas the two dimensionless retention times are controlled by two single-loop PID controllers. To feed the MPC with the required internal concentration profiles, a moving horizon estimator (MHE) is developed and added to the control scheme. This new control scheme, MHE + MPC/PID, has been extensively investigated in three case studies. A hypothetical case where the internal concentration profiles are completely available to the MPC, where they come from the MHE processing noiseless measurements, and where they are from MHE processing measurements with added normally distributed random noise. In each of these cases, the control system successfully follows the change in the references over a wide range from complete to incomplete separation, with changes of the product purities from 99 % to 90 % have been investigated. That is due to the use of fully blown chromatographic models used in MHE and MPC, which are solved numerically by the popular mixing cell model. To further improve the computational performance of this control scheme, the space-time solution element/conservation element (CE/SE) method is considered as a numerical tool.

Due to its special formulation, the CE/SE method allows the resolution of steep concentration fronts with only a few dozen discrete elements. In studies of various chromatographic processes, ranging from a simple batch process to a ternary 8-zone centre-cut SMB process with both linear and Langmuir isotherms, it is found to be 10 to 350 times faster than the mixing cell model. Therefore, the CE/SE method is preferred as a numerical method for the proposed control concept.

Finally, the control of SMB processes with the proposed control scheme, for both linear and Langmuir isotherms, has been investigated using the CE/SE method. It is shown that in both cases, i.e. linear and nonlinear adsorption isotherms, reference tracking and disturbance rejection work well if the isotherm parameters are known. In practice, however, isotherm parameters may change due to the adsorbent ageing. As a challenging test scenario, a 10 % step change of the Henry coefficients of the nonlinear Langmuir adsorption isotherms is considered. To handle this scenario, the MHE is augmented with the ability to estimate Henry coefficients online to automatically readjust the operating conditions. It is shown that the extended control system can handle this scenario if the measurement noise is negligible. Lastly, limitations of this approach for highly sensitive processes with steep concentration fronts and measurement noise are also demonstrated for the above challenging test scenario.

Zusammenfassung

Die Chromatographie ist ein leistungsstarkes Trennverfahren, das vor allem für schwierige Trennungen temperaturempfindlicher Substanzen oder von Substanzen mit ähnlichen Eigenschaften eingesetzt wird, die sich mit anderen Methoden nur schwer trennen lassen. Unter den verschiedenen chromatographischen Verfahren hat das Simulation-Moving-Bed-Verfahren (SMB), bei dem es sich um eine kontinuierliche Variante des grundlegenden chromatographischen Verfahrens handelt, große praktische Anwendung gefunden.

Das SMB-Verfahren ermöglicht eine deutlich höhere Produktivität als das chromatographische Batch-Verfahren und senkt gleichzeitig den Lösungsmittelverbrauch, wodurch die Kosten pro Produktionseinheit gesenkt werden. Sein Hauptnachteil ist seine Komplexität. Der Prozess ist von Natur aus nichtlinear, sowohl aufgrund des beteiligten chromatographischen Systems, das aus den gelösten Stoffen, dem Adsorbens und dem Desorbens besteht, als auch aufgrund der Wechselwirkung zwischen diesen Komponenten. Darüber hinaus reagieren SMB-Verfahren an ihrem wirtschaftlich optimalen Betriebspunkt sehr empfindlich auf Störungen. In der Praxis werden SMB-Prozesse meist ohne Regelung betrieben, wobei der gewählte Betriebspunkt nicht am Optimum, sondern an einem suboptimalen Punkt liegt, der sich in einem Betriebsbereich mit höherer Robustheit gegenüber Störungen, aber geringerer Produktivität befindet. Deshalb ist der Einsatz automatischer Regelsysteme für das SMB-Verfahren besonders attraktiv.

Aus regelungstechnischer Sicht stellen SMB-Prozesse eine große Herausforderung dar – sie zeichnen sich durch eine komplexe hybride Dynamik aus, die zyklischen Betrieb, lange Einschwingzeiten, gekoppelte Regelkanäle und begrenzte Messinformationen umfasst. Dies erfordert ein tiefgehendes Verständnis des Prozesses. Mathematische Modellierung und Computersimulationen sind unverzichtbare Werkzeuge für die Auslegung, Optimierung und Regelung solcher Prozesse und kommen auch in der vorliegenden Arbeit zum Einsatz.

Diese Dissertation baut auf den Untersuchungen zur Regelung von SMB-Prozessen auf, die zunächst von Marco Fütterer (Fütterer, 2010) durchgeführt und anschließend von Paul Suvarov (Suvarov, 2016) in Magdeburg und in Mons weitergeführt wurden. Sie entwickelten eine einfache, aber leistungsfähige Regelungsstrategie von Suvarov, die experimentell erfolgreich validiert wurde für SMB-Prozesse, die im linearen Bereich der Adsorptionsisothermen betrieben werden. Gleichzeitig zeigte diese Strategie jedoch einige Schwächen beim Betrieb im nichtlinearen Bereich der Isothermen, der im Mittelpunkt der vorliegenden Arbeit steht, insbesondere bei chromatographischen Systemen mit Langmuir-Isothermen.

Die betrachtete Pilotanlage verfügt über vier Regelgrößen – die beiden Produktreinheiten und zwei dimensionslose Verweilzeiten zur Steuerung der Regeneration in den äußeren Zonen. Die Stellgrößen sind die Durchflussmengen in den einzelnen Zonen der SMB-Anlage. Das ursprüngliche Regelkonzept von Suvarov wird zunächst durch Hinzufügen von zwei Entkopplern erweitert, um die

Kreuzkopplung in den Produktregelkanälen zu reduzieren und so ein dezentrales Regelkonzept zu schaffen. Um zudem eine schnellere Reaktion zu erreichen, werden die PI-Regler durch PID-Regler ersetzt. Die Ergebnisse zeigten, dass diese Regelungsstrategie im Bereich des Arbeitspunktes, für den die Regler und Entkopplungsglieder ausgelegt wurden, eine sehr gute Leistung erbringt. Die Leistung verschlechtert sich jedoch, wenn der Betrieb nicht in der Nähe dieses Punktes erfolgt. Darüber hinaus ist die Fähigkeit zur Bewältigung von Eingangsbeschränkungen begrenzt.

Daher wurde eine andere Regelungsstrategie entwickelt, die auf einer online optimierenden Regelung basiert. Insbesondere wird ein neuartiges semi-zentrales Regelkonzept vorgeschlagen, bei dem die beiden Produktreinheiten durch einen nichtlinearen modellprädiktiven Regler (MPC) geregelt werden, während die beiden dimensionslosen Retentionszeiten durch zwei separate Regelkreis-PID-Regler geregelt werden. Um den MPC mit den für seinen Betrieb erforderlichen internen Konzentrationsprofilen zu versorgen, wurde ein Moving-Horizon-Schätzer (MHE) entwickelt und in das Regelkonzept integriert. Dieses neue Regelkonzept, MHE + MPC/PID, wurde in drei Fallstudien umfassend untersucht. Untersucht wurden ein hypothetischer Fall, in dem die internen Konzentrationsprofile dem MPC vollständig zur Verfügung stehen, ein Fall, in dem sie aus der Verarbeitung rauschfreier Messwerte durch den MHE stammen, sowie ein Fall, in dem sie aus der Verarbeitung von Messwerten mit hinzugefügtem normalverteiltem Zufallsrauschen durch den MHE stammen. In jedem dieser Fälle folgt das Regelsystem erfolgreich den Änderungen der Sollwerte über einen weiten Bereich von vollständiger bis unvollständiger Trennung mit Änderungen der Produktreinheiten von 99 % bis 90 %. Dies ist auf die Verwendung rigoroser chromatographischer Modelle in MHE und MPC zurückzuführen, die numerisch mittels des gängigen m. H. finite Volumina 1. Ordnung gelöst werden. Um die Rechenleistung dieses Regelkonzepts weiter zu verbessern, wird die sogenannte conservation element/solution element method (CE/SE) im Folgenden als verbessertes numerisches Lösungsverfahren betrachtet.

Aufgrund ihrer speziellen Formulierung ermöglicht die CE/SE-Methode die Auflösung steiler Konzentrationsfronten mit einer relativ geringen Anzahl an diskreten Elementen. In Untersuchungen verschiedener chromatographischer Prozesse – von einem einfachen Batch-Prozess bis hin zu einem ternären 8-Zonen-SMB-Prozess mit Centre-Cut und sowohl linearen als auch Langmuir-Isothermen – hat sich gezeigt, dass sie 10- bis 350-mal schneller ist als das Mischzellenmodell. Daher wird die CE/SE-Methode als numerische Methode für das vorgeschlagene Regelungskonzept bevorzugt.

Abschließend wurde die Regelung von SMB-Prozessen mit dem vorgeschlagenen Regelkonzept sowohl für lineare als auch für Langmuir-Isothermen unter Verwendung der CE/SE-Methode untersucht. Es zeigt sich, dass in beiden Fällen, d.h. bei linearen und nichtlinearen Adsorptionsisothermen, die Referenzverfolgung und die Störgrößenunterdrückung gut funktionieren, sofern die Isothermenparameter bekannt sind. In der Praxis können sich die Isothermenparameter jedoch

aufgrund der Alterung des Adsorptionsmittels ändern. Als anspruchsvolles Testszenario wird eine stufenweise Änderung der Henry-Koeffizienten der nichtlinearen Langmuir-Adsorptionsisothermen um 10 % betrachtet. Um dieses Szenario zu bewältigen, wird das MHE um die Fähigkeit erweitert, die Henry-Koeffizienten online zu schätzen, um die Betriebsbedingungen automatisch anzupassen. Es zeigt sich, dass das erweiterte Regelsystem dieses Szenario bewältigen kann, sofern das Messrauschen vernachlässigbar ist. Schließlich werden anhand des oben genannten anspruchsvollen Testszenarios auch die Grenzen dieses Ansatzes für hochempfindliche Prozesse mit steilen Konzentrationsfronten und Messrauschen aufgezeigt.

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Nomenclature

Acronyms and abbreviations

AE	algebraic equation
ANN	artificial neural network
BC	boundary condition
CE	control effort
CE/SE	space-time conservation element/solution element method
CFL	Courant-Friedrichs-Lewy
CSS	cyclic steady state
CST	continuously-stirred tank
Eq.	Equation
Eqs.	Equations
HPLC	high-performance liquid chromatography
IC	initial condition
LDF	linear driving force
MC	mixing cell model
MHE	moving horizon estimator
MOL	method of lines
NLP	nonlinear programming
(N)MPC	(nonlinear) model predictive controller
OA	online analytical analyser, HPLC
ODE	ordinary differential equation
PDAE	system of partial differential and algebraic equations
PDE	partial differential equation
PI	proportional–integral controller
PID	proportional–integral–derivative controller

RS	root-squared control error
SMB	simulated moving bed chromatographic process
SQP	sequential quadratic programming
TMB	true moving bed chromatographic process
UV	ultraviolet optical measuring instrument

Latin letters

a	supplementary power coefficient in the CE/SE method formulation, $a = 1$
b_i	retention factor in the Langmuir isotherm expression, $[\text{L}\cdot\text{g}^{-1}]$
$C_{i,p}$	liquid phase concentration, $[\text{g}\cdot\text{L}^{-1}]$
$\mathbf{C}_p$	vector containing the liquid phase concentrations of all components, $[\text{g}\cdot\text{L}^{-1}]$
$\bar{C}_{\text{Ex},i,k}$	average extract concentration of the i -th component for the k -th cycle, $[\text{g}\cdot\text{L}^{-1}]$, measured variable
$\bar{C}_{\text{Ra},i,k}$	average raffinate concentration of the i -th component for the k -th cycle, $[\text{g}\cdot\text{L}^{-1}]$, measured variable
CE	control efforts
CFL	Courant-Friedrichs-Lewy number, [-]
$\overline{\text{CFL}}$	Courant-Friedrichs-Lewy number in the reversed CE/SE method, [-]
D_{ax}	axial dispersion coefficient, $[\text{m}^2\cdot\text{s}^{-1}]$
D_{col}	column diameter, [m]
$D(s)$	decoupler transfer function
e	control error, [-]
F	column factor, [-]
f	algebraic function
$\mathbf{f}$	vector of fluxes
$\mathbf{f}_{t,\lambda}^n$	numerical analogue of the derivative $\frac{\partial \mathbf{f}}{\partial t}$ at the point (λ, n)
$\mathbf{f}_{z,\lambda}^n$	numerical analogue of the derivative $\frac{\partial \mathbf{f}}{\partial z}$ at the point (λ, n)
$G(s)$	plant transfer function
H_i	adsorption Henry coefficient, [-]
I	integral part of the control signal
$\mathbf{I}_\mu$	identity matrix of size μ
J_{MHE}	estimation cost function
J_{MPC}	control cost function
K	plant gain

K_D	PID controller derivative gain
K_I	PID controller integral gain
K_P	PID controller proportional gain
k_{m_i}	mass transfer coefficient, [s ⁻¹]
L_{col}	column length, [m]
$\mathbf{M}$	mass matrix in the mixing cell model
m	dimensionless flowrate ratio of the liquid to the solid phase (m value), [-], manipulated variable
m_2, m_3	PID controllers outputs for zones II and III before adding the decouplers action, [-]
Δm	rate of change of the manipulated variables (m values), [-]
N	number of theoretical separation stages
N_{ch}	length of the control horizon
N_{col}	number of chromatographic columns constituting the SMB chromatographic plant
N_{comp}	number of components
N_{cyc}	number of simulated cycles
N_{eh}	length of the estimation horizon
N_{ph}	length of the prediction horizon
N_t	number of the time grid points
N_z	number of the spatial grid points in CE/SE method
N_{zone}	number of the zones in the TMB and SMB processes
$\mathbf{P}$	weight matrix in the MHE cost functions
$\mathbf{p}_{\text{CE/SE}}$	vector of source terms in the CE/SE method
$\mathbf{p}_{\text{MC}}$	vector of source terms in the mixing cell model
Pu	product purity, [%], process variable
$\hat{P}u$	product purity predicted by the MPC, [%]
Q	flowrate of the liquid passing through the cascade of CSTs in the mixing cell model, [m ³ ·s ⁻¹]
$Q_{d,\text{TMB}}$	flowrate of the liquid phase within the d -th zone in the TMB process, [m ³ ·s ⁻¹]
Q_{De}	desorbent flowrate, [m ³ ·s ⁻¹], manipulated variable
Q_{Ex}	extract flowrate, [m ³ ·s ⁻¹], manipulated variable

Q_{ext}	external flowrate, $[\text{m}^3 \cdot \text{s}^{-1}]$, manipulated variable
Q_{Fe}	feed flowrate, $[\text{m}^3 \cdot \text{s}^{-1}]$, manipulated variable
$Q_{\text{int},d}$	internal flowrate within the d -th zone in the SMB process, $[\text{m}^3 \cdot \text{s}^{-1}]$, manipulated variable
Q_{Ra}	raffinate flowrate, $[\text{m}^3 \cdot \text{s}^{-1}]$, manipulated variable
Q_{sol}	flowrate of the solid phase in the TMB process, $[\text{m}^3 \cdot \text{s}^{-1}]$
$\mathbf{Q}$	covariance matrix of the measurement noise
ΔQ_{int}	rate of change of the manipulated variables, $[\text{m}^3 \cdot \text{s}^{-1}]$
$q_{i,p}$	solid phase concentration, $[\text{g} \cdot \text{L}^{-1}]$
$q_{i,p}^*$	solid phase concentration at the interphase in equilibrium with the liquid phase, $[\text{g} \cdot \text{L}^{-1}]$
$\mathbf{q}$	vector containing the solid phase concentrations of all components, $[\text{g} \cdot \text{L}^{-1}]$
RS	root-squared control error
S_{col}	column cross-sectional area, $[\text{m}^2]$
s	complex frequency-domain variable
$\mathbf{s}$	supplementary vector in the CE/SE method formulation
t	time coordinate, $[\text{s}]$
t_d	wave retention time in physical units, $[\text{s}]$
t_{sim}	simulation duration of the batch chromatographic processes in Chapter 5 $[\text{s}]$
t_{sw}	switching time (cycle duration), $[\text{s}]$
Δt	time step size, $[\text{s}]$
$\mathbf{u}$	vector of state variables
$\mathbf{u}_{t,\lambda}^n$	numerical analogue of the derivative $\frac{\partial \mathbf{u}}{\partial t}$ at the point (λ, n)
$\mathbf{u}_{z,\lambda}^n$	numerical analogue of the derivative $\frac{\partial \mathbf{u}}{\partial z}$ at the point (λ, n)
$\mathbf{u}_{z-, \lambda}^n$	value of $\mathbf{u}_{z,\lambda}^n$ calculated to the left of the point with coordinates (λ, n)
$\mathbf{u}_{z+, \lambda}^n$	value of $\mathbf{u}_{z,\lambda}^n$ calculated to the right of the point with coordinates (λ, n)
$\tilde{\mathbf{u}}$	supplementary vector in CE/SE method formulation
V_{col}	column volume, $[\text{m}^3]$
ΔV_{col}	elementary column volume, $[\text{m}^3]$
v	interstitial velocity of the liquid phase, $[\text{m} \cdot \text{s}^{-1}]$
W_0	supplementary function in the CE/SE method formulation
w	weight factor in the MPC cost function, $[-]$

x	supplementary variable in the CE/SE method formulation
y	arbitrary process variable
$\hat{y}$	arbitrary process variable predicted by the MPC
$\mathbf{y}$	vector containing all of the measured variables
z	spatial coordinate, [m]
z_d	distance travelled by the concentration waves in the respective zones, [m]
Δz	spatial step size, [m]

Greek letters

$\alpha_{i,p}^{\text{liq}}$	coefficient of the approximating sigmoid function for the liquid phase, [g·L ⁻¹]
$\alpha_{i,p}^{\text{sol}}$	coefficient of the approximating sigmoid function for the solid phase, [g·L ⁻¹]
α_{rel}	relative selectivity (separation factor), [-]
α^{sat}	gain-reduction factor, [-]
$\beta_{i,p}$	power coefficient of the approximating sigmoid function, [m ⁻¹]
$\gamma_{i,p}$	power coefficient of the approximating sigmoid function, [m]
ε	total column void fraction, [-]
ζ	supplementary variable in the triangle tip calculation for systems with Langmuir isotherms
η	measurement error of the measuring device, [%]
θ	adjusted time, [-]
$\sigma, \sigma_1, \sigma_2$	supplementary variables in the triangle tip calculation for systems with Langmuir isotherms
τ	dimensionless retention time, [-], measured and process variable
$\hat{\tau}$	dimensionless retention time predicted by the MPC, [-]
ω_d	coordinates of the triangle tips for systems with Langmuir isotherms, [-]

Subscripts and superscripts

0	initial value at the beginning of the cycle, serves as an initial condition for the PDAE system in Eq. (2.1) and for the PDEs in Eq. (2.2); initial guess for the optimisation variables in the MPC and MHE optimisation procedures
A, B, C	different components
d	zone index ($d = \text{I, II, ..., } N_{\text{zone}}$)
De	desorbent stream

est	estimated variable
Ex	extract stream or extract product
ext	external flowrate
Fe	feed stream or feed mixture
fil	filtered variable
I, II, III, IV	column numbers and zone numbers
i, l	component indices ($i, l = A, B, \dots, N_{\text{comp}}$)
in	column inlet
init	initial operating point
int	internal flowrate
j	equilibrium cell index in the mixing cell model ($j = 1, 2, \dots, N$)
k, g	cycle indices ($k, g = 1, 2, \dots, N_{\text{cyc}}$)
liq	liquid (mobile) phase
Δm	rate of change of the manipulated variables
max	upper bound for the optimisation variables
meas	measured variable
min	lower bound for the optimisation variables
n	time coordinate index ($n = 1, 2, \dots, N_t$)
out	column outlet
p, h	column indices ($p, h = I, II, \dots, N_{\text{col}}$)
Pu	product purity
ΔQ	rate of change of the manipulated variables
r	current cycle
(R)	(R)-bicalutamide
Ra	raffinate stream or raffinate product
ref	reference value
ref'	updated reference value
(S)	(S)-bicalutamide
sat	saturation
sol	solid (stationary) phase
T	transpose operation
true	value of measured variables without the measurement noise
$z = 0^-$	value just before the column inlet, serves as a boundary condition for the

	PDAE system in Eq. (2.1) and for the PDEs in Eq. (2.2)
λ	spatial coordinate index in the CE/SE method ($\lambda = 1, 2, \dots, N_z$)
μ	size of an identity matrix
τ	dimensionless retention time

Chapter 1

Introduction

1.1 Chromatographic separation and chromatographic processes

The demand for high-purity end products has been increasing over the last decade, particularly in the pharmaceutical and biotechnology sectors. The usual processes for obtaining such products are distillation, extraction, and chromatography. In the chemical industry, distillation is considered the workhorse of downstream separation and purification processes due to its broad applicability and the wide range of products obtained by this separation method. Extraction is also widely used industrially. Due to the fact that chromatography is more expensive than distillation and extraction, it is mainly used in the production of products and raw materials that cannot be separated easily by either of these processes, their separation is economically inefficient, or for other technological or environmental reasons. For example, chromatography has been successfully used to separate substances with a high degree of purity which have similar physicochemical properties (e.g., racemic mixtures of enantiomers) or temperature-sensitive substances (e.g., biopolymers), since the process is usually carried out at ambient temperature (Schmidt-Traub et al., [2020](#)). So far, chromatography has been successfully applied industrially for separation in both large-scale production, such as the chemical, petrochemical, and food industries, as well as middle- and small-scale production in the cosmetic, biotechnology, and pharmaceutical industries. It is particularly widely used in the latter two due to the high cost of the end products (e.g., active pharmaceutical ingredients (APIs) and other biotechnological products such as proteins) and the possibility of obtaining these products

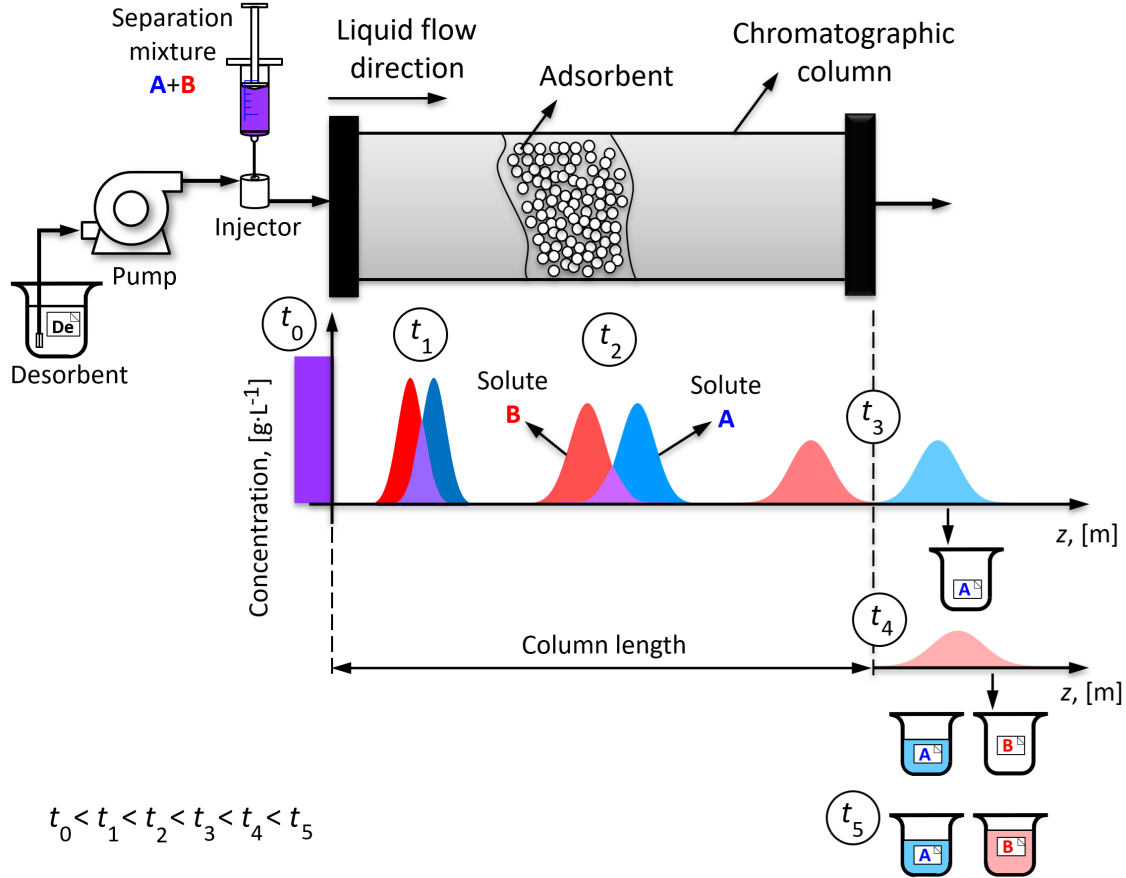


Figure 1.1: Batch chromatographic process.

with very high purity, which are often subject to stringent regulatory requirements by specialised authorities such as the European Union Drugs Agency (EUDA) in Europe or the Food and Drug Administration (FDA) in the USA.

Chromatography was first introduced in the early twentieth century by the Russian botanist Tswett (Tswett, 1905, Tswett, 1906). He used this process to isolate plant-based pigments. The idea of the process can be represented graphically by the chromatographic column shown in Figure 1.1. This process is a batch one and is the basic chromatographic process that serves as the basis for more complex semi-continuous and continuous processes. There are three main constituents of this process. First, a feed mixture containing the components that need to be separated from each other; these dissolved components are also called solutes. Second, a stationary phase on whose surface the adsorption of the solutes takes place; it is called an adsorbent or just a sorbent. Third, a mobile phase that transports the solutes through the adsorbent and is called a desorbent, eluent, or solvent. The adsorbent can be solid or liquid, while the desorbent is a fluid (gas, liquid, or supercritical fluid). Together, these three units form a chromatographic system. Depending on

the mobile phase, the process is called gas chromatography (GC), liquid chromatography (LC), or supercritical chromatography (SCC). Gas chromatography is widely used for analytical purposes and is also used for air purification. However, in the remainder, only liquid chromatography on a preparative scale with a solid adsorbent will be considered, since the results of the study can be directly applied to gas chromatography as well. Moreover, the terms liquid (or fluid) phase and mobile phase, as well as solid phase and stationary phase, will be used interchangeably. Other auxiliary units include the column itself (most often made of stainless steel), pumps, valves, pipes and fittings, injection devices, detectors and measuring devices, control units, etc. A chromatographic column packed with adsorbent is also often referred to as a chromatographic bed.

The liquid phase is pulse-injected into the chromatographic column, and the dissolved components to be separated move along the column at different relative velocities. As a result, if the column is long enough and has a sufficient number of theoretical separation stages, the individual target components appear at the column outlet one after the other and can be collected separately. The solid phase is usually silica, activated carbon, zeolite, cross-linked organic polymers, or chiral stationary phases, and it is selected so that the separated components have different strengths of interaction with its active centres. Due to that, components with a higher affinity to the solid phase remain longer in an adsorbed state (the red component in Figure 1.1) on the adsorbent surface and move at a lower relative velocity than those components with a lower affinity to the solid phase, which stay predominantly in the liquid bulk (the blue component in Figure 1.1). Thus, components with lower affinity to the solid phase elute from the column first and are often called fast, whereas those with higher affinity to the solid phase elute later and are called slow. The driving force for mass transfer between the liquid and solid phases is the difference between the equilibrium and instantaneous concentrations of the participating solutes under given conditions. The mathematical representation of the thermodynamic equilibria between the solute concentration in the liquid phase and onto the adsorbent surface at constant temperature is called an adsorption isotherm, and it is one of the most important characteristics of any chromatographic system.

The batch process is low in productivity and has very high solvent consumption because, after each separation operation, the chromatographic column must be cleaned with a pure solvent to remove any residual solute. Therefore, this process is used for spe-

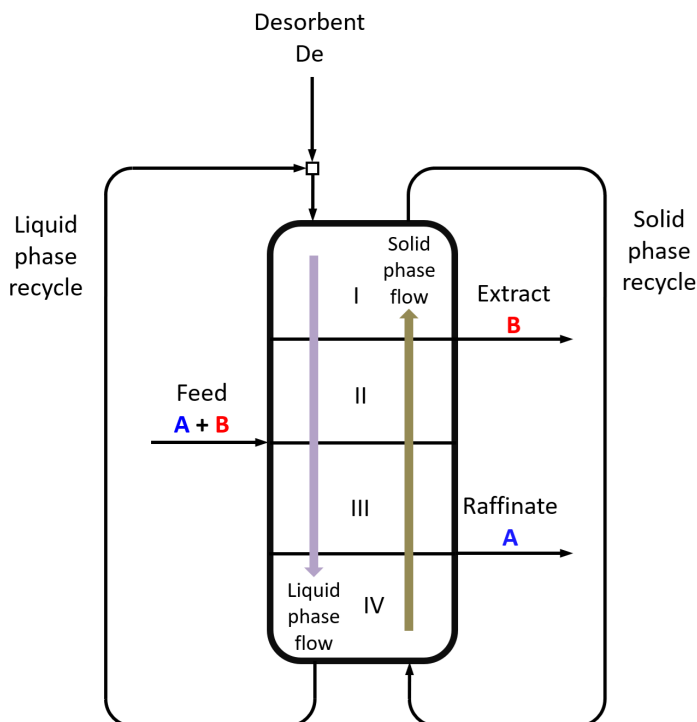


Figure 1.2: Schematic representation of the true moving bed concept.

cialised small-scale separations, as well as an analytical method. To increase productivity and simultaneously reduce solvent consumption, semi-continuous and continuous processes have been developed. The semi-continuous processes are multicolumn processes in which columns are connected in series or in parallel, and they have a particular application in the biotechnological industry. Such processes are, e.g, multicolumn countercurrent solvent gradient purification (MCSGP) and pressure swing adsorption (PSA).

However, continuous processes have become the most widespread as a result of their higher productivity and economic efficiency compared to all other chromatographic processes. The first practical implementation of such a continuous process was in the 1960s (Broughton et al., 1970) and is called a simulated moving bed (SMB). The SMB is based on the true moving bed (TMB) process, shown in Figure 1.2, in which the liquid and solid phases move countercurrently to each other with appropriate velocities in a chromatographic column. If these velocities are well chosen, the more retained component remains adsorbed on the solid phase and moves with it in the opposite direction to the liquid phase. In contrast, the less retained component remains in the liquid phase, enabling separation of the two components. A problem with TMB that prevents its practical realisation is the difficulties associated with the movement of the solid phase. For this reason, the SMB process shown in Figure 1.3 has been adopted, which is a practical implementation of the

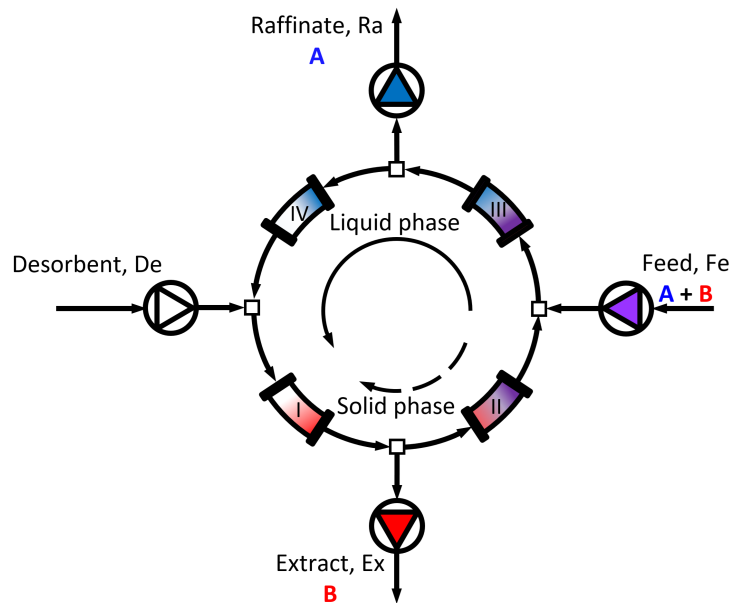


Figure 1.3: Schematic representation of the classic binary simulated moving bed chromatographic process.

TMB process in which the solid phase remains essentially stationary. To achieve counter-current motion between the two phases, the solid phase is partitioned into several columns (chromatographic beds) that are connected in a ring topology, and at a specific interval of time, called switching time, all columns are repositioned in the opposite direction to the movement of the liquid flow, with the inlet and outlet ports remaining stationary. This is where the name simulated moving bed comes from. The technical implementation of the repositioning of the columns is either by placing them on a carousel that rotates one position opposite the direction of the liquid flow (this is the Knauer process), or by using multi-way valves that open and close simultaneously, thus simulating the movement of the inlet and outlet ports in the same direction as the liquid flow, while the columns remain immobile (this is the Novasep process). Due to the presence of rotating parts, the Knauer process is used for laboratory installations and in small-scale production, while Novasep is used in large-scale production. Despite the different technical implementations, from a mathematical modelling perspective, these two SMB processes differ very little, so only the Knauer SMB process will be considered in the remainder. SMB processes do not reach a steady state due to repetitive column switching; instead, they reach a cyclic steady state (CSS), characterised by constant cycle-averaged process variables, e.g., product purities.

The principle of operation of the classic binary SMB process is as follows. The SMB plant is divided into separate zones or sections by the inlet and outlet ports, each with a specific task. Each zone may have one or more separate chromatographic columns. Figure

1.3 shows a variant with only one column in each zone. The initial mixture is introduced into the SMB plant through the feed inlet between zones II and III, where the less retained component remains predominantly in the liquid phase and is directed with the liquid flow to one of the outlet ports called raffinate, while the more retained component is predominantly adsorbed into the solid phase and, upon switching, moves with it to the other outlet port, called the extract. In zone I, the more retained component is desorbed from the solid phase by injecting fresh solvent through the desorbent inlet at the beginning of this zone. Zones II and III are the zones where the separation of the individual components takes place, which is why these two zones are called separation zones; zones I and IV are called regenerative because in zone I, the solid phase is cleaned of the slower component, while in zone IV, the liquid phase is cleaned of the faster component.

During the design phase of the SMB processes, the size and number of chromatographic columns, the type of adsorbent, and the desorbent are selected; these are the so-called design parameters, which, once selected, are difficult or expensive to change. In addition to these, there are the so-called operating parameters, which may change during SMB plant operation depending on various criteria, such as productivity, recovery of the key component, and the specified purity of the products. These operating parameters are the liquid-phase flowrates within each zone of the SMB plant, called internal flowrates, and the switching time, and they must be appropriately selected for the SMB plant to operate as desired.

To determine appropriate operating conditions in accordance with the given design criteria, one needs to have a good understanding of the behaviour and characteristics of the underlying chromatographic system and the SMB plant. These include knowledge of the nature of the interphase interaction between the liquid and solid phases and, more precisely, the types of adsorption isotherms and their parameters, as well as many transport and kinetic coefficients, such as axial dispersion and mass transfer coefficients, etc. Among the characteristics of the SMB plant, one of the most important parameters are the dead or void volumes resulting from the process equipment. Since it is not always possible to know all of these quantities precisely, their measurement is inaccurate or difficult, or they change their values either due to changes in the operating conditions (e.g., change in ambient temperature) or over time with the operation of the plant (e.g., ageing of the adsorbent), the operation of such SMB chromatographic processes is a challenging task.

In real practice, the operating parameters are typically determined a priori, offline, using a well-known and widely accepted methodology such as the triangle theory (Mazzotti et al., 1994, Mazzotti et al., 1997), and the resulting operating point is then applied to the SMB plant and maintained in manual mode. At the optimal economic operating point with maximum productivity and minimum desorbent and energy consumption, SMB plants are particularly sensitive to disturbances and model errors. That is why SMB processes are operated with a certain safety margin, with the actual applied operating point being suboptimal. Therefore, the use of process control for SMB processes is very attractive, as a suitable control concept can reliably bring the process to the desired (optimal) operating point via safe trajectories and maintain this point by effectively rejecting disturbances that affect the plant's operation. This thesis addresses precisely that problem, developing an efficient control strategy for the SMB chromatographic process using limited measurement information from the SMB plant.

1.2 State of the art and motivation

In the industry, SMB processes are usually operated in open-loop due to their complexity and insufficient knowledge of some process parameters, e.g., isotherm parameters or dead volumes, which are difficult to measure accurately or vary over time. For these reasons and because the SMB process is very sensitive to disturbances at its economic optimum, it is predominantly operated suboptimally with respect to its economic indicators, such as productivity and desorbent consumption. It is not surprising that, under these circumstances, the design, operations, and control of the SMB process have attracted growing interest in recent years. Along with the development of new more advanced techniques for the operation of the process (e.g. power feed process (Zhang et al., 2003, Kawajiri and Biegler, 2006), ModiCon process (Schramm et al., 2002, Schramm, Kienle, et al., 2003, Schramm, Kaspereit, et al., 2003), VariCol process (Ludemann-Hombourger et al., 2000, Toumi et al., 2002, Toumi et al., 2003), outlet stream swing operation (Gomes and Rodrigues, 2007), Intermittent-SMB (Katsuo and Mazzotti, 2010), flow focusing operation (Lee et al., 2020a, Lee et al., 2020b), etc.) there is also a growing interest in applying automatic control to this process, and particularly any kind of online optimising control such as model predictive control (MPC).

In MPC, a model of the process is used in an online optimisation procedure that finds

the optimum of a process indicator, which may be technical and/or economical, called a cost function, over a finite future horizon, subject to technological, safety-related, ecological, or economical constraints. The outcome of this procedure is an optimal control trajectory in which only the first move is applied to the plant. After that, the horizon moves forward with one sampling step and the procedure is repeated again using the new information from the process.

MPC has been successfully applied to classical single-column batch and multi-column semi-continuous chromatographic processes, such as MCSGP and PSA. These processes are particularly interesting because they are widely used downstream processes in the biotechnological industries. Extensive research has been conducted by Pistikopoulos and Papathanasiou, and their co-workers, on the multi-parametric model predictive control (mpMPC) of MCSGP and PSA processes, e.g., in (Papathanasiou et al., 2016) and (Kharjuria and Pistikopoulos, 2011). Eslami et al., 2022 used MPC to control affinity and ion-exchange chromatography for protein separation. A recent review of the application of MPC in biopharmaceutical production is available in (Eslami and Jungbauer, 2024).

In a series of publications (Erdem et al., 2004a, Grossmann et al., 2008, Grossmann et al., 2010), Morari, Mazzoti, Morbidelli and their co-workers proposed MPC of SMB processes with locally linearised models and with an online adaptation that relies on fast measurement information from UV sensors (Erdem et al., 2004a), on cycle-averaged concentration measurements (cycle to cycle control) (Grossmann et al., 2008), and a unified multi-rate method with both measurements (Grossmann et al., 2010). Another approach was proposed by Engell and co-workers in (Toumi and Engell, 2004) and (Toumi et al., 2007), where online optimising control was developed for a 3-zone reactive SMB process for the production of fructose from glucose. In these works, computational effort associated with the detailed model is reduced using advanced numerical methods. Lee et al., 2020a, and Lee et al., 2020b proposed a repetitive online optimisation technique of the cyclic steady states. Apart from the classical operation mode in which the internal flow rates do not change within the cycle, two more advanced operation modes were considered too – the outlet stream swing operation and the flow focusing operation. The proposed technique was first investigated theoretically in a simulation study (Lee et al., 2020a) and afterwards it was verified experimentally (Lee et al., 2020b) for the separation of a racemic mixture of bicalutamides.

In previous work in Magdeburg and Mons (Fütterer, 2010, Suvarov et al., 2014, Suvarov et al., 2016, Suvarov et al., 2019), a control concept called adaptive cycle-to-cycle control was developed for binary separation problems based on fundamental insight into the process dynamics. Process dynamics is characterised by propagating concentration fronts, i.e., nonlinear wave propagation. The proposed concept has a two-layer control structure. The first layer are simple feedback PI controllers that control product purities, and the second layer is a parameter estimator that online estimates optimal operating conditions based on the movement of concentration fronts within the SMB plant, providing feed-forward action.

An overview with many references on advanced process control strategies for different chromatographic separation processes is provided by Engell and Kienle in Chapter 9 in (Schmidt-Traub et al., 2020).

The mathematical relation describing the thermodynamic equilibrium between the solid and liquid phases is called an adsorption isotherm and contains several parameters that must be known for the conceptual design and the control of the SMB process. However, these are not always available, and their values change over time. Moreover, SMB chromatographic processes are distributed parameter systems, and state estimators are required by MPC strategies to alleviate the lack of measurement of the time-varying spatial profiles. In this connection, only a relatively small number of contributions pertaining to the SMB process could be found in the literature. In the control scheme based on a linear reduced-order model proposed in (Erdem et al., 2004a, Erdem et al., 2004b), a Kalman filter based on the concentration measurements of the extract and raffinate outlet streams was used. Küpper and Engell, 2006 used decentralised extended Kalman filters to estimate concentrations and adsorption parameters for nonlinear adsorption isotherms. Alamir and Corriu, 2003 proposed a moving horizon estimator (MHE) with concentration profile approximations by truncated exponential functions, and in (Corriu and Alamir, 2006), a hybrid nonlinear state observer. Küpper et al. proposed in (Küpper et al., 2009a, Küpper et al., 2009b) a moving horizon state and parameter estimator based on the discretised state space model of the SMB plant. The investigated system was described by extended Langmuir isotherms, and the estimator performance was tested for different scenarios, including plant-model mismatch and change of the adsorption Henry coefficients. The estimator could reconstruct the internal profiles with a comparatively faster convergence

than an extended Kalman filter.

The starting point of the present thesis was the work done by Suvarov in (Suvarov et al., 2014, Suvarov et al., 2016, and Suvarov et al., 2019) as described above. The experimental validation of this control concept performed very well (Suvarov, 2016) for separations in the linear concentration range, but showed some weaknesses for feeds with higher concentrations in the nonlinear range. A more sophisticated control strategy, such as a multivariable feedback controller, might improve the control performance.

The objective of the thesis was to improve the existing control strategy proposed by Suvarov for binary SMB processes in the nonlinear region of adsorption isotherms. For this purpose, a decoupled control system is investigated, and, afterwards, more advanced control schemes are applied, namely, centralised multivariable control, including one based on online optimisation (i.e. MPC).

For this purpose, first, a simulation tool was written that is based on a modular principle, thus allowing simulations of different chromatographic processes and configurations (from single-column batch processes to ternary centre-cut SMB processes) with different adsorption isotherms (linear, Langmuir, and bi-Langmuir), expressed through different models (ideal equilibrium model and mass transfer models). Mathematical modelling of chromatographic processes leads to a system of PDAEs. These PDAEs only have analytical solutions in a limited number of special cases; therefore, numerical methods must be used to solve them. For this purpose, the widely used mixing cell model is used, as well as the conservation element/solution element (CE/SE) method (Chang, 1993, Chang, 1995, Chang and Wang, 2002).

The next step was to conduct a study of a binary SMB process with Langmuir adsorption isotherms in the nonlinear range of the isotherms. For this purpose, the control scheme proposed by Suvarov was first modified by replacing the PI controllers with PID controllers for faster response, and second by adding two static decouplers to the two controllers within the separation zones. When properly tuned, these two decouplers should alleviate cross-coupling effects within the two separation zones of the SMB plant.

However, since the applicability of this approach is limited to the linear range, another control strategy based on online optimisation was considered. In it, the two product purities are controlled by a nonlinear MPC, whereas the two single-loop PID regulators in the two regeneration zones remain in place. That led to a novel semi-centralised con-

trol scheme for the control of binary SMB processes. NMPC was preferred because it can handle process nonlinearity, interconnected control channels, physical constraints, and measurement information delays efficiently. For the MPC to perform its online optimisation task, the spatially distributed time-varying internal concentration profiles must be available to the predictor. These profiles are almost impossible to obtain through measurements. Therefore, an estimator that would reconstruct the internal concentration profiles from the available scarce measurement information was synthesised. The chosen estimator was the MHE, which is a dual concept of the MPC. In it, the concentration profiles in each zone of the SMB process are approximated by basis functions. In a subsequent step, the MHE was extended with the ability to estimate a key parameter of the SMB process, namely the adsorption Henry coefficient, whose change acts as a disturbance on the process. The predictor and estimation models in the MPC and MHE are solved numerically, as in the virtual model of the SMB process, either by the mixing cell model or by the CE/SE method. In this way, it is possible to run simulations both without plant-model mismatch and with it, thus making it possible to account for both types of uncertainty – structural and parametric.

1.3 Publication list

The results of the study have been presented in three peer-reviewed publications¹ and one publication which is in a preparation phase.

The first article was published in the open-access journal *Processes* (Chernev et al., 2020) and deals with the numerical simulation of various chromatographic processes using the mixing cell model and the conservation element/solution element method, and compares them in terms of solution accuracy and computational time. The investigated chromatographic processes range from simple batch processes to ternary centre-cut eight-zone processes using both the ideal-equilibrium and the mass transfer model with linear driving force for systems described by both linear and Langmuir adsorption isotherms.

The second publication was a conference paper presented at the 26th International Conference on System Theory, Control and Computing (ICSTCC) in Sinaia, Romania. The conference proceedings were published in the IEEE Xplore database (Chernev et al., 2022).

¹All publications are used in full or partially in the current thesis according to the conditions for non-commercial usage by the respective publishers.

This paper addresses model predictive control of an SMB process for a system operating in the linear range of the adsorption isotherms, using the conservative element/solution element method as the numerical method.

The third paper was an article published in the *Computers & Chemical Engineering* journal (Chernev et al., 2024). The paper discusses a newly proposed control scheme for the control of an SMB chromatographic process that combines model predictive control and classical PID control. Also, a moving horizon estimator was proposed that reconstructs, online, the internal concentration profiles needed by the predictor from the scarce measurement information. The investigated system is with Langmuir isotherms in the nonlinear range, and the numerical method used is the mixing cell model.

The fourth article, currently under preparation (Chernev et al., 2026), addresses the control of SMB chromatographic processes with Langmuir isotherms and uses the CE/SE method as a numerical method for simulations. Moreover, the MHE is extended in order to be able to estimate the physical parameters of the SMB process in addition to the internal concentration profiles; particularly in this study, the estimated parameters are the adsorption Henry coefficients. This allows the proposed control scheme to be used for both reference tracking and disturbance rejection control scenarios, and the latter case is also investigated in this publication. The results and discussion section of this publication are presented in Chapter 6.2 of the thesis.

The results of the study were also presented at the Benelux control meeting in Brussels in 2022 and at several internal workshops.

Here is a bibliographic list of the published contributions:

1. Chernev, V. P., Vande Wouwer, A., Kienle, A. (2020). Efficient Simulation of Chromatographic Processes Using the Conservation Element/Solution Element Method. *Processes*, 8(10), p. 1316. <https://doi.org/10.3390/pr8101316>.
2. Chernev, V.P., Santos, L.O., Vande Wouwer, A., Kienle, A., (2022). Model Predictive Control of Simulated Moving Bed Chromatographic Processes Using Conservation Element/Solution Element Method. *In: Proceedings of the 26th International Conference on System Theory, Control and Computing (ICSTCC)*, Sinaia, Romania, 19-21 October 2022. pp. 355-361. <https://doi.org/10.1109/ICSTCC55426.2022.9931774>.
3. Chernev, V.P., Kienle, A., Vande Wouwer, A. (2024). Moving horizon estimation and control of binary simulated moving bed chromatographic processes with Langmuir

isotherms. *Computers & Chemical Engineering*, 189, p. 108804. <https://doi.org/10.1016/j.compchemeng.2024.108804>.

4. Chernev, V.P., Kienle, A., Vande Wouwer, A. (2026). Moving horizon estimation and control of binary simulated moving bed chromatographic processes using the conservation element/solution element method. *In preparation*.

1.4 Outline of the thesis

This chapter briefly presents the chromatographic process and its importance from an industrial point of view; the basic concepts and terminology; different processes and configurations; the challenges associated with their operation, especially SMB processes; and why these processes are suitable candidates for process control.

Chapter 2 presents the most widely used mathematical models of chromatographic processes.

Chapter 3 presents control of an SMB binary process with nonlinear adsorption isotherms using decoupled linear controllers, specifically PID control combined with static decouplers for the product purity control loops.

Chapter 4 presents a new scheme for semi-centralised control. This control scheme is based on nonlinear MPCs responsible for product purity and independent PID controllers in the regeneration zones. Since the MPC requires real-time information on internal concentration profiles, which cannot be obtained from the sparse measurements, a moving horizon estimator (MHE) is also proposed, first validated in open-loop and then integrated into the control scheme. The mixing cell model is used as a numerical method.

Chapter 5 presents the CE/SE method and compares it with the mixing cell model in terms of accuracy and computational efficiency for different processes with different adsorption isotherms.

Chapter 6 presents the proposed control scheme, using the CE/SE method as a numerical method. First, this approach is applied to an SMB process with linear adsorption isotherms, and then to one with Langmuir isotherms. In the Langmuir isotherm case, the reference tracking problem is first considered, and then the MHE is extended to estimate the adsorption Henry coefficient, H , in real time. For this purpose, the open-loop verification of the MHE is presented, followed by its use for disturbance rejection control when

the disturbance is a change in H .

Finally, Chapter [7](#) presents conclusions and prospects for future work in the field of control of SMB chromatographic processes.

Chapter 2

Theoretical background

2.1 Mathematical modelling

Mathematical modelling and simulation, in conjunction with the experimental investigation of the chromatographic processes, provide a better understanding of the complex phenomena of these processes. Mathematical models are obtained from first principles and are used in the design phase and for optimisation and control purposes of different chromatographic processes. Because of the complexity of these processes when starting their modelling, several assumptions are made, resulting in various models ranging from the equilibrium models to the general rate model. In the current thesis, two models are used: the ideal-equilibrium model, which assumes thermodynamic equilibrium between the solid and the fluid phases, and the mass transfer (or transport) model, which takes into account the finite mass transfer resistance between both phases.

The mathematical model of any chromatographic process is obtained from the material balances around a chromatographic bed, as shown in Figure 2.1, and in the most general case, leads to a partial differential algebraic system of equations (PDAEs). The starting point for modelling all chromatographic processes (from batch one to the more complex continuous SMB processes) in the current work is the mass transfer model with a linear driving force (LDF), which assumes a finite mass transfer resistance between the solid and the fluid phase. A detailed derivation of the governing equations could be found elsewhere, e.g. (Schmidt-Traub et al., 2020 and Guiochon and Lin, 2003); here, only the resulting PDAE system is presented. The PDAE system of the LDF model consists of three equations - one partial differential equation (PDE), one ordinary differential equation (ODE), and

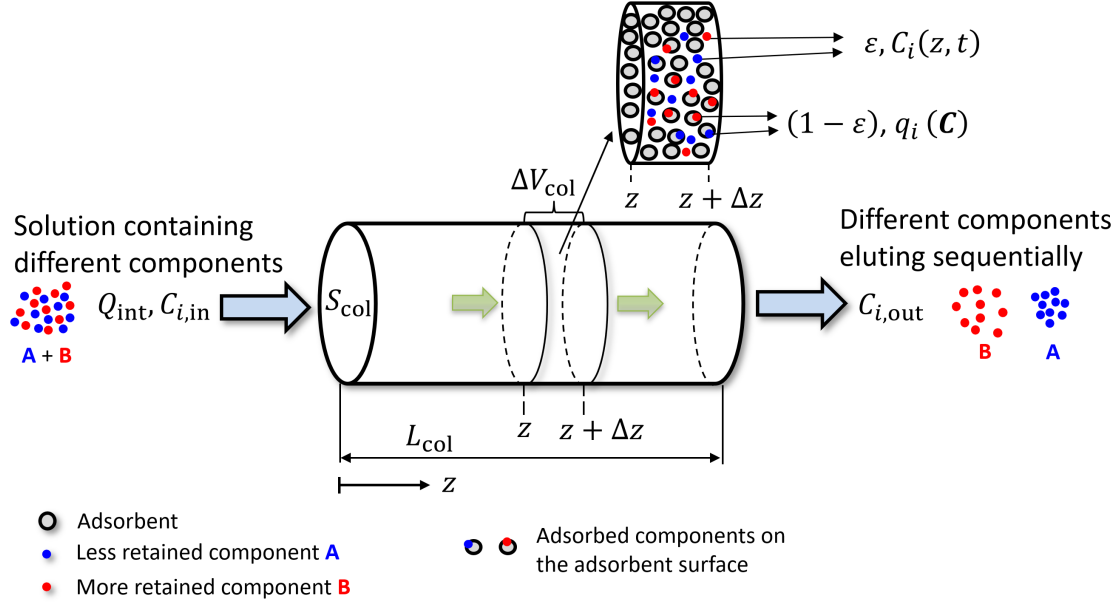


Figure 2.1: Simplified schematic representation of a chromatographic bed used for the derivation of the chromatographic governing equations.

one algebraic equation (AE) for each of the components and each of the columns in the system.

$$\begin{cases} \varepsilon_p \frac{\partial C_{i,p}}{\partial t} + (1 - \varepsilon_p) \frac{\partial q_{i,p}}{\partial t} + \varepsilon_p v_p \frac{\partial C_{i,p}}{\partial z} = \varepsilon_p D_{ax,p} \frac{\partial^2 C_{i,p}}{\partial z^2}, \\ \frac{\partial q_{i,p}}{\partial t} = k_{m,i,p} (q_{i,p}^*(C_p) - q_{i,p}), \\ q_{i,p}^* = f(C_p), \end{cases} \quad \begin{aligned} i &= A, B, \dots, N_{comp}; \\ p &= I, II, \dots, N_{col}, \end{aligned} \quad (2.1)$$

where ε is the column void fraction [-], v the interstitial velocity of the liquid phase [$\text{m} \cdot \text{s}^{-1}$], D_{ax} the axial dispersion coefficient [$\text{m}^2 \cdot \text{s}^{-1}$], k_m the mass transfer coefficient between the two phases [s^{-1}], C the liquid-phase concentration [$\text{g} \cdot \text{L}^{-1}$], q the solid-phase concentration [$\text{g} \cdot \text{L}^{-1}$], q^* the solid-phase concentration at the interphase boundary in equilibrium with the liquid phase [$\text{g} \cdot \text{L}^{-1}$], $\mathbf{C}$ a vector containing the liquid phase concentrations of all components [$\text{g} \cdot \text{L}^{-1}$], t and z are the time and spatial coordinates, respectively, i and p are the component and column indices, respectively, N_{comp} is the number of the participating solutes, and N_{col} is the total number of chromatographic columns that constitute the separation system. This model formulation assumes isothermal operation, a constant void fraction, and a constant mobile-phase velocity inside each of the columns. The apparent axial dispersion coefficient D_{ax} lumps together all effects leading to band broadening in addition to the finite mass transfer resistance, which has a similar effect. In the remainder,

only the limiting case when $D_{\text{ax},p} \rightarrow 0$ is considered, which is valid for efficient columns with a high number of theoretical separation stages N . In the limiting case where the mass transfer is instantaneous (i.e., $k_{m_i,p} \rightarrow \infty$ and $q_{i,p} \rightarrow q_{i,p}^*$) and there is negligible dispersion, the ideal equilibrium model (Schmidt-Traub et al., 2020) is obtained.

$$\varepsilon_p \frac{\partial C_{i,p}}{\partial t} + (1 - \varepsilon_p) \frac{\partial q_{i,p}^*(C_p)}{\partial t} + \varepsilon_p v_p \frac{\partial C_{i,p}}{\partial z} = 0, \quad i = \text{A, B, } \dots, N_{\text{comp}}; \quad p = \text{I, II, } \dots, N_{\text{col}}. \quad (2.2)$$

For solving a PDAE system, it is also necessary to have initial conditions (IC) and boundary conditions (BC). The following initial and boundary conditions are used when the PDAE system in Eq. (2.1) and the PDEs in Eq. (2.2) with $D_{\text{ax}} = 0$ are solved

$$C_{i,p}(z, t = 0) = C_{i,p}(z, 0) = C_{0,i,p}, \quad i = \text{A, B, } \dots, N_{\text{comp}}; \quad p = \text{I, II, } \dots, N_{\text{col}}, \quad (2.3)$$

$$q_{i,p}(z, t = 0) = q_{i,p}(z, 0) = q_{0,i,p}, \quad i = \text{A, B, } \dots, N_{\text{comp}}; \quad p = \text{I, II, } \dots, N_{\text{col}}, \quad (2.4)$$

$$C_{i,p}(z = 0, t) = C_{i,p}(0, t) = C_{z=0^-,i,p}, \quad i = \text{A, B, } \dots, N_{\text{comp}}; \quad p = \text{I, II, } \dots, N_{\text{col}}. \quad (2.5)$$

Eqs. (2.3)-(2.4) give the initial conditions, C_0 and q_0 are the liquid and solid phase concentrations within the chromatographic columns, respectively, at the beginning of the batch process or at the beginning of each cycle for the SMB processes $[\text{g} \cdot \text{L}^{-1}]$, while Eq. (2.5) gives the boundary conditions, and $C_{z=0^-}$ is the liquid phase concentration immediately before the inlet of the respective chromatographic column $[\text{g} \cdot \text{L}^{-1}]$. The boundary conditions are of Dirichlet type and for the binary 4-zone SMB process with one column per zone, they are derived from the nodal total and component material balances (Eqs. (2.6)-(2.13)). Nodes are the points at which the inlet and outlet ports are connected to the SMB plant and are designated by white squares in Figure 1.3. Those nodal material balances are as follows

- Desorbent node

$$Q_{\text{De}} = Q_{\text{int,I}} - Q_{\text{int,IV}}, \quad (2.6)$$

$$Q_{\text{De}} C_{i,\text{De}}(t) = Q_{\text{int,I}} C_{i,\text{I}}(0, t) - Q_{\text{int,IV}} C_{i,\text{IV}}(L_{\text{col}}, t), \quad i = \text{A, B, } \dots, N_{\text{comp}}. \quad (2.7)$$

When only clean desorbent is fed to SMB plant $C_{i,\text{De}}(t) = 0$ for each of the components i .

- Extract node

$$Q_{\text{Ex}} = Q_{\text{int,I}} - Q_{\text{int,II}}, \quad (2.8)$$

$$C_{i,\text{Ex}}(t) = C_{i,\text{I}}(L_{\text{col}}, t) = C_{i,\text{II}}(0, t), \quad i = \text{A, B, } \dots, N_{\text{comp}}. \quad (2.9)$$

- Feed node

$$Q_{\text{Fe}} = Q_{\text{int,III}} - Q_{\text{int,II}}, \quad (2.10)$$

$$Q_{\text{Fe}} C_{i,\text{Fe}}(t) = Q_{\text{int,III}} C_{i,\text{III}}(0, t) - Q_{\text{int,II}} C_{i,\text{II}}(L_{\text{col}}, t), \quad i = \text{A, B, } \dots, N_{\text{comp}}. \quad (2.11)$$

- Raffinate node

$$Q_{\text{Ra}} = Q_{\text{int,III}} - Q_{\text{int,IV}}, \quad (2.12)$$

$$C_{i,\text{Ra}}(t) = C_{i,\text{III}}(L_{\text{col}}, t) = C_{i,\text{IV}}(0, t), \quad i = \text{A, B, } \dots, N_{\text{comp}}. \quad (2.13)$$

In these expressions, Q_{De} , Q_{Ex} , Q_{Fe} , and Q_{Ra} are the external flowrates of the desorbent, extract, feed, and raffinate streams, respectively $[\text{m}^3 \cdot \text{s}^{-1}]$, $Q_{\text{int},p}$ is the internal flowrate of the liquid phase in each of the columns $[\text{m}^3 \cdot \text{s}^{-1}]$, $C_{i,\text{De}}(t)$, $C_{i,\text{Ex}}(t)$, $C_{i,\text{Fe}}(t)$, and $C_{i,\text{Ra}}(t)$ are the instantaneous liquid phase concentration of the desorbent, extract, feed, and raffinate streams, respectively $[\text{g} \cdot \text{L}^{-1}]$, L_{col} is the length of the chromatographic columns $[\text{m}]$, $C_{i,p}(0, t)$ and $C_{i,p}(L_{\text{col}}, t)$ are the instantaneous liquid phase concentrations at the inlet and at the outlet, respectively, of each of the columns $[\text{g} \cdot \text{L}^{-1}]$.

The boundary conditions for the first and third columns are as follows (i.e. for the desorbent and feed nodes)

$$C_{z=0^-,i,\text{I}}(t) = C_{i,\text{I}}(0, t) = \frac{Q_{\text{int,IV}} C_{i,\text{IV}}(L_{\text{col}}, t)}{Q_{\text{int,I}}}, \quad i = \text{A, B, } \dots, N_{\text{comp}}, \quad (2.14)$$

$$C_{z=0^-,i,\text{III}}(t) = C_{i,\text{III}}(0, t) = \frac{Q_{\text{Fe}} C_{i,\text{Fe}} + Q_{\text{int,II}} C_{i,\text{II}}(L_{\text{col}}, t)}{Q_{\text{int,III}}}, \quad i = \text{A, B, } \dots, N_{\text{comp}}, \quad (2.15)$$

whereas for all other columns

$$C_{z=0^-,i,p}(t) = C_{i,p}(0, t) = C_{i,p-1}(L_{\text{col}}, t), \quad i = \text{A, B, } \dots, N_{\text{comp}}; \quad p = \text{II, IV}. \quad (2.16)$$

The interstitial velocities in Eqs. (2.1)-(2.2) are computed by the following expression

$$v_p = \frac{Q_{\text{int},p}}{\varepsilon_p S_{\text{col},p}}, \quad p = \text{I, II, } \dots, N_{\text{col}}. \quad (2.17)$$

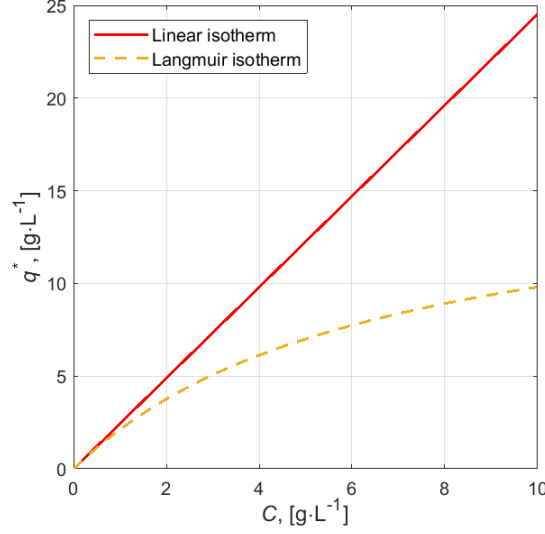


Figure 2.2: Comparison between linear, Eq. (2.19), and Langmuir adsorption isotherms, Eq. (2.18), for a single component adsorption. $H = 2.45$, $b = 0.15 \text{ L} \cdot \text{g}^{-1}$.

where S_{col} is the cross-sectional area of the chromatographic columns $[\text{m}^2]$.

The algebraic equation in the PDAE system in Eq. (2.1) and the function $q_{i,p}^*(C_p)$ in Eq. (2.2) are called adsorption isotherms (Guiochon et al., 2006), and they express the state of thermodynamic equilibrium between the liquid and the solid phases. A popular isotherm model is the so-called Langmuir isotherm, which expresses the nonlinear competitive adsorption behaviour of the participating species and is given by

$$q_{i,p}^*(C_p) = \frac{H_{i,p} C_{i,p}}{1 + \sum_{l=A}^{N_{\text{comp}}} b_{l,p} C_{l,p}}, \quad i = A, B, \dots, N_{\text{comp}}; \quad p = \text{I}, \text{II}, \dots, N_{\text{col}}. \quad (2.18)$$

In the low concentration range of the Langmuir isotherms, they can be substituted by the linear ones

$$q_{i,p}^*(C_p) = H_{i,p} C_{i,p}, \quad i = A, B, \dots, N_{\text{comp}}; \quad p = \text{I}, \text{II}, \dots, N_{\text{col}}. \quad (2.19)$$

In the last two equations, H is the adsorption Henry coefficient $[-]$, b is the retention factor $[\text{L} \cdot \text{g}^{-1}]$ and l is an additional component index. The values of the parameters H and b must be determined experimentally, e.g. by the frontal analysis method or retention time method. Figure 2.2 shows a comparison between the linear and the Langmuir isotherms for just one component, i.e. concentrations of all the components in the separation mixture are considered to be equal to zero except one.

Since, in the vast majority of cases, SMB processes are built from chromatographic

columns with identical dimensions and properties, it is assumed that the quantities L_{col} , D_{col} , ε , $k_{m,i}$, H_i , and b_i and the ones dependent on them like S_{col} , V_{col} , F , and α_{rel} are equal for all of the columns constituting the SMB plant, and because of that the column indices will be omitted for them in the remainder of the thesis.

2.2 Mixing cell model

An analytical solution of the PDAE system in Eq. (2.1) and the PDE in Eq. (2.2) is only possible for a limited number of cases; therefore, they are solved numerically. Possible solution methods are finite element methods, finite volume methods, or other more advanced numerical methods and techniques (LeVeque, 1992, Vande Wouwer et al., 2014). Due to its simplicity, the so-called mixing cell model is widely used in many commercial codes for the simulation of chromatographic processes, as in the present work. It is a first-order backward difference finite volume method, and the spatially discretised PDAE system can be integrated in time with the built-in MATLAB ODE solvers, such as `ode45`, which implements the explicit Runge-Kutta method with variable step size (MATLAB, 2022a).

A chromatographic column with a very high number of theoretical separation stages, N , could be represented as a sequence of continuously-stirred tanks (CST), each with an equal volume of $\frac{V_{\text{col}}}{N}$ (cf. Figure 2.3), through which a constant liquid flow passes. In each of these CSTs, also called a mixing (or equilibrium) cell, the adsorbent and the separation mixture are in equilibrium, and the liquid phase travels from one mixing cell to the next. If the number of these mixing cells is high enough (usually more than 100), the solutions from the cell model and the ideal-equilibrium model are practically identical, but the computational load of the mixing cell model is considerably smaller.

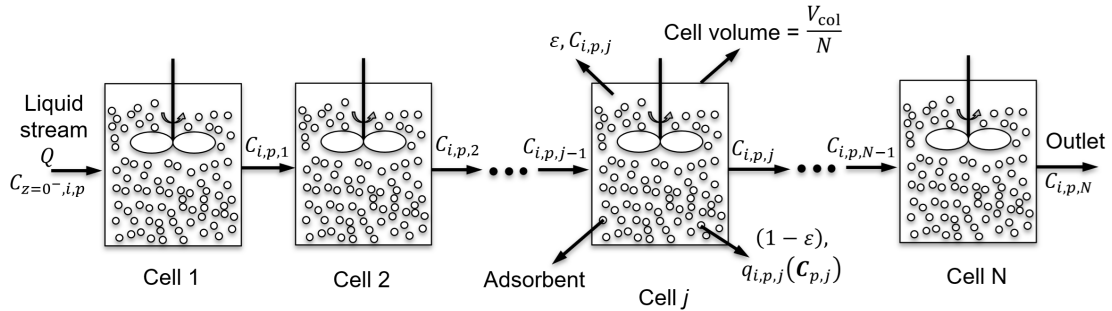


Figure 2.3: Schematic representation of the mixing cell model used for the numerical solution of the governing PDAE system.

The component mass balance of solute i in the j -th CST in the p -th column is

$$\begin{aligned} i &= A, B, \dots, N_{\text{comp}}; \\ \varepsilon \frac{V_{\text{col}}}{N} \frac{dC_{i,p,j}}{dt} + (1 - \varepsilon) \frac{V_{\text{col}}}{N} \frac{dq_{i,p,j}(C_{p,j})}{dt} &= Q(C_{i,p,j-1} - C_{i,p,j}), \quad p = \text{I, II}, \dots, N_{\text{col}}; \\ j &= 1, 2, \dots, N, \end{aligned} \quad (2.20)$$

here V_{col} is the volume of the chromatographic column [m^3], Q is the volume flowrate of the liquid passing through the cascade of mixing cells [$\text{m}^3 \cdot \text{s}^{-1}$], N is the number of theoretical separation stages, i.e. number of the CSTs, and j is the CST index. For the first CST the value of $C_{i,p,j-1}$ comes from the boundary condition

$$C_{i,p,j-1} = C_{z=0^-,i,p}, \quad i = A, B, \dots, N_{\text{comp}}; \quad p = \text{I, II}, \dots, N_{\text{col}}; \quad j = 1. \quad (2.21)$$

For a binary separation system with two solutes, A and B, and Langmuir adsorption isotherms (Eq. (2.18)), applying the chain differentiation rule in Eq. (2.20) for the derivative $\frac{dq_{i,p,j}(C_{p,j})}{dt}$ gives the following equation for component A

$$\begin{aligned} \varepsilon \frac{V_{\text{col}}}{N} \frac{dC_{A,p,j}}{dt} + (1 - \varepsilon) \frac{V_{\text{col}}}{N} \frac{H_A(1 + b_B C_{B,p,j})}{\left(1 + \sum_{l=A}^B b_l C_{l,p,j}\right)^2} \frac{dC_{A,p,j}}{dt} - (1 - \varepsilon) \frac{V_{\text{col}}}{N} \frac{H_A b_B C_{A,p,j}}{\left(1 + \sum_{l=A}^B b_l C_{l,p,j}\right)^2} \frac{dC_{B,p,j}}{dt} \\ = Q(C_{A,p,j-1} - C_{A,p,j}), \quad p = \text{I, II}, \dots, N_{\text{col}}; \quad j = 1, 2, \dots, N. \end{aligned} \quad (2.22)$$

Taking into account that the column factor, F , [-] is defined as

$$F = \frac{1 - \varepsilon}{\varepsilon}, \quad (2.23)$$

and grouping the members, Eq. (2.22) becomes

$$\begin{aligned} \left(1 + \frac{F H_A(1 + b_B C_{B,p,j})}{\left(1 + \sum_{l=A}^B b_l C_{l,p,j}\right)^2}\right) \frac{dC_{A,p,j}}{dt} - \frac{F H_A b_B C_{A,p,j}}{\left(1 + \sum_{l=A}^B b_l C_{l,p,j}\right)^2} \frac{dC_{B,p,j}}{dt} = \frac{Q}{\varepsilon} \frac{N}{V_{\text{col}}} (C_{A,p,j-1} - C_{A,p,j}), \\ p = \text{I, II}, \dots, N_{\text{col}}; \quad j = 1, 2, \dots, N. \end{aligned} \quad (2.24)$$

By analogy, for component B, we have

$$-\frac{FH_B b_A C_{B,p,j}}{\left(1 + \sum_{l=A}^B b_l C_{l,p,j}\right)^2} \frac{dC_{A,p,j}}{dt} + \left(1 + \frac{FH_B(1 + b_A C_{A,p,j})}{\left(1 + \sum_{l=A}^B b_l C_{l,p,j}\right)^2}\right) \frac{dC_{B,p,j}}{dt} = \frac{Q}{\varepsilon} \frac{N}{V_{\text{col}}} (C_{B,p,j-1} - C_{B,p,j}),$$

$$p = \text{I, II, ..., } N_{\text{col}}; \quad j = 1, 2, \dots, N.$$
(2.25)

Eqs. (2.24)-(2.25) can be cast in the following matrix-vector form

$$\mathbf{M}_{p,j} \frac{d\mathbf{C}_{p,j}}{dt} = \mathbf{p}_{\text{MC},p,j}, \quad p = \text{I, II, ..., } N_{\text{col}}; \quad j = 1, 2, \dots, N, \quad (2.26)$$

where $\mathbf{M}_{p,j}$ is called a mass matrix, and it is

$$\mathbf{M}_{p,j} = \begin{bmatrix} 1 + \frac{FH_A(1 + b_B C_{B,p,j})}{\left(1 + \sum_{l=A}^B b_l C_{l,p,j}\right)^2} & -\frac{FH_A b_B C_{A,p,j}}{\left(1 + \sum_{l=A}^B b_l C_{l,p,j}\right)^2} \\ -\frac{FH_B b_A C_{B,p,j}}{\left(1 + \sum_{l=A}^B b_l C_{l,p,j}\right)^2} & 1 + \frac{FH_B(1 + b_A C_{A,p,j})}{\left(1 + \sum_{l=A}^B b_l C_{l,p,j}\right)^2} \end{bmatrix},$$

$$p = \text{I, II, ..., } N_{\text{col}}; \quad j = 1, 2, \dots, N, \quad (2.27)$$

$\mathbf{C}_{p,j}$ is a vector that contains the liquid concentrations of all components

$$\mathbf{C}_{p,j} = \begin{bmatrix} C_{A,p,j} \\ C_{B,p,j} \end{bmatrix}, \quad p = \text{I, II, ..., } N_{\text{col}}; \quad j = 1, 2, \dots, N, \quad (2.28)$$

and $\mathbf{p}_{\text{MC},p,j}$ is a vector of the source terms

$$\mathbf{p}_{\text{MC},p,j} = \begin{bmatrix} \frac{Q}{\varepsilon} \frac{N}{V_{\text{col}}} (C_{A,p,j-1} - C_{A,p,j}) \\ \frac{Q}{\varepsilon} \frac{N}{V_{\text{col}}} (C_{B,p,j-1} - C_{B,p,j}) \end{bmatrix}, \quad p = \text{I, II, ..., } N_{\text{col}}; \quad j = 1, 2, \dots, N. \quad (2.29)$$

Eq. (2.26) defines a system of first-order ODEs and can be solved by any standard integrator, e.g. **ode45** or **ode15s** in MATLAB.

When the adsorption isotherms are linear, then the equations are not coupled and for the i -th component we have

$$\begin{aligned}
i &= \text{A, B, } \dots, N_{\text{comp}}; \\
\frac{dC_{i,p,j}}{dt} &= \frac{Q}{\varepsilon} \frac{N}{V_{\text{col}}} \frac{1}{1 + FH_i} (C_{i,p,j-1} - C_{i,p,j}), & p &= \text{I, II, } \dots, N_{\text{col}}; \\
j &= 1, 2, \dots, N.
\end{aligned} \tag{2.30}$$

Eq. (2.30) is a first-order ODE too and can also be solved by any integrator.

2.3 Triangle theory and its application for the selection of the SMB operating points

As already mentioned, the design phase of an SMB process involves selecting the adsorbent, desorbent, and chromatographic columns. Once these parameters have been selected, it is difficult or costly to change them. The degrees of freedom that can be adjusted during the operation of the SMB process, depending on specific criteria such as product purity, productivity, or solvent consumption, are called operating parameters, and a combination of the set of all possible values of these parameters is called an operating point. There are five operating parameters in the classic binary SMB process: the flow rates within each of the four zones of the SMB plant and the switching time. The most popular methodologies for determining the operating point are the standing wave method (Mallmann et al., 1998) and the triangle theory presented by the group around Mazzotti and Morbidelli (Mazzotti et al., 1997). The latter has become de facto a standard for selecting the operating point for SMB processes.

The triangle theory is based on the TMB process, aiming at complete separation, taking as a basis the ideal equilibrium model Eq. (2.2) with its assumptions, without taking into account the dead volumes in the SMB system and the nonlinearities arising from the kinetics of the chromatographic columns and the dependence of the column efficiency on the mobile phase flow rate. As a result, the operating conditions selected by this method are suboptimal from an economic point of view, but, given its simplicity and the small amount of information required, it has found wide practical application. To determine the operating point for linear isotherms, only the parameters of the isotherms (i.e., adsorption Henry coefficients, H), and for systems with Langmuir isotherms, these are H , the retention factors, b , and the feed concentration, C_{Fe} are required. This methodology has been extended to systems with bi-Langmuir isotherms (Gentilini et al., 1998), for cases

where the aim is reduced purity (Kaspereit et al., 2007), and also taking into account dead volumes when there is some information about them – Katsuo et al., 2009 and Nowak, 2013.

In the TMB process, the flowrate ratio between the liquid and solid phases is given by the expression

$$m_d = \frac{\text{Liquid phase flowrate (TMB)}}{\text{Solid phase flowrate (TMB)}} = \frac{Q_{d,\text{TMB}}}{Q_{\text{sol}}}, d = \text{I, II, } \dots, N_{\text{zone}}, \quad (2.31)$$

where m_d [-] is a dimensionless quantity and is also called an m value, $Q_{d,\text{TMB}}$ are the flowrates of the liquid phase in each of the four zones of the TMB process (cf. Figure 1.2) [$\text{m}^3 \cdot \text{s}^{-1}$], Q_{sol} is the flowrate of the solid phase [$\text{m}^3 \cdot \text{s}^{-1}$], d is the zone index, and N_{zone} is the number of the zones within the TMB process. If it can be assumed that the SMB plant consists of infinite number of chromatographic columns and that they are switched synchronously, then according to Storti et al., 1993, the two processes – TMB and SMB are equivalent, and then for the SMB process, the m values are defined as

$$m_d = \frac{\text{Liquid phase flowrate (SMB)}}{\text{Solid phase flowrate (SMB)}} = \frac{Q_{\text{int},d} t_{\text{sw}} - \varepsilon V_{\text{col}}}{(1 - \varepsilon) V_{\text{col}}}, d = \text{I, II, } \dots, N_{\text{zone}}, \quad (2.32)$$

where $Q_{\text{int},d}$ is the internal flowrate of the liquid phase in each of the zones of the SMB process [$\text{m}^3 \cdot \text{s}^{-1}$] and t_{sw} is the switching time [s]. For SMB processes with only one chromatographic column per zone, which are considered in the thesis, $N_{\text{col}} = N_{\text{zone}}$, and the indices p and d have the same meaning.

The flow rate ratios in zones II and III, m_{II} and m_{III} , are responsible for the separation, while those in zones I and IV, m_{I} and m_{IV} , are responsible for the regeneration of the solid and liquid phases, respectively. In order to achieve total separation and complete regeneration, the following inequalities must be satisfied for a separation system with linear adsorption isotherms if A is the weaker adsorbing component and B is the stronger adsorbing component

$$m_{\text{I}} \geq H_{\text{B}}, \quad (2.33)$$

$$H_{\text{A}} \leq m_{\text{II}} \leq m_{\text{III}} \leq H_{\text{B}}, \quad (2.34)$$

$$-\frac{\varepsilon}{1 - \varepsilon} \leq m_{\text{IV}} \leq H_{\text{A}}. \quad (2.35)$$

The graphical representation of the m value selection procedure is very convenient and

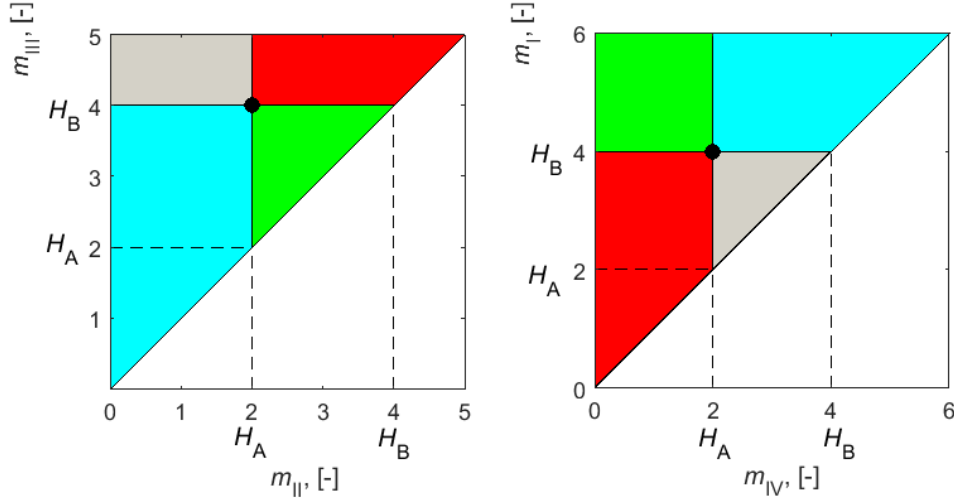


Figure 2.4: Triangle diagram for a system described by linear adsorption isotherms, Eq. (2.19). $H_A = 2$, $H_B = 4$, $C_{A, Fe} = C_{B, Fe} = 0.5 \text{ g}\cdot\text{L}^{-1}$.

provides insight into the separation and regeneration behaviour of the separation system. One graph shows the plane for selecting m_{II} and m_{III} , and another shows the plane for selecting m_I and m_{IV} . These graphs are shown in Figure 2.4. Since the region of total separation in the case of linear isotherms is a triangle, this is where the name of this theory comes from.

In the separation diagram (plane $m_{II} - m_{III}$), the meaning of the different regions is as follows. If the operating point lies within the green region, there is complete separation between the two components: the raffinate is completely free of the more retained component B, while the extract is completely free of the less retained component A. In the blue region, only the raffinate is pure, while the extract is contaminated with A. In the red region, only the extract is pure, while the raffinate is contaminated with B. In the grey region, both products are contaminated. The region below the diagonal $m_{II} = m_{III}$ is infeasible unless there is a flow reversal. The optimal operating point is at the tip of the triangle (black circle), where we have total separation, maximum productivity, and minimum solvent consumption. The further the operating point is from the tip of the triangle and closer to the diagonal, the greater the solvent consumption, the lower the productivity, and, although the final products are separated, they are highly diluted. On the diagonal

itself, the feed flow rate is zero.

In the regeneration diagram (plane $m_{IV} - m_I$), the regions are as follows: in the green region, there is complete regeneration of the solid phase of solute B in zone I and of the liquid phase of solute A in zone IV. In the blue region, there is incomplete regeneration of the liquid phase of solute A; in the red region, there is incomplete regeneration of the solid phase of solute B; and in the grey region, neither phase is fully regenerated. When the liquid phase is not fully regenerated, solute A appears in zone I, which may eventually lead to contamination of the extract with A, even when the condition for total separation in Eq. (2.34) is satisfied. Similarly, if the solid phase is not fully regenerated during the current cycle, after switching, solute B appears in zone IV in the next cycle, which may eventually lead to contamination of the raffinate with B. An event in which a solute appears in a zone where it should not be, due to incomplete regeneration, is called a breakthrough. We have minimum desorbent consumption while, at the same time, complete regeneration of both phases occurs when the operating point lies at the tip of the triangle (black circle) on the $m_{IV} - m_I$ diagram.

The separation and regeneration regions depend only on the isotherm parameters for linear isotherms. In contrast, for Langmuir isotherms, they depend not only on these parameters but also on the feed concentrations. Figure 2.5 shows a typical triangular diagram for a separation system with Langmuir adsorption isotherms. In this case, the region of total separation in the $m_{II} = m_{III}$ diagram is no longer an ideal triangle. The lower the feed concentration, the closer the green region is to the shape of an ideal triangle, and the horizontal curve shifts upwards. Vice versa, the higher the feed concentration, the more this curve shifts downwards and the area of the green region decreases, thus making it challenging to select a working point with complete separation. As the feed concentration increases, the line defined by ω_{IV} also shifts to the left, thus reducing the region of complete regeneration. The individual regions have the same meaning as in the case of linear isotherms. The coordinates of the two tips are (Mazzotti et al., 1997, Migliorini et al., 1998)

$$\omega_I = H_B, \quad (2.36)$$

$$\omega_{II} = \frac{H_A}{H_B} \sigma_1, \quad (2.37)$$

$$\omega_{III} = \frac{\sigma_1 (\sigma_2 (H_B - H_A) + H_B (H_B - \sigma_2))}{H_A (H_B - \sigma_2)}, \quad (2.38)$$

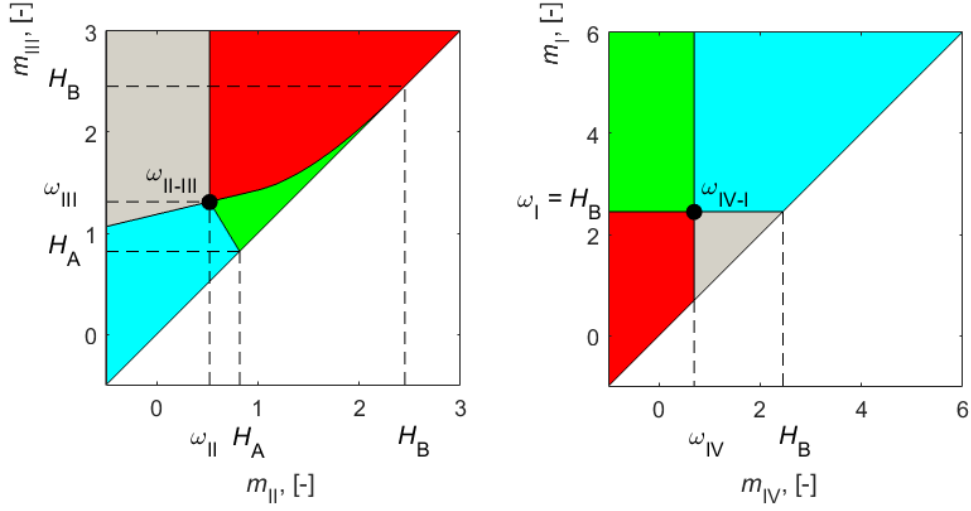


Figure 2.5: Triangle diagram for a system described by Langmuir adsorption isotherms, Eq. (2.18). $H_A = 0.82$, $H_B = 2.45$, $b_A = 0.03 \text{ L}\cdot\text{g}^{-1}$, $b_B = 0.15 \text{ L}\cdot\text{g}^{-1}$, $C_{A, \text{Fe}} = C_{B, \text{Fe}} = 5 \text{ g}\cdot\text{L}^{-1}$.

$$\omega_{\text{IV}} = \frac{1}{2} \left(\zeta - \sqrt{\zeta^2 - 4H_A m_{\text{III}}} \right), \quad (2.39)$$

where

$$\zeta = H_A + m_{\text{III}} + b_A C_{A, \text{Fe}}(m_{\text{III}} - m_{\text{II}}). \quad (2.40)$$

In Eqs. (2.37)-(2.38) σ_1 and σ_2 ($\sigma_1 > \sigma_2 > 0$) are the roots of the following quadratic equation

$$(1 + b_A C_{A, \text{Fe}} + b_B C_{B, \text{Fe}}) \sigma^2 - (H_A (1 + b_B C_{B, \text{Fe}}) + H_B (1 + b_A C_{A, \text{Fe}})) \sigma + H_A H_B = 0. \quad (2.41)$$

In practice, when the desired operating point is at the tip of the triangle in both cases (linear and nonlinear isotherms), a suboptimal operating point is chosen with some safety margin. Then, a value for t_{sw} is selected based on the desired productivity, the maximum allowable pressure drop, or the time required to perform the online measurements. Once the m values are selected and the switching time, t_{sw} , is specified, the internal flowrates Q_{int} can be calculated for each zone of the SMB plant using Eq. (2.32), and after that, the external flowrates taking into account the total nodal material balances (Eqs. (2.6), (2.8),

(2.10), and (2.12)) that are maintained by the four pumps.

2.4 Product purity and retention time measurements of the SMB process

The concentration measurements of the two outlet streams are done with an online analytical analyser (OA), namely, high-performance liquid chromatography (HPLC). A small amount of each of the outlet streams is collected during each cycle, and then after the end of the cycle, the samples are analysed by the HPLC, these are denoted by the two coloured cylinders in Figure 2.6b. These cycle-averaged concentrations are given by

$$\bar{C}_{i,\text{Ex},k} = \frac{1}{t_{\text{sw}}} \int_0^{t_{\text{sw}}} C_{i,\text{Ex},k}(t) dt, \quad i = \text{A}, \text{B}, \dots, N_{\text{comp}} \quad (2.42)$$

$$\bar{C}_{i,\text{Ra},k} = \frac{1}{t_{\text{sw}}} \int_0^{t_{\text{sw}}} C_{i,\text{Ra},k}(t) dt, \quad i = \text{A}, \text{B}, \dots, N_{\text{comp}} \quad (2.43)$$

where $\bar{C}_{i,\text{Ex}}$ and $\bar{C}_{i,\text{Ra}}$ are the averaged outlet concentrations for one cycle of the extract and the raffinate, respectively $[\text{g} \cdot \text{L}^{-1}]$, $C_{i,\text{Ex}}(t)$ and $C_{i,\text{Ra}}(t)$ are the instant outlet concentrations of the extract and the raffinate during the cycle, respectively $[\text{g} \cdot \text{L}^{-1}]$, t_{sw} is the switching time (or the cycle duration) [s] and k is the cycle index. The integration in Eqs. (2.42)-(2.43) is achieved by the trapezoidal rule over time points determined by the integrator **ode45**.

The purity of the two products is the most important indicator of any SMB process, which is why it is a control variable. The product purities for each cycle k are determined by the key component for each outlet. For the extract, this is the more retained component B, while for the raffinate, this is the less retained component A:

$$Pu_{\text{Ex},k} = \frac{\bar{C}_{\text{B},\text{Ex},k}}{\bar{C}_{\text{A},\text{Ex},k} + \bar{C}_{\text{B},\text{Ex},k}} \times 100\%, \quad (2.44)$$

$$Pu_{\text{Ra},k} = \frac{\bar{C}_{\text{A},\text{Ra},k}}{\bar{C}_{\text{A},\text{Ra},k} + \bar{C}_{\text{B},\text{Ra},k}} \times 100\%, \quad (2.45)$$

where Pu is the respective product purity [%].

The other measured and controlled quantities are the dimensionless retention times, τ , in the two regeneration zones I and IV. τ_{I} shows how long the desorption wave of the more retained component B resides in the first zone during each cycle, while τ_{IV} shows how long

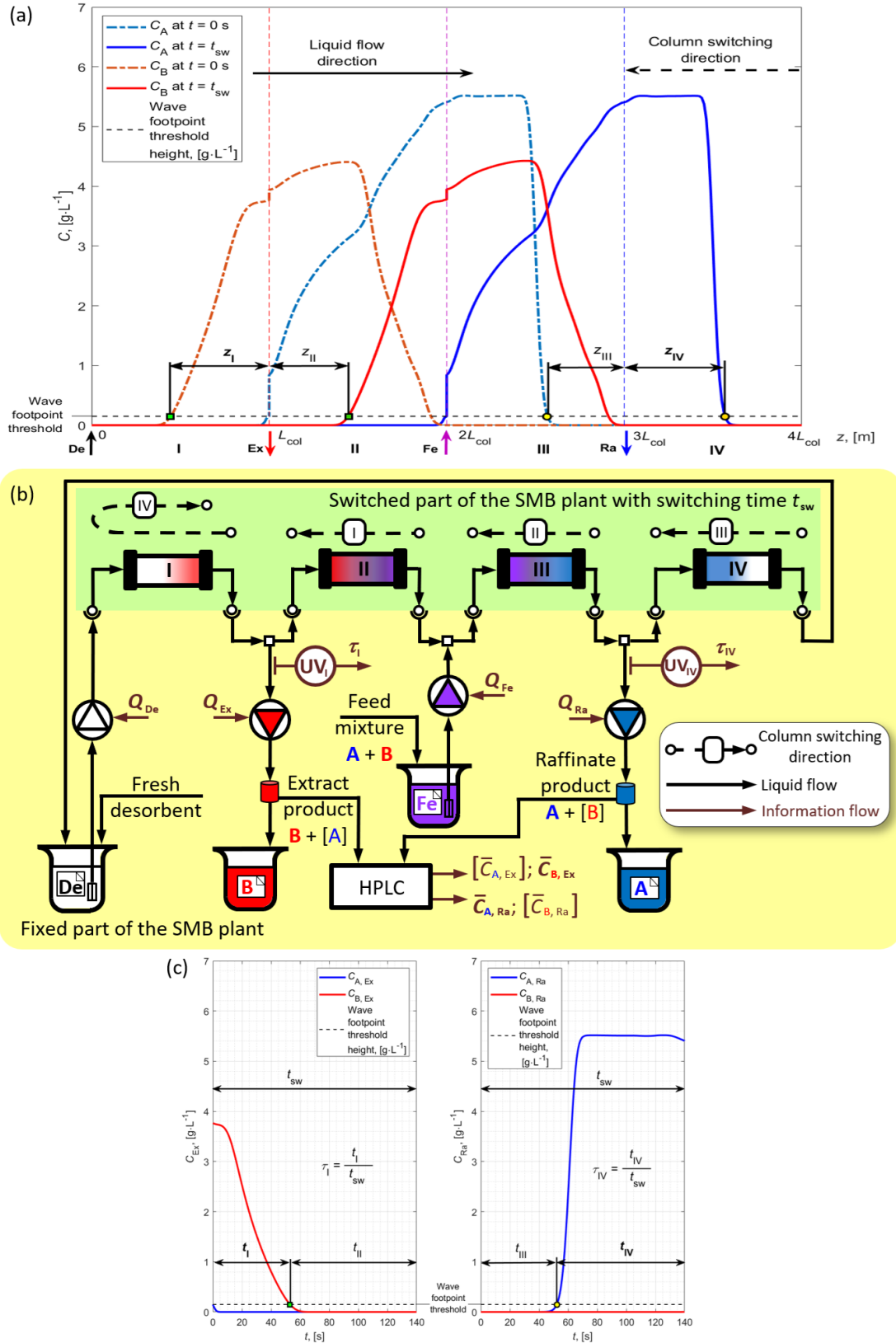


Figure 2.6: Concentration profiles and process and instrumentation diagram of a binary SMB plant. (a) Typical internal concentration profiles along the SMB plant for a system with Langmuir isotherms when cyclic steady state is reached. Dash-dotted curves – concentration profiles at the beginning of the cycle, solid curves – at the end of the cycle. (b) Process and instrumentation diagram. (c) Outlet concentration profiles corresponding to the internal ones from (a), left diagram – extract, right diagram – raffinate. In (a) and (c), green squares – footprint of the desorption wave of the more retained component B, yellow circles – footprint of the adsorption wave of the less retained component A.

the adsorption wave of the less retained component A resides in the fourth zone. The value of τ is between 0 and 1. A value of 1 means that the wave breaks through its respective zone and the products might become contaminated. If $\tau_I = 1$, then the solid phase in zone I is not cleaned from component B, and it appears subsequently in zone IV after the column switching, and in this way, eventually B contaminates the raffinate product. Similarly, if $\tau_{IV} = 1$, then the liquid phase in zone IV is not cleaned from component A, which leads to its appearance in zone I and eventually to contamination of the extract product with A. The movement of these two waves is detected by two UV sensors positioned on each of the two outlet ports, as shown in Figure 2.6b. When the concentration measured by the UV sensor drops below (on the extract port) or jumps above (on the raffinate port) some specified value, called a wave footprint threshold (Figure 2.6a), then this time is recorded. The duration from the beginning of the current cycle until the footprint of the desorption wave of the more retained component passes through the UV sensor on the extract port in absolute units is t_I (left-hand side diagram in Figure 2.6c). In contrast, the duration from the passing of the footprint of the adsorption wave of the less retained component through the UV sensor on the raffinate port until the end of the current cycle is t_{IV} (right-hand side diagram in Figure 2.6c). Then, the dimensionless times, τ , are defined as

$$\tau_d = \frac{t_d}{t_{sw}}, \quad d = I, IV. \quad (2.46)$$

The value of the footprint threshold should be chosen as small as possible to be representative of the exact position of the concentration wave, but at the same time, it should be higher than the measurement noise.

The described model (Eqs. (2.1)-(2.19)) together with the measurement setup (Eqs. (2.42)-(2.46)) is used for the creation of a software tool written in MATLAB for the simulation of different chromatographic processes. This tool serves both as a virtual SMB chromatographic plant as well as a model inside the predictive controller and the moving horizon estimator. The configuration of the software tool allows simulations without plant-model mismatch as well as simulations with structural and parametric uncertainties. Structural uncertainties are related to the dead volumes in the fittings and the other equipment. Parametric uncertainties may arise from the difference between the real values of the adsorption (H and b) and kinetic parameters (D_{ax} and k_m) or process variables (C_{Fe}) and their model analogues. These uncertainties will affect the prediction of the distributed

parameter model, but they could be compensated using parameter adaptation or state and parameter estimation. From a control perspective, the SMB process is very challenging because it is a continuous system with discrete events (column switchings), complex non-linear dynamics, interconnected control channels, and long settling times. Moreover, in practice, there is often insufficient real-time measurement information.

Chapter 3

PID control of SMB processes

SMB chromatographic separations are widely used for difficult separations. However, due to their complexity, SMB processes are typically operated in open-loop mode in most real-world cases nowadays. Furthermore, there is often a lack of reliable information for some of the crucial process parameters, such as adsorption isotherm parameters or column dead volumes. Combined with the fact that, at their economic optimal operating point, these processes are sensitive to disturbances, leads to the fact that they are operated at suboptimal conditions for robustness reasons. All this leads to an increased interest in applying some form of process control to SMB chromatographic processes. In the present chapter, a decentralised control scheme based on PID controllers is investigated.

3.1 Decentralised PID control motivation

As already mentioned, in previous work done by Fütterer and Suvarov in Magdeburg and Mons (Fütterer, [2010](#), Suvarov et al., [2014](#), Suvarov et al., [2016](#), and Suvarov, [2016](#)) on the control of binary SMB processes, a new adaptive control concept called adaptive cycle-to-cycle control was developed. This concept consists of simple linear feedback controllers (PI controllers) for disturbance rejection and an online recursive estimator for feedforward action. The concept is called cycle-to-cycle because the control inputs are applied to the SMB plant at the beginning of each switching interval, and the control action remains constant for the rest of the interval. At the end of the switching interval, new control inputs are calculated and applied to the plant again after the next switching. An adaptive feature is provided by the parameter estimator.

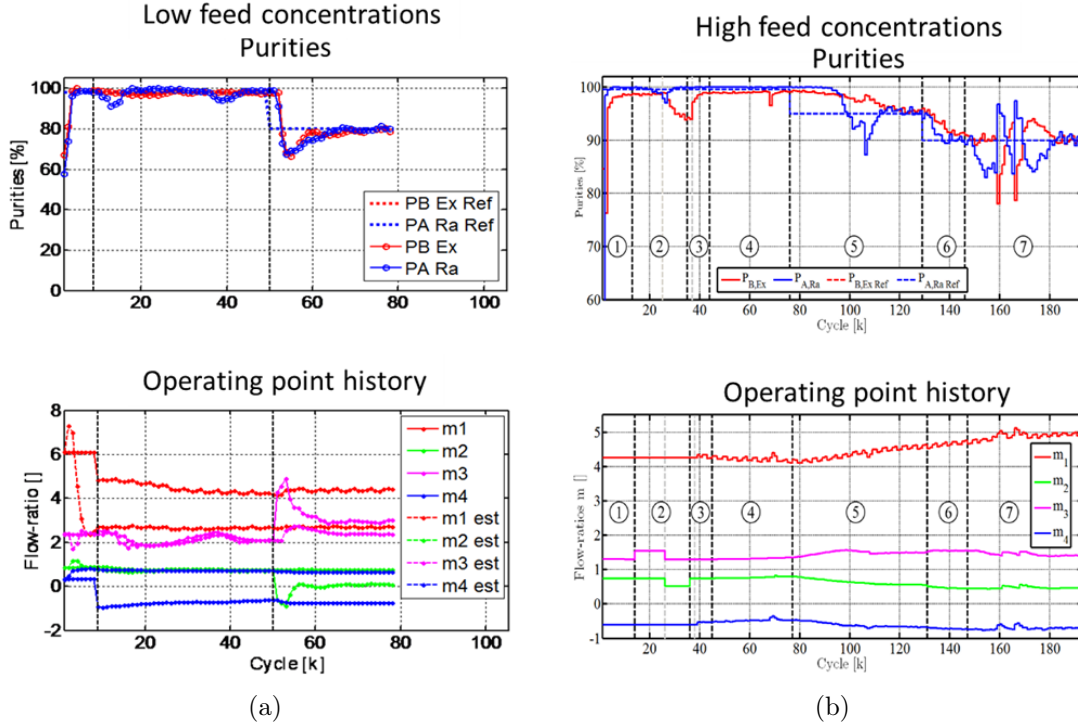


Figure 3.1: Adaptive cycle-to-cycle control of a binary SMB process – experimental validation of the control concept based on parameter estimators and PI controllers.

(a) Low feed concentrations, $C_{i,Fe} = 0.5 \text{ g} \cdot \text{L}^{-1}$. Taken from (Suvarov et al., 2016).

(b) High feed concentrations, $C_{i,Fe} = 5 \text{ g} \cdot \text{L}^{-1}$ (Suvarov, unpublished).

Upper diagrams – product purities evolution; lower diagrams – m values evolution.

The proposed concept was validated experimentally for the separation of a racemic bicalutamide mixture, which is an active pharmaceutical ingredient used in prostate cancer medications. The measured variables were the positions of the moving concentration fronts, measured by two UV detectors, and the purities of the two product outlets, measured by an online HPLC system. Control variables were the internal flowrates Q_{int} in each zone and the switching time t_{sw} . The control part consisted of four simple single-loop PI controllers, which essentially controlled the position of the concentration fronts in the regeneration zones of the SMB plant, i.e., zones I and IV, as well as the two product purities. A significant part of the proposed control concept is the parameter estimator, which finds online the optimal operating conditions for total separation, i.e. minimum desorbent consumption and maximum productivity. The estimator is based on information about the moving concentration fronts inside the SMB plant (Suvarov et al., 2019). The estimator used the retention times τ_d measured from the UV detectors in zones I and IV and obtained from a simple wave footpoint model in zones II and III to estimate online the optimal operating conditions for total separation in terms of the m values, which are dimensionless flowrates in each zone.

This control concept was validated experimentally for both low feed concentrations within the linear range of the adsorption isotherms (cf. Figure 3.1a) and high feed concentrations within the nonlinear range of the adsorption isotherms (cf. Figure 3.1b). As shown in Figure 3.1a, this simple control concept is effective and performs very well at low feed concentrations. In the first 10 cycles, the parameter estimator finds the optimal operating conditions for total separation in an open-loop, which were then applied to the plant. After that cycle, the control loops were closed and total separation was maintained. In the 60th cycle, the set point was changed to reduced purity separation, i.e. at 80 % for both of the product streams, and the controllers were able to track these reference changes only for about 10 cycles. However, for the validation at high feed concentration (Figure 3.1b), the results showed that the control concept had problems. The reference changes were tracked down by the controllers, but at the cost of considerably longer regulation time (about 50 cycles) and higher overshoots. That could be explained by the fact that concentration fronts are much steeper at high feed concentrations, making the SMB process more sensitive to disturbances, and by the cross-coupling between the control channels, which cannot be neglected anymore. This clearly shows that a control scheme consisting solely of single-loop independent PI controllers is not suitable for nonlinear conditions. In such cases, a more advanced control strategy is required. It was conjectured that decoupling could improve the control performance.

3.2 Multivariable control

In order to investigate the performance of a multivariable control scheme for an SMB process, a scheme without the parameter estimator is considered in the following simulation study. The control scheme is shown in Figure 3.2.

When operating such processes, it is very convenient to consider m values as manipulated variables, because triangular diagrams make it easy to see under what conditions the system is operating and what values of the process variables could be expected. A change in one of the m values leads to a direct change in the state of the SMB plant in that zone and, consequently, in the controlled variables. Thus, a change in m_I leads to a change in the conditions in regeneration zone I, a change in m_{II} leads to a direct change in separation zone II, and due to cross-coupling, to a lesser extent in the other separation zone III, the same applies to m_{III} —its change leads to a direct change in the conditions

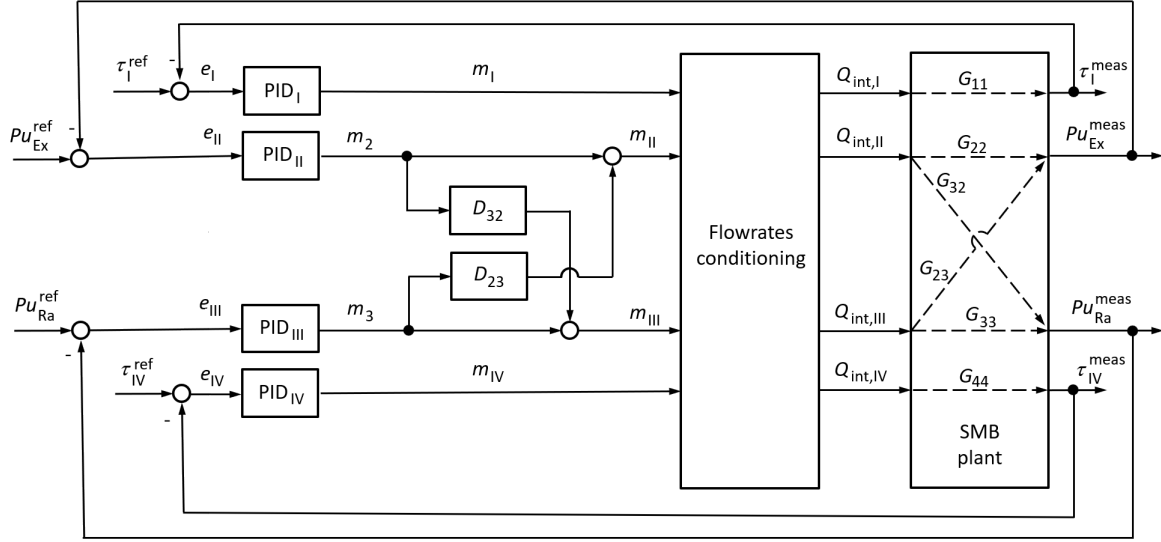


Figure 3.2: Control block-diagram for the binary SMB process with decoupling of the separation control channels.

in zone III and, to a lesser extent, in zone II, and finally, the change in m_{IV} leads to changes in regeneration zone IV. Thus, the control channels in the two regeneration zones are $m_I \rightarrow \tau_I$ and $m_{IV} \rightarrow \tau_{IV}$, while in the two separation zones they are $m_{II} \rightarrow Pu_{Ex}$ and $m_{III} \rightarrow Pu_{Ra}$, and the cross-coupling effects in the separation zones must also be accounted for in some way. Any other distribution of the control channels is inefficient and, therefore, in practice, when controlling SMB processes, it is not necessary to perform a relative gain array analysis to determine the loop pairing. In open-loop operation of SMB processes, the m values are typically determined offline beforehand and maintained in manual mode. However, when using any form of process control, these values are calculated by the control system. Once the m values have been obtained, the physical operating parameters of the SMB process are also determined, namely the four internal flowrates in each plant zone and the switching time. Since there are four m values and five operating parameters, one of them is assumed to be constant, and using this value and the already known m values, the remaining four parameters are computed using Eq. (2.32). Typically, the switching time is kept constant, with the selected value accounting for the maximum pressure drop in the SMB system and/or the minimum time required to complete all online concentration measurements. In this way, all four internal flow rates are known; using them, the external flow rates are computed from the total nodal material balance equations (Eqs. (2.6), (2.8), (2.10), (2.12)), which are maintained by the four pumps, which effectively act as the actuators of the control system.

For the decoupling of the control channels within the separation zones of the SMB plant, two static decouplers are considered (Figure 3.2). The decoupler D_{32} should be chosen in such a way that it eliminates the influence of m_{III} over the extract purity and vice versa for D_{23} and the raffinate purity. The decoupler transfer functions, D_{ph} , are

$$D_{32}(s) = -\frac{G_{32}(s)}{G_{33}(s)}, \quad (3.1)$$

$$D_{23}(s) = -\frac{G_{23}(s)}{G_{22}(s)}. \quad (3.2)$$

where G_{ph} are SMB plant transfer functions, which are given in Appendix A. For a static decoupling we have

$$D_{32}(0) = -\frac{G_{32}(0)}{G_{33}(0)} = -\frac{K_{32}}{K_{33}}, \quad (3.3)$$

and

$$D_{23}(0) = -\frac{K_{23}}{K_{22}}. \quad (3.4)$$

For the investigation of the decoupled control scheme, an SMB system governed by Langmuir isotherms is considered. The SMB plant configuration consists of one chromatographic column per zone, and the physical parameters of the separation system corresponding to high feed concentrations in the nonlinear range of the adsorption isotherms, together with the parameters of the SMB plant, are summarised in Table 3.1

For the initial operating point close to the triangle tip, the SMB plant gains are

$$K_{22} = 1.37, \quad (3.5)$$

Table 3.1: Parameters of the binary SMB chromatographic plant with Langmuir adsorption isotherms.

Quantity	Value	Quantity	Value
H_A , [-]	0.82	H_B , [-]	2.45
b_A , [$\text{L}\cdot\text{g}^{-1}$]	0.03	b_B , [$\text{L}\cdot\text{g}^{-1}$]	0.15
k_{m_A} , [s^{-1}]	10	k_{m_B} , [s^{-1}]	10
$C_{A,\text{Fe}}$, [$\text{g}\cdot\text{L}^{-1}$]	5	$C_{B,\text{Fe}}$ [$\text{g}\cdot\text{L}^{-1}$],	5
L_{col} , [m]	0.147	D_{col} , [m]	0.025
ε , [-]	0.78	t_{sw} , [s]	140
N , [-]	101	Column configuration	1-1-1-1
Initial operating point, m_{init} , [-] [5.09, 0.59, 1.285, 0.16] ^T			

$$K_{23} = -0.15, \quad (3.6)$$

$$K_{33} = -5.67, \quad (3.7)$$

$$K_{32} = -0.11. \quad (3.8)$$

Thus, the two decouplers are

$$D_{32}(0) = -\frac{-0.11}{-5.67} = -0.02, \quad (3.9)$$

$$D_{23}(0) = -\frac{-0.15}{1.37} = 0.11. \quad (3.10)$$

The four PID controllers are realised with a bumpless transfer functionality for the transfer from manual to automatic control modes, and anti-windup mechanism. The outputs of the controllers are the m values, which are the ratios of the liquid to the solid flowrates in each zone of the SMB plant. The internal flowrates, $Q_{\text{int},p}$, are computed from Eq. (2.32) and applied to the SMB plant by the four pumps through the external flowrates. The control law of the four controllers is the classical PID algorithm and it is

$$m_{p,r} = K_{P,p}e_{p,r-1} + K_{I,p} \sum_{k=1}^{r-1} e_{p,k} + K_{D,p}(e_{p,r-1} - e_{p,r-2}), \quad p = \text{I}, \dots, \text{IV}, \quad (3.11)$$

here r is an index denoting the current cycle and $e_{p,r-1}$ is the control error from the previous $(r-1)$ th cycle

$$e_{p,r-1} = \tau_{p,r-1}^{\text{ref}} - \tau_{p,r-1}^{\text{meas}}, \quad p = \text{I}, \text{IV}, \quad (3.12)$$

$$e_{\text{II},r-1} = Pu_{\text{Ex},r-1}^{\text{ref}} - Pu_{\text{Ex},r-1}^{\text{meas}}, \quad (3.13)$$

$$e_{\text{III},r-1} = Pu_{\text{Ra},r-1}^{\text{ref}} - Pu_{\text{Ra},r-1}^{\text{meas}}. \quad (3.14)$$

Here $K_{P,p}$, $K_{I,p}$ and $K_{D,p}$ are the proportional, integral, and derivative gains of the four PID controllers, respectively, τ_p^{ref} and τ_p^{meas} are the reference and measured values of τ_{I} and τ_{IV} , respectively, $Pu_{\text{Ex}}^{\text{ref}}$ and $Pu_{\text{Ra}}^{\text{ref}}$ are the reference values of the extract and raffinate products, respectively, and $Pu_{\text{Ex}}^{\text{meas}}$ and $Pu_{\text{Ra}}^{\text{meas}}$ are the values of the extract and raffinate products obtained by Eq. (2.44) and Eq. (2.45), respectively. To be in the physically feasible region of the $m_{\text{II}} - m_{\text{III}}$ and $m_{\text{IV}} - m_{\text{I}}$ diagrams (Figure 3.5), the following conditions for

Table 3.2: Control parameters for the decoupled control scheme.

Quantity	Value	Quantity	Value	Quantity	Value	Quantity	Value
$K_{P,I}$, [-]	-7.12	$K_{P,II}$, [-]	3.11	$K_{P,III}$, [-]	-3.38	$K_{P,IV}$, [-]	2.53
$K_{I,I}$, [-]	-9.18	$K_{I,II}$, [-]	2.27	$K_{I,III}$, [-]	-4.79	$K_{I,IV}$, [-]	0.81
$K_{D,I}$, [-]	-0.14	$K_{D,II}$, [-]	1.06	$K_{D,III}$, [-]	-0.59	$K_{D,IV}$, [-]	1.54
		$D_{32}(0)$, [-]	-0.02	$D_{23}(0)$, [-]	0.11		
Wave footprint threshold, [g·L ⁻¹]				0.15			

the newly computed controls must hold

$$m_{II,r} \leq m_{III,r}. \quad (3.15)$$

$$m_{IV,r} \leq m_{I,r}. \quad (3.16)$$

If the last two inequalities are not satisfied, then $m_{II,r} = m_{III,r}$ and $m_{IV,r} = m_{I,r}$. The four PID controllers are tuned with the Skogestad's tuning rule (Skogestad, 2003) and all control parameters are summarised in Table 3.2.

The simulation scenario is chosen in such a way that it is of practical interest for operating real SMB processes. Both total and incomplete separations are considered, along with the start-up procedure and transitions, with considerable changes in the references. When allowed, incomplete separation conditions are attractive because, under such conditions, productivity is much higher compared to total separation. The change of the references can be seen in Table 3.3.

The simulation results are presented in Figures 3.3-3.5. The plant is started-up in open-loop mode for the 19 cycles until a cyclic state for the chosen operating point is reached, then the control loops are closed with references close to the current values for τ_I

Table 3.3: Change of the references for the decoupled control scheme.

Cycles, k	Pu^{ref} , [%]		τ^{ref} , [-]	
	Extract	Raffinate	I	IV
1 - 19	Open loop			
20 - 45	95	95	0.45	0.68
46 - 60	99	99	0.45	0.68
61 - 80	90	90	0.52	0.64
81 - 100	95	95	0.6	0.6

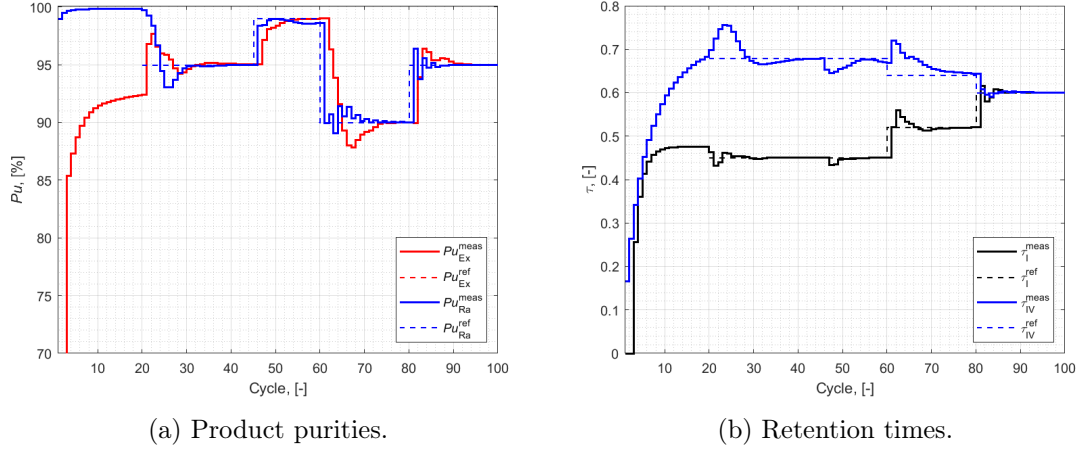


Figure 3.3: Process variables for the decoupled control scheme.

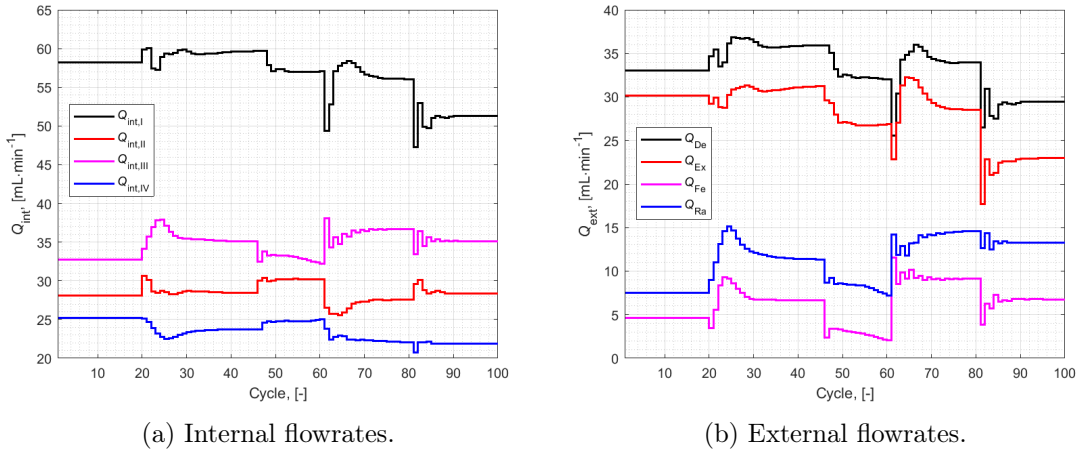


Figure 3.4: Manipulated variables for the decoupled control scheme.

and τ_{IV} . The control quality is very good for the references corresponding to the operating point for which the controllers and decouplers were tuned. However, in the cases where we are far from this point, the quality of the control is not satisfactory. In order to overcome this issue, the controllers and the decouplers should be retuned for each new operating point, which is not in the vicinity of the initial one, i.e. gain-scheduling control and a look-up table with tuning parameters are necessary. In such a way, the biggest advantage of the simple control scheme with PID controllers is lost. That is why in the following chapters, more advanced centralised control concepts based on online optimisation, namely model predictive control, are considered.

3.3 Control with active input constraints

To ensure the safe and efficient operation of the SMB process, some constraints are imposed on its manipulated variables. The constraints are determined both by the char-

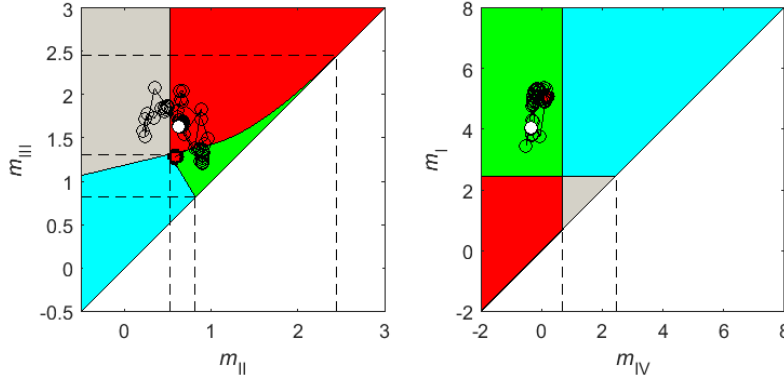


Figure 3.5: Change of the operating point (m values) in the triangle diagram for the decoupled control scheme. Red circle – initial operating point (1st cycle), white circle – last operating point.

acteristics of the SMB process itself and by the physical limitations of the actuators in the control system. The maximum flow rate is limited by the pressure drop that the chromatographic columns can withstand. In addition, the pumps that are the actuators of the SMB plant have a recommended operating range, beyond which their operating characteristics become nonlinear or the accuracy of operation decreases. Therefore, their operation in these areas is not recommended. Since the maximum internal flow rate is in the first zone, immediately after the desorbent inlet, it determines the maximum allowable pressure drop in the system.

To evaluate the performance of the control system near the input boundaries, a test case with an active constraint on the maximum flow rate is considered. The results are presented in Figures 3.6a and 3.7a where $Q_{\text{int,I}}^{\text{max}} = 64 \text{ mL} \cdot \text{min}^{-1}$ is denoted by the dashed black line and the real $Q_{\text{int,I}}$ applied to the SMB plant is denoted by the solid black curve. At the 40th cycle, the reference of τ_1^{ref} is changed from 0.45 to 0.37. In order for the control system to follow that change in the reference, it computes values for $Q_{\text{int,I}}$ beyond the maximum of $64 \text{ mL} \cdot \text{min}^{-1}$, thereby making the imposed constraint active. At this moment, the actuator becomes saturated and further increase of $Q_{\text{int,I}}$ does not affect τ_1 . As a result, two phenomena occur.

The first is that the new reference has not been reached due to the physical limitations

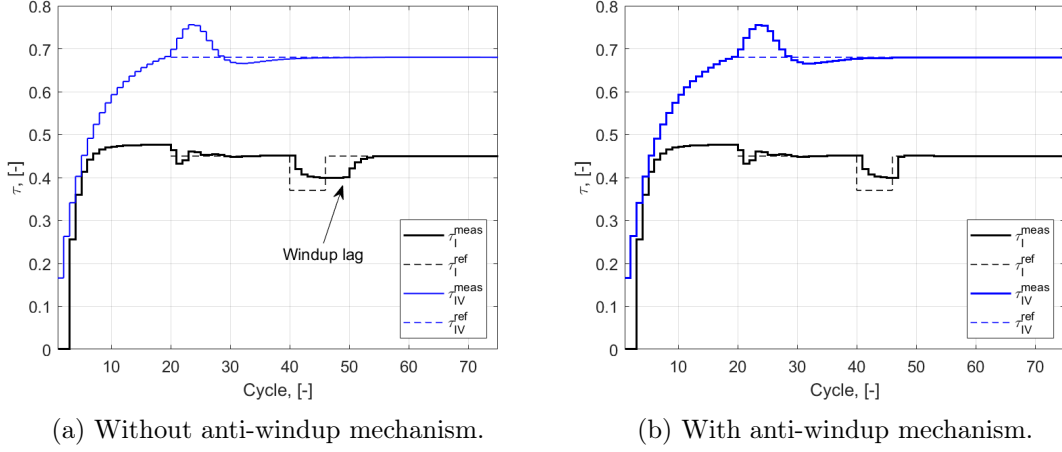


Figure 3.6: Dimensionless retention times for the control with active constraint of $Q_{\text{int},I}$.

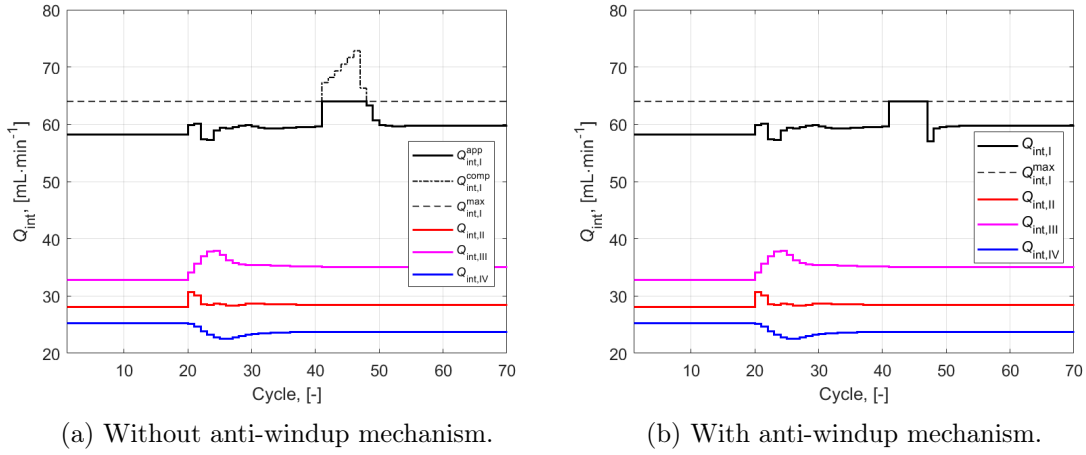


Figure 3.7: Internal flowrates for the control with active constraint of $Q_{\text{int},I}$.

of the SMB plant. Possible ways to overcome this problem include changing the hardware equipment, for example, by using chromatographic columns that allow a larger pressure drop or pumps with a wider operating range; or by avoiding operating in regimes where these limits are reached. Unfortunately, in practice, these possible solutions are either too expensive or technically too challenging. Another possible solution is to use a process control method that implicitly incorporates the input constraints already during the computation procedure of the control signal, such as MPC.

The second observed phenomenon is that when the reference τ_I^{ref} returns to 0.45 in the 46th cycle, the control system does not react immediately; it takes some time before doing so in the 50th cycle. That is because, due to actuator saturation, the existing control error causes the integral part of the signal to increase from cycle to cycle. With each subsequent cycle, the calculated output of the PID controller increases, as shown by the black dash-dotted curve in Figure 3.7a. After changing the reference from 0.37 to 0.45,

the error changes sign, and the integral component starts to decrease, requiring some time to return the integral of the control error to a value at which the flowrate is less than the maximum and the actuator to exit its saturated state. This phenomenon is called integral windup, and the associated delay a windup lag.

Various techniques have been proposed in the literature to tackle the integral windup (Åström and Hägglund, 2006) — back-calculation and tracking, control signal clamping, integrator clamping, etc. In the present work, the method of clamping the control signal is used. In practice, the integral component of the control action is computed using Eqs. (3.17) and (3.18)

$$I_{p,r} = \begin{cases} I_{p,r-1} + K_{I,p} e_{p,r-1}, & \text{if } m_p^{\min} < m_{p,r} < m_p^{\max} \\ I_{p,r-1} + K_{I,p} e_{p,r-1} + (m_p^{\text{sat}} - m_{p,r-1}), & \text{if } m_{p,r} \leq m_p^{\min} \text{ or } m_{p,r} \geq m_p^{\max} \end{cases},$$

$$p = \text{I, IV},$$
(3.17)

$$I_{p,r} = \begin{cases} I_{p,r-1} + K_{I,p} e_{p,r-1}, & \text{if } m_p^{\min} < m_{p,r} < m_p^{\max} \\ I_{p,r-1} + K_{I,p} e_{p,r-1} + (m_p^{\text{sat}} - m_{p,r-1} - \alpha^{\text{sat}} m_{ph,r-1}), & \text{if } m_{p,r} \leq m_p^{\min} \text{ or } m_{p,r} \geq m_p^{\max} \end{cases},$$

$$p = \text{II, III}$$
(3.18)

Here I is the integral of the computed control action by the PID controllers, $m^{\min}$ and $m^{\max}$ are the minimum and maximum allowed values of the process variables beyond which the actuators saturate, m^{sat} is the saturation value of control signal and is equal to either $m^{\min}$ or $m^{\max}$ depending on which constrained is violated, and α^{sat} is a gain-reduction factor, ($0 < \alpha^{\text{sat}} < 1$). When the input constraints are not active, i.e. the computed control signal lies between the minimum and maximum allowed values, the expression in the first row of both formulas is used to compute the integral part. In contrast, only when the input constraints are active, the second row of the two formulas is used in both formulas to compute the integral part of the control action. Thus, when saturation is reached, the error between the saturated and computed control variables is removed from the integral part, I . In this way, the integrator is kept at the saturation limit, and as a result, when the

reference changes in the opposite direction, the control system is able to respond quickly. In addition to that, when saturation is reached, the control signal is also clamped

$$m_{p,r} = m_p^{\min}, \text{ if } m_{p,r} \leq m_p^{\min}, \quad (3.19)$$

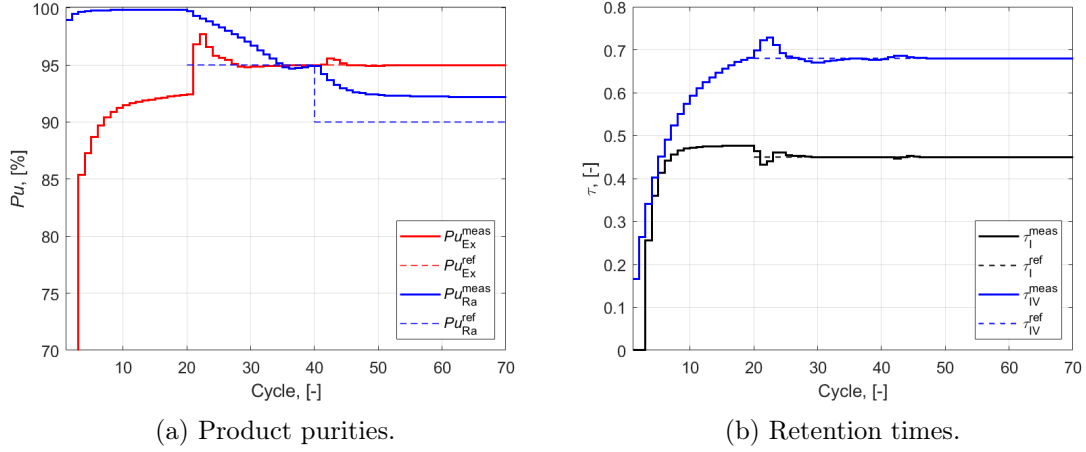
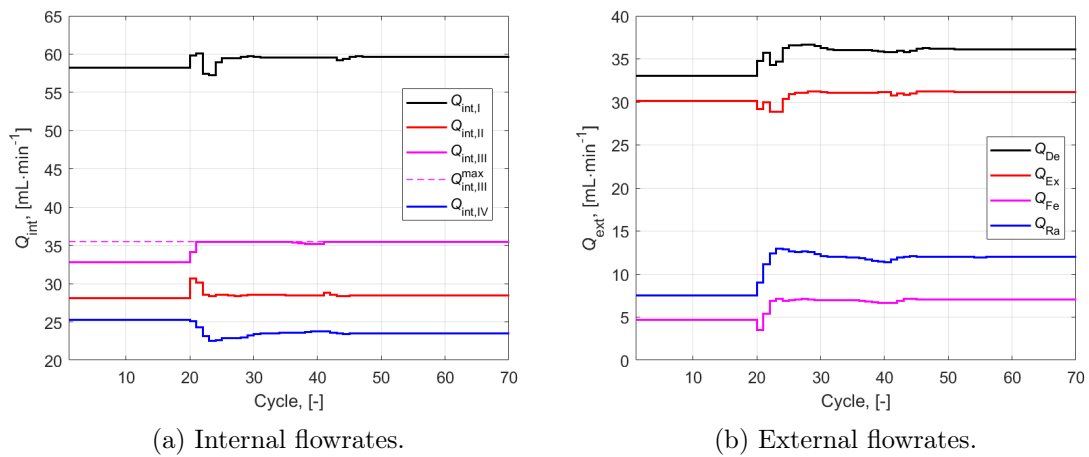
$$m_{p,r} = m_p^{\max}, \text{ if } m_{p,r} \geq m_p^{\max}. \quad (3.20)$$

Figures 3.6b and 3.7b show the simulation results of the same test problem with the anti-windup mechanism enabled. As can be seen, the reference at $\tau_I^{\text{ref}} = 0.35$ is still not reached, but the windup lag has been eliminated, and the control system leaves the saturation state and reacts immediately after the last change of τ_I^{ref} .

A case with active input constraints within the separation zone is also considered. For this purpose, $Q_{\text{int,III}}^{\max} = 35.5 \text{ mL} \cdot \text{min}^{-1}$; this constraint may arise from technological reasons or may be a result of a pump malfunction. In this case, the influence of the corresponding decoupler must also be considered when calculating the saturation error. That is realised by the last term in the brackets of Eq. (3.18). The idea of reducing the decoupler signal by the factor α^{sat} is presented in Hoyo et al., 2023. In this publication, the authors investigated several systems with feedforward control and found that the optimal value of α^{sat} lies in the range 0.6-0.8. Therefore, $\alpha^{\text{sat}} = 0.6$ is used in the present study case. The results are presented in Figures 3.8 and 3.9.

Two cycles after the control loops are closed, the constraint imposed on $Q_{\text{int,III}}$ becomes active; nevertheless, both purities manage to reach their references. For the raffinate purity, this takes 15 cycles. After reaching the setpoint $Pu_{\text{Ra}}^{\text{ref}}$, the constraint ceases to be active, and the saturated state of the actuator is immediately exited thanks to the anti-windup mechanism. However, the control action remains close to $Q_{\text{int,III}}^{\max}$. In the 40th cycle, $Pu_{\text{Ra}}^{\text{ref}}$ changes from 95 % to 90 %, at which point the actuator becomes saturated again, and the constraint becomes active. Unfortunately, after this second saturation, the control system is unable to lead the raffinate purity to its new reference.

An online optimising control, such as MPC, could be advantageous over this traditional control scheme because it performs better near the physical limits of the SMB process. Due to the fact that MPC natively incorporates the constraints into its real-time optimisation procedure. This issue will be discussed in the following chapter.

Figure 3.8: Process variables for the control with active constraint of $Q_{int,III}$.Figure 3.9: Manipulated variables for the control with active constraint of $Q_{int,III}$.

Chapter 4

Optimisation-based control of SMB processes using the cell method

Chapter 4 was published in parts in Computers & Chemical Engineering, (2024), 189, Chernev, V.P., Kienle, A., Vande Wouwer, A. Moving horizon estimation and control of a binary simulated moving bed chromatographic process with Langmuir isotherms. p. 108804².

4.1 Moving horizon estimator

For the predictive controller to compute the control action for the next cycle, the internal concentration profiles, which serve as an initial condition for the PDAE system (Eq. (2.1)), need to be available before the beginning of this cycle. A direct measurement is impossible as it would require the placement of an array of sensors along the chromatographic columns which would be technically challenging and very expensive. For this reason, the concentration profiles are obtained from the limited measurement information by state observers or estimators. In the present work, moving horizon estimators (MHE) are considered as they are in line with model predictive control (they are the dual concept of MPC) and they can handle nonlinear distributed parameter systems. In MPC, the manipulated control inputs are computed through real-time optimisation over a future horizon based on the process model and the available measurement information up to the present time. To facilitate implementation, the control inputs are usually discretised using

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the control vector parametrisation. In a dual way, MHE estimates the initial condition of a receding time window extending in the past through optimisation based on the dynamic model and past values of the input and output signals. Since the measurement information from the SMB plant is available once at the end of each cycle and the control input is applied to the plant again at the beginning of each cycle and then kept constant until the end of the cycle, it is natural to consider a sampling interval for MPC and MHE corresponding to the cycle duration.

Estimating the internal concentration profiles would, a priori, require a large number of variables (since there are two distinct components in two phases over four chromatographic columns), which is why, in this study, the unknown profiles are approximated using basis functions. In this way, the estimated variables can be reduced to a few coefficients in these basis functions. Two types of basis functions were tested, i.e., Lagrangian and sigmoid functions of the wave type. The results showed that the sigmoid functions in Eq. (4.1) approximate the internal concentration profiles more closely. This is due to the fact that the sigmoid function represents an analytical solution of the corresponding travelling wave problem under idealising conditions in an infinitely extended spatial domain (Fechtner and Kienle, 2021). In the remainder, therefore, sigmoid functions are used to approximate the concentration profiles.

$$\begin{cases} C_{0,i,p}^{\text{est}}(r - N_{\text{eh}} + 1) = \frac{\alpha_{i,p}^{\text{liq}}}{1 + \exp(\beta_{i,p}(z - \gamma_{i,p}))}, \\ q_{0,i,p}^{\text{est}}(r - N_{\text{eh}} + 1) = \frac{\alpha_{i,p}^{\text{sol}}}{1 + \exp(\beta_{i,p}(z - \gamma_{i,p}))}, \end{cases} \quad i = \text{A, B, } \dots, N_{\text{comp}}; \quad p = \text{I, II, } \dots, N_{\text{col}}. \quad (4.1)$$

In these expressions, the coefficients $\alpha_{i,p}^{\text{liq}}$, $\alpha_{i,p}^{\text{sol}}$, $\beta_{i,p}$ and $\gamma_{i,p}$ are the four parameters to estimate, while $C_{0,i,p}^{\text{est}}$ and $q_{0,i,p}^{\text{est}}$ are the resulting liquid and solid concentration profiles at the beginning of the estimation horizon ($r - N_{\text{eh}} + 1$), N_{eh} is the length of the estimation horizon. In a sigmoid function, α represents the height, β the steepness of the curve and γ the position of the midpoint along the z -axis. Since the internal concentration profiles for the liquid and the solid phases differ from each other only by their height, there are two sets of coefficients α per component in each of the columns – one for the liquid and one for the solid phase. However, there are only one β and one γ coefficient because the forms and positions of the liquid and solid profiles are similar. The total number of estimated parameters is $4N_{\text{comp}}N_{\text{zone}}$, and in our particular case, for a binary separation with 1

column per zone, $4 \cdot 2 \cdot 4 = 32$ parameters to estimate. As in the linear case, the number of measurements should be at least equal to the number of estimated parameters. This is obviously a necessary but not sufficient condition. Sufficiency would require that the mapping from the measurements to the estimated parameters is regular. For the present nonlinear hybrid system with distributed parameters, proving regularity is difficult. It was found empirically that the estimation horizon N_{eh} should be at least 6 cycles. For every cycle there are 6 measurements – 2 extract concentrations ($\bar{C}_{A,Ex}$ and $\bar{C}_{B,Ex}$), 2 raffinate concentrations ($\bar{C}_{A,Ra}$ and $\bar{C}_{B,Ra}$), and 2 dimensionless retention times (τ_I and τ_{IV}), which gives a total number of 36 measurements to identify the 32 unknown parameters.

The MHE optimisation problem is formulated as:

$$\min_{\alpha_{i,p}^{liq}, \alpha_{i,p}^{sol}, \beta_{i,p}, \gamma_{i,p}} J_{MHE} = \sum_{k=r-N_{eh}+1}^r (\mathbf{y}_k^{meas} - \mathbf{y}_k^{est})^T \mathbf{P}_k (\mathbf{y}_k^{meas} - \mathbf{y}_k^{est}), \quad (4.2a)$$

$$\begin{aligned} \text{s.t. } \alpha_{i,p}^{liq, \min} &\leq \alpha_{i,p}^{liq} \leq \alpha_{i,p}^{liq, \max}, \\ \alpha_{i,p}^{sol, \min} &\leq \alpha_{i,p}^{sol} \leq \alpha_{i,p}^{sol, \max}, \\ \beta_{i,p}^{\min} &\leq \beta_{i,p} \leq \beta_{i,p}^{\max}, \\ \gamma_{i,p}^{\min} &\leq \gamma_{i,p} \leq \gamma_{i,p}^{\max}, \end{aligned} \quad (4.2b)$$

$$i = A, B, \dots, N_{comp}; \quad p = I, II, \dots, N_{col},$$

where J_{MHE} is the value of the moving horizon estimation cost function, $\mathbf{y}$ is a vector containing all of the measurements, $\mathbf{y} = (\bar{C}_{A,Ex}, \bar{C}_{B,Ex}, \bar{C}_{A,Ra}, \bar{C}_{B,Ra}, \tau_I, \tau_{IV})^T$, $\mathbf{P}_k$ is a weight matrix and k is the cycle index. In the deterministic case, $\mathbf{P}_k$ is chosen as the identity matrix ($\mathbf{P}_k = \mathbf{I}_6$). In contrast, when there is additive measurement noise, the weight matrix is the inverse of the noise covariance matrix ($\mathbf{P}_k = \mathbf{Q}_k^{-1}$).

$$\mathbf{Q}_k = \text{diag}((\eta_{HPLC} \bar{C}_{A,Ex,k}^{meas})^2, (\eta_{HPLC} \bar{C}_{B,Ex,k}^{meas})^2, (\eta_{HPLC} \bar{C}_{A,Ra,k}^{meas})^2, (\eta_{HPLC} \bar{C}_{B,Ra,k}^{meas})^2, (\eta_{UVI} \tau_{I,k}^{meas})^2, (\eta_{UVIV} \tau_{IV,k}^{meas})^2), \quad (4.3)$$

where η [%] is a constant relative error of the measuring device.

The initial values of the optimisation variables α^{liq} , α^{sol} , β and γ for the r -th cycle are the optimal ones from the previous $(r-1)$ th cycle, whereas for the very first cycle at which the estimator is turned on, they are given in Appendix B.

$i = \text{A, B}$ – components
 $p = \text{I, II, III, IV}$ – columns

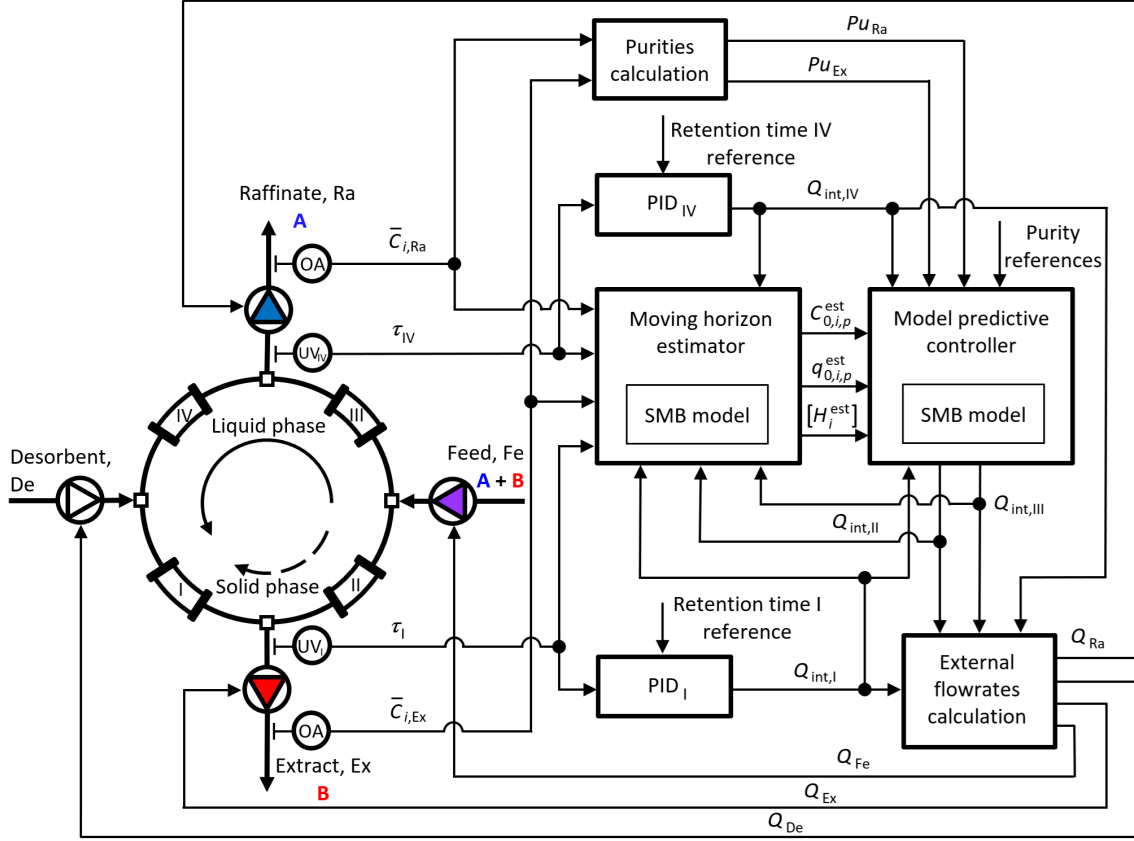


Figure 4.1: SMB process configuration with the moving horizon estimator and the combined NMPC/PID control strategy.

For the validation of the MHE, a hypothetical system with two solutes is considered. The investigated chromatographic system is the same as that in Chapter 3 and the values of its parameters are summarised in Table 3.1. The feed concentration is high enough to be in the nonlinear range of the Langmuir isotherms. The switching time interval is chosen so as to allow the HPLC measurements to occur within it. About 140 seconds are needed for measuring two samples (one from the extract and one from the raffinate outlet) (Lee et al., 2020b).

The simulation scenario is a start-up of the process with empty columns, i.e., $C_{0,i,p} = q_{0,i,p} = 0$ for the first cycle, for all components and all columns. The column configuration of the SMB plant is one column per zone, as shown in Figure 4.1. The number of equilibrium cells is 101, which allows for reasonable resolution of the steep concentration fronts typical of systems described by nonlinear Langmuir isotherms.

The operating point is changed manually by changing the m values, which are the flowrate ratios of the mobile to the stationary phase. The internal flow rates are computed

Table 4.1: Operating points for the open-loop MHE validation.

Cycles, k	m_I , [-]	m_{II} , [-]	m_{III} , [-]	m_{IV} , [-]
1 - 20	5.09	0.59	1.28	0.16
21 - 50	5.09	0.23	1.50	0.16
51 - 80	5.09	0.62	1.28	0.16
81 - 110	3.22	0.62	1.28	0.39

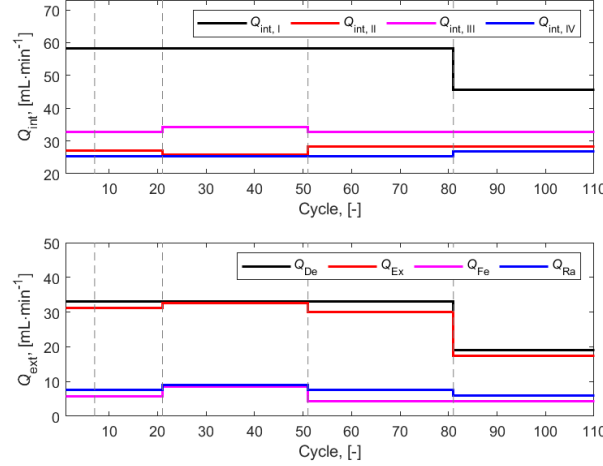


Figure 4.2: Change of the operating point for the open-loop validation of the MHE in both cases without and with additive measurement noise. The corresponding m values are given in Table 4.1.

Upper diagram – internal flowrates, lower diagram – external flowrates.

based on Eq. (2.32) and are imposed by the pumps modulating the external flow rates. The latter are computed from the total nodal material balances (Eqs. (2.6), (2.8), (2.10), (2.12)). The operation conditions for each cycle are shown in Table 4.1 and in Figure 4.2.

The estimator is turned on in the 7th cycle when the number of passed cycles is higher than the length of the estimation horizon, N_{eh} which is six. There is no difference in the values for the parameters of the virtual SMB plant and those in the model inside the estimator.

The moving horizon estimator is tested in open-loop in two case studies. The first case assumes ideal measurements without noise, whereas the second case considers additive measurement noise. All simulations in this chapter are performed with Mathworks MATLAB R2022b on a desktop PC with Intel Core i9-12900K (3.20 GHz) CPU and 32 GB RAM under the Microsoft Windows 11 Pro operating system. The optimisation procedure is performed with the built-in MATLAB Optimization Toolbox function for constrained NLP optimisation - **fmincon** with the active set method (which is an SQP method) and

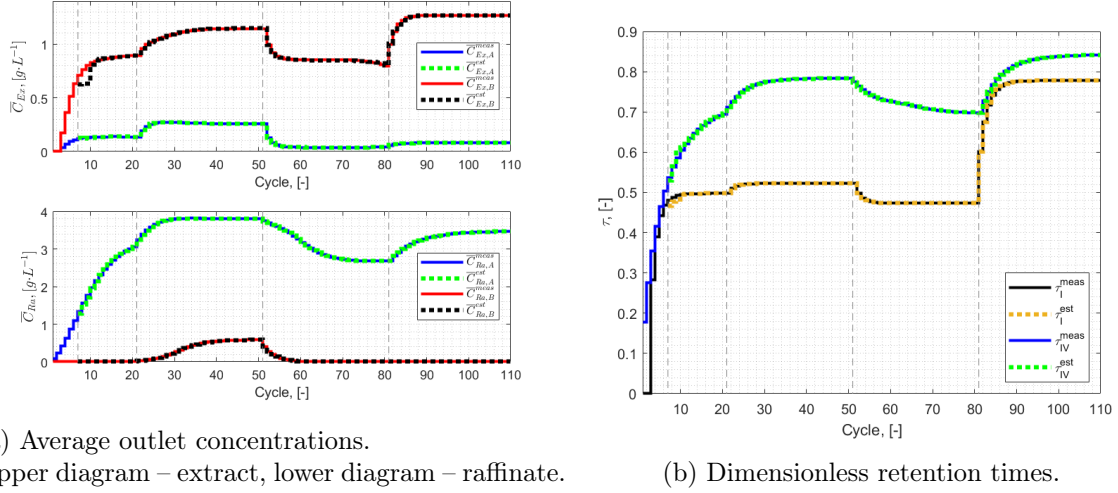


Figure 4.3: Comparison between the measured and the estimated process outputs for the open-loop MHE validation in the case with noiseless measurements. $N_{eh} = 6$.

with the default values for the stopping criteria (MATLAB, 2022b).

The simulation results in the deterministic case (no measurement noise) are presented in Figures 4.3-4.6. The MHE needs only 5 cycles to converge (Figure 4.3) even though the SMB plant has not reached its cyclic steady state yet for the given operating point. Afterwards, it closely follows the measurements until the end of the simulation, even when the operating point of the plant has been changed. What is measured by the HPLC is the averaged outlet concentrations of the r -th cycle after its end. For this simulation study, these average outlet concentrations are calculated with Eqs. (2.42)-(2.43) and they are presented in Figure 4.3a for both outlets. The product purities corresponding to the average outlet concentrations are computed with Eqs. (2.44)-(2.45) and are shown in Figure 4.4, and the operating point changes corresponding to Table 4.1 are shown in Figure 4.2. It can be concluded from the optimal value of the estimation cost function, J_{MHE} , (Figure 4.5), which is a measure of the performance of the estimator, that the quality of the estimation depends on the shape of the internal concentration profiles. When the profiles are well approximated by the sigmoid functions (cycles 81-110), the estimation error is small. Even when the shape of the profiles differs from the ideal sigmoid function, the estimation is quite satisfactory. Typical liquid phase internal concentration profiles at the end of the 67th cycle are shown in Figure 4.6 (solid curves), and compared with the estimated profiles (dashed curves).

Next, a Gaussian measurement noise with a constant relative error of 1% is considered for the HPLC analyser and the two UV devices (this error level is representative of the

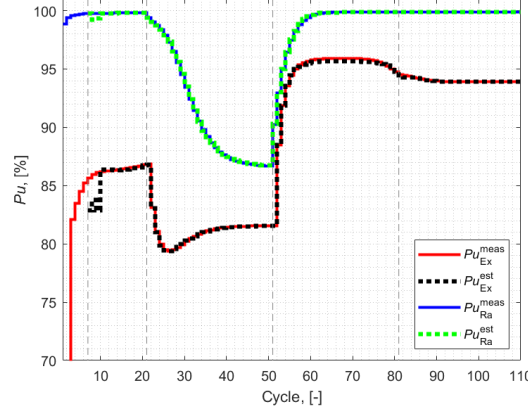


Figure 4.4: Comparison between the measured and the estimated product purities corresponding to the average outlet concentrations from Figure 4.3a for the open-loop MHE validation in the case with noiseless measurements. $N_{eh} = 6$.

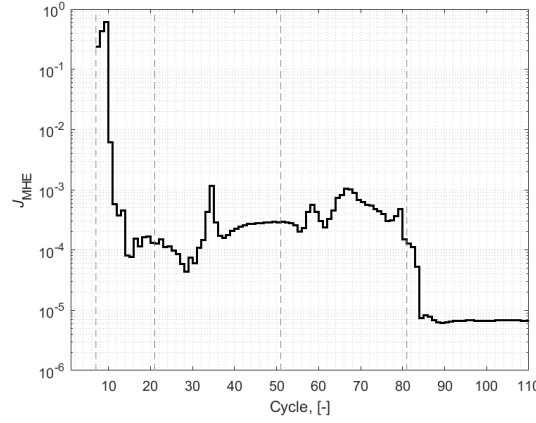


Figure 4.5: Estimation cost function optimal value at each cycle for the open-loop MHE validation in the case with noiseless measurements. $N_{eh} = 6$.

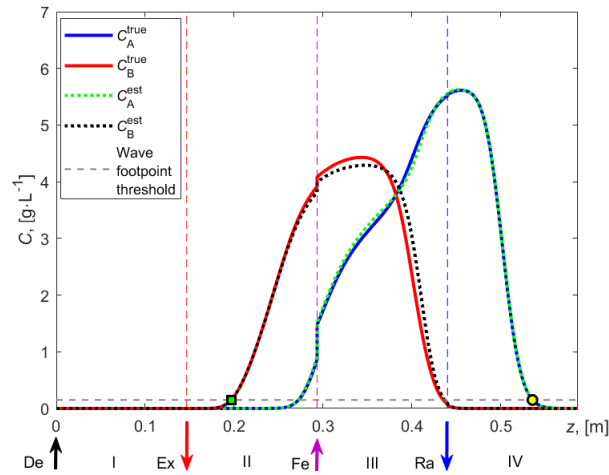
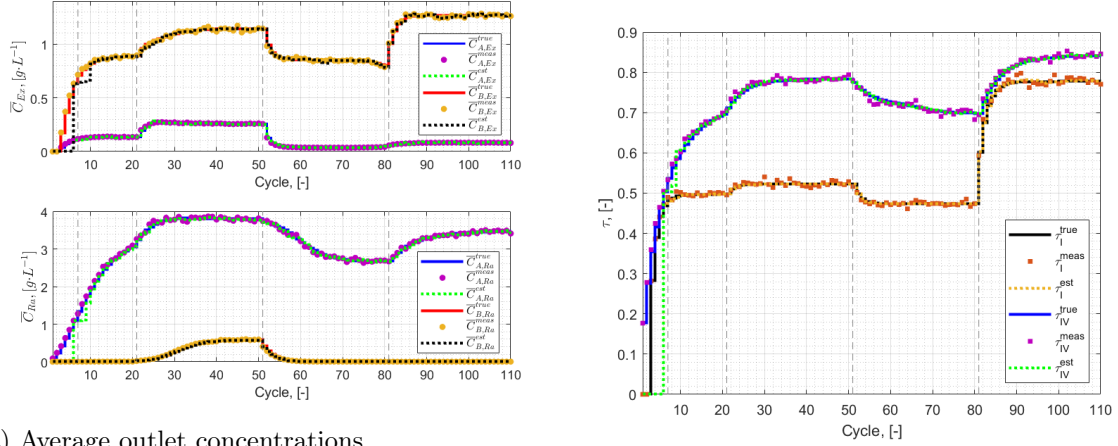


Figure 4.6: Comparison between the true and the estimated liquid phase internal concentration profiles at the end of the 67th cycle in the case with noiseless measurements. $N_{eh} = 6$.

Green square – footpoint of the desorption wave of the more retained component B, yellow circle – footpoint of the adsorption wave of the less retained component A.



(a) Average outlet concentrations.

Upper diagram – extract, lower diagram – raffinate.

(b) Dimensionless retention times.

Figure 4.7: Comparison between the measured and the estimated process outputs for the open-loop MHE validation in the case with additive measurement noise (1% relative error). $N_{eh} = 6$.

devices currently available on the market). The results are presented in Figure 4.7. Direct comparison of the values of J_{MHE} with the noise-free case study cannot be made because of the weight matrix $\mathbf{P}$. As expected, the estimation quality decreases due to the noise but the performance of the estimator is still very good.

4.2 Combined PID and NMPC control concept

The control of SMB chromatographic processes is challenging because of their complex nonlinear dynamics and long settling times. Moreover, SMB are continuous systems with discrete events (switching of the columns) and with interconnected control channels. A centralised control, such as nonlinear model predictive control (NMPC), was considered initially to tackle the process control, but with unsatisfactory results. The control structure was similar to that in Figure 4.1, where the four internal flowrates are used as manipulated variables controlling the two product purities and the position of the concentration waves in the regeneration zones. Six tuning parameters were considered, namely 4 weight factors (one for each term of the control cost function except one) plus the prediction and the control horizons. Finding suitable values for all these tuning parameters was a hard task even for the case where the concentration profiles are known to the controller, and of course, even harder for the case where the profiles are estimated using MHE.

On the other hand, as seen in Chapter 3, the dynamics of the two retention times in the regeneration zones are weakly coupled to those in the separation zones. It is therefore

possible to control them with decoupled linear controllers, such as PID controllers. This is how the idea of a hybrid control scheme combining classical PID control with NMPC emerged.

The structure of the proposed control strategy is shown in Figure 4.1. The average concentrations of the two products are measured by an online analytical analyser (high-performance liquid chromatography), and the product purities are computed according to Eqs. (2.44)-(2.45). The movement of the concentration waves inside the columns is tracked by two UV sensors placed on each of the outlet ports and the two dimensionless retention times τ_I and τ_{IV} can be determined. The manipulated variables are the four internal flowrates Q_{int} within each zone of the SMB plant whereas the switching time t_{sw} is kept constant. The internal flowrates are determined through the nodal material balances (Eqs. (2.6), (2.8), (2.10), (2.12)) by acting on the external flows established by the four pumps placed on each of the external ports. In this configuration, the internal flowrates in zones I and IV are determined by two single-loop PID controllers for the two dimensionless retention times τ_I and τ_{IV} , whereas NMPC of the two product purities computes the two internal flowrates in zones II and III.

The applied strategy is a cycle-to-cycle control, where the control action is applied to the SMB plant at the beginning of each cycle and is kept constant until the end of this cycle, where new measurement and estimation information is available and the procedure can be repeated.

The most frequent disturbances to the SMB chromatographic processes are changes in the feed concentrations, temperature fluctuations, which in turn lead to a change of the isotherm parameters, and in the long run, the ageing of the adsorbent, which again leads to a change of the isotherm parameters.

4.2.1 Control of the dimensionless retention times with PID controllers

The idea of controlling the dimensionless retention times (and in this way the concentration wave positions) in the regeneration zones is to prevent breakthroughs, which eventually would lead to contamination of the products. In this way, τ_I and τ_{IV} need to be stabilised around some specific values, ensuring a sufficient safety margin. The latter dimensionless retention times are weakly coupled with the internal flow rates inside zones II and III, and simple PID controllers are sufficient for the task. The control action for the

current r -th cycle is computed according to the classical PID control law

$$m_{p,r} = K_{P,p} e_{p,r-1} + K_{I,p} \sum_{k=1}^{r-1} e_{p,k} + K_{D,p} (e_{p,r-1} - e_{p,r-2}), \quad p = \text{I, IV}, \quad (4.4)$$

where $e_{p,r-1}$ is the control error from the previous $(r-1)$ th cycle

$$e_{p,r-1} = \tau_{p,r-1}^{\text{ref}} - \tau_{p,r-1}^{\text{meas}}, \quad p = \text{I, IV}, \quad (4.5)$$

$K_{P,p}$, $K_{I,p}$ and $K_{D,p}$ are the proportional, integral and derivative gains, respectively of the two PID controllers, and τ_p^{ref} and τ_p^{meas} are the reference and measured values of τ_{I} and τ_{IV} , respectively. To be in the physically feasible region of the $m_{\text{IV}} - m_{\text{I}}$ diagram (Figure 4.12), the following condition for the newly computed controls must hold

$$m_{\text{IV},r} \leq m_{\text{I},r}. \quad (4.6)$$

If this is not true, then $m_{\text{IV},r} = m_{\text{I},r}$. Finally, the internal flowrates $Q_{\text{int,I}}$ and $Q_{\text{int,IV}}$, which are the real manipulated variables, are computed from Eq. (2.32). The two PID controllers are implemented with an anti-windup mechanism and a bumpless transfer functionality for the transfer from manual to automatic control when the control loops are closed. In this way, considerable jumps of the manipulated and, respectively, of the process variables are avoided.

The m_{I} (or $Q_{\text{int,I}}$)– τ_{I} control loop must have negative gains because when the flowrate in the first zone is increased this pushes the concentration wave to the zone end, thus decreasing the retention time τ_{I} and vice versa. Of the two dimensionless retention times, τ_{IV} has slower dynamics and is more strongly coupled to $Q_{\text{int,III}}$ than τ_{I} to $Q_{\text{int,II}}$ making its control more challenging. The PID controllers are tuned with Skogestad's tuning rule (Skogestad, 2003).

Once the concentration waves are secured in their respective regeneration zones, control of the two product purities becomes an easier task since product contamination arising from breakthroughs has been alleviated.

4.2.2 Control of the product purities with NMPC

NMPC computes the manipulated variables in the two separation zones ($Q_{\text{int,II}}$ and $Q_{\text{int,III}}$) by minimising in real-time the following cost function

$$\begin{aligned} \min_{Q_{\text{int},p,g}} J_{\text{MPC}} = & \sum_{k=r}^{r+N_{\text{ph}}-1} \left((Pu_{\text{Ex},k}^{\text{ref}} - \hat{P}u_{\text{Ex},k})^2 + w_{Pu} (Pu_{\text{Ra},k}^{\text{ref}} - \hat{P}u_{\text{Ra},k})^2 \right) \\ & + w_{\Delta m} \sum_{k=r}^{r+N_{\text{ch}}-1} \sum_{h=\text{II}}^{\text{III}} \Delta m_{h,k}^2, \quad p = \text{II}, \text{III}, \end{aligned} \quad (4.7a)$$

subject to the following nodal material balances and lower and upper bounds for $Q_{\text{int},p}$

$$\begin{aligned} Q_{\text{Ex},g} &= Q_{\text{int,I},g} - Q_{\text{int,II},g} \geq 0, \quad \forall g = r, r+1, \dots, N_{\text{ch}}, \\ Q_{\text{Fe},g} &= Q_{\text{int,III},g} - Q_{\text{int,II},g} \geq 0, \quad \forall g = r, r+1, \dots, N_{\text{ch}}, \\ Q_{\text{Ra},g} &= Q_{\text{int,III},g} - Q_{\text{int,IV},g} \geq 0, \quad \forall g = r, r+1, \dots, N_{\text{ch}}, \\ Q_{\text{int},p,g}^{\min} &\leq Q_{\text{int},p,g} \leq Q_{\text{int},p,g}^{\max}, \quad \forall g = r, r+1, \dots, N_{\text{ch}}; \quad p = \text{II}, \text{III}, \end{aligned} \quad (4.7b)$$

where J_{MPC} is the value of the nonlinear model predictive control cost function, $Pu_{\text{Ex}}^{\text{ref}}$ and $Pu_{\text{Ra}}^{\text{ref}}$ are the reference values of the extract and raffinate products, respectively, and $\hat{P}u_{\text{Ex}}$ and $\hat{P}u_{\text{Ra}}$ are the values predicted by the model (Eqs. (2.1)-(2.19) and (2.42)-(2.46)), Δm is the rate of change of the manipulated variables between two consecutive cycles, w_{Pu} and $w_{\Delta m}$ are the weight factors for the purity and the manipulated variables terms, respectively, N_{ph} is the length of the prediction horizon, N_{ch} is the length of the control horizon ($N_{\text{ch}} \leq N_{\text{ph}}$), and h and g are additional column and cycle indices, respectively.

In this formulation of the MPC problem, there are only 4 tuning parameters – the two horizon lengths (N_{ph} and N_{ch}) and the two weight factors (w_{Pu} and $w_{\Delta m}$). However, finding a suitable tuning set is still delicate, especially when the MPC is coupled with the MHE.

The outcome of the MPC optimisation procedure (Eqs. (4.7a)-(4.7b)) is a sequence of internal flowrates $Q_{\text{int,II}}$ and $Q_{\text{int,III}}$ from the current cycle (r -th) until the end of the control horizon. Only the first values (those for the r -th cycle) are actually applied to the SMB plant. The initial values of the manipulated variables for the next iteration are the optimal values from the previous cycle. For the very first cycle at which the control loops are closed, the initial values of the manipulated variables for the entire control horizon are equal to the open-loop ones from the previous cycle. As for the MHE optimisation

procedure, the active set method implemented in the built-in MATLAB Optimization Toolbox function **fmincon** is used with the default stopping criteria (MATLAB, 2022b).

The performance of the proposed control strategy is evaluated in three case studies. First, the hypothetical case where all internal concentrations are available to the controller, then the case where the MPC is coupled with MHE in a noise-free situation, and finally when the measurements are noisy.

4.3 Control based on the exact internal concentration profiles

In this hypothetical case study, the internal concentration profiles can be measured with N evenly distributed noise-free sensors, corresponding to the number of equilibrium separation stages. These sensors deliver information at the end of each cycle.

The values of the control parameters are given in Table 4.2. Horizon lengths (N_{ph} and N_{ch}) ranging from 1 to 8 were investigated and suitable choices from a computational point of view are N_{ph} equal to 2 or 3. In the following, the results for $N_{\text{ph}} = 2$ are presented. The value of the purity weight factor, w_{Pu} , is found by trial and error starting with the values for the case with linear isotherms (Chernev et al., 2022). The upper bound for the internal flow rates is determined from the maximum pressure drop within each chromatographic column and the minimum is the lowest flow rate that the pumps can maintain.

The simulation scenario is similar to that in Chapter 3.2. The simulations start with empty columns and under open-loop conditions with the first operating point from Table 4.1. At the 20th cycle, the loops are closed with 95 % references for both purities and references for τ_I and τ_{IV} close to their actual values. At the 35th cycle, the purity references are changed to complete separation (99 %), and at the 50th cycle, they are changed by

Table 4.2: Control parameters for the semi-centralised control scheme based on both exact internal concentration profiles, discussed in the current subchapter, and MHE processing noiseless measurements, discussed in Chapter 4.7.

Quantity	Value	Quantity	Value
$K_{P,I}$, [-]	-7.12	$K_{P,IV}$, [-]	2.53
$K_{I,I}$, [-]	-9.18	$K_{I,IV}$, [-]	0.81
$K_{D,I}$, [-]	-0.14	$K_{D,IV}$, [-]	1.54
N_{ph} , [-]	2	N_{ch} , [-]	2
w_{Pu} , [-]	8	$w_{\Delta m}$, [-]	0, 0.03, 0.05
$Q_{\text{int}}^{\min}$, [mL·min ⁻¹]	1	$Q_{\text{int}}^{\max}$, [mL·min ⁻¹]	100

a larger quantity ($\approx 9.1\%$) from 99 to 90 %. The two retention time references are changed too. Finally, at the 65th cycle, the purity references are changed back to 95 % while the references of both retention times are set to 0.6. The exact values of the references are shown in Table 4.3 and are also used for the other two case studies in this chapter. Although SMB chromatographic processes are mostly used for total separations and obtaining products with high purity, reduced-purity separations, if acceptable, can be of interest when higher productivity is required. Note that the changes that are operated (particularly at cycle 50) are severe tests in the case of nonlinear isotherms.

Table 4.3: Change of the references for the semi-centralised control scheme.

Cycles, k	$Pu^{\text{ref}}, [\%]$		$\tau^{\text{ref}}, [-]$	
	Extract	Raffinate	I	IV
1 - 19	Open loop			
20 - 34	95	95	0.45	0.68
35 - 49	99	99	0.45	0.68
50 - 64	90	90	0.52	0.64
65 - 80	95	95	0.6	0.6

The results of the simulations are shown in Figures 4.8-4.12 for three different values of the weight factor for the rate of change of the manipulated variables, $w_{\Delta m} = 0, 0.03, 0.05$.

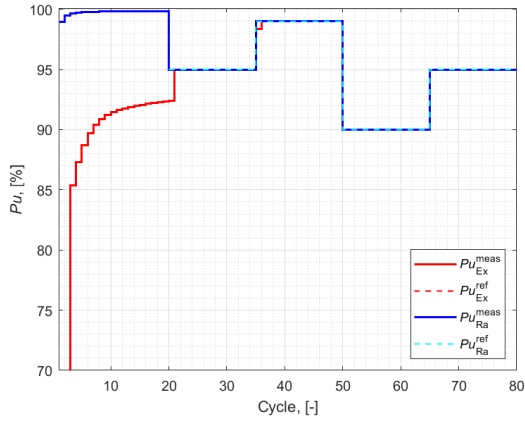
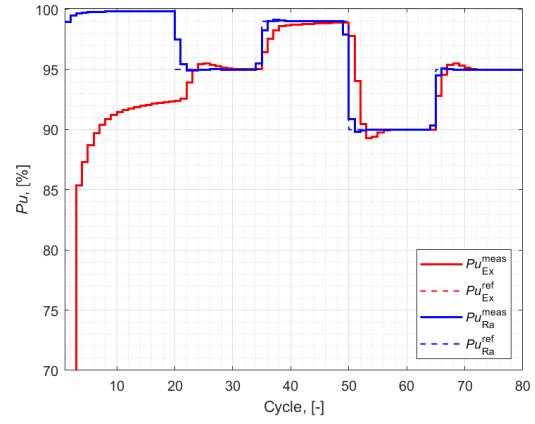
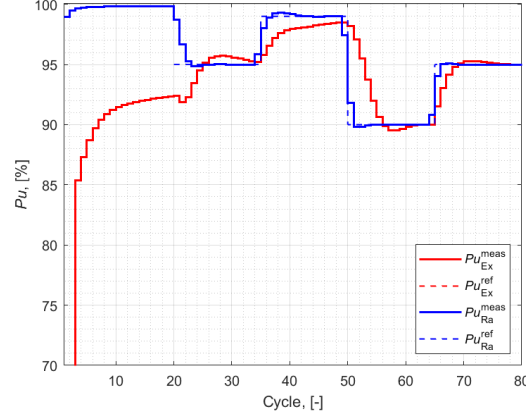
(a) $w_{\Delta m} = 0$. Experiment E1 in Table 4.4(b) $w_{\Delta m} = 0.03$. Experiment E2 in Table 4.4(c) $w_{\Delta m} = 0.05$. Experiment E3 in Table 4.4

Figure 4.8: Process variables – product purities, for control based on the exact internal concentration profiles. Values of the tuning parameters – $w_{P_u} = 8$, $N_{ph} = 2$, $N_{ch} = 2$.

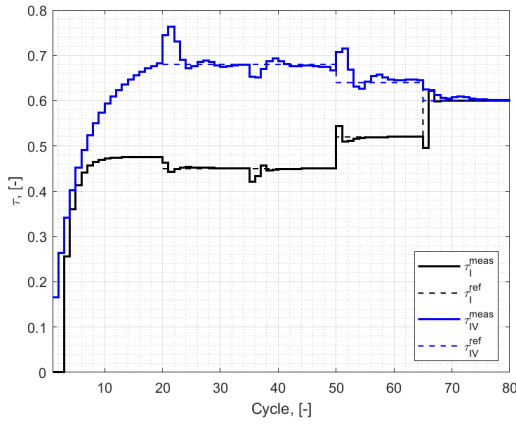
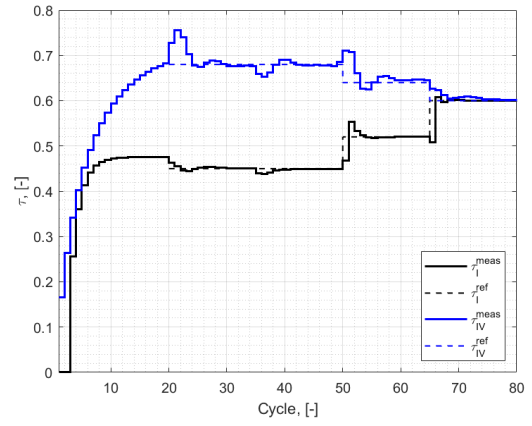
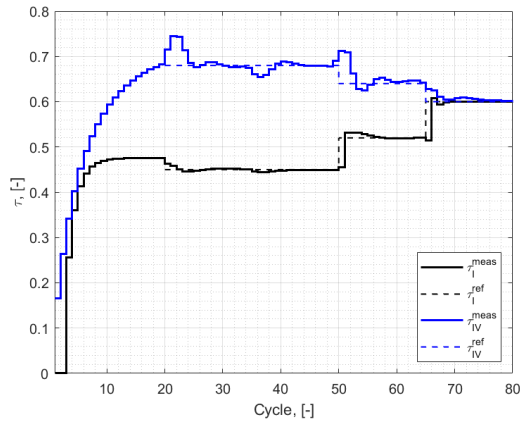
(a) $w_{\Delta m} = 0$.(b) $w_{\Delta m} = 0.03$.(c) $w_{\Delta m} = 0.05$.

Figure 4.9: Process variables – dimensionless retention times, for control based on the exact internal concentration profiles. Values of the tuning parameters – $w_{Pu} = 8$, $N_{ph} = 2$, $N_{ch} = 2$.

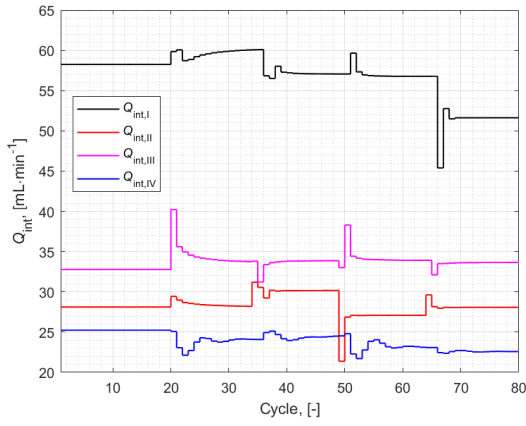
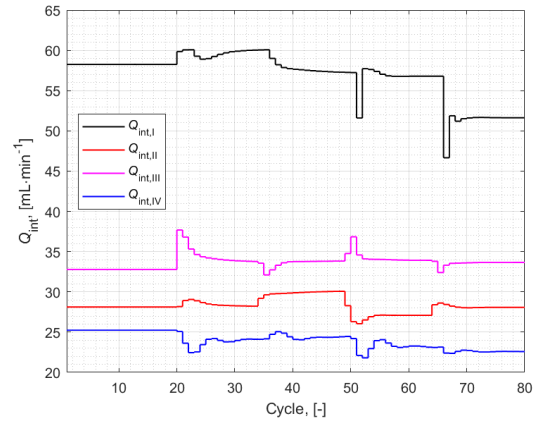
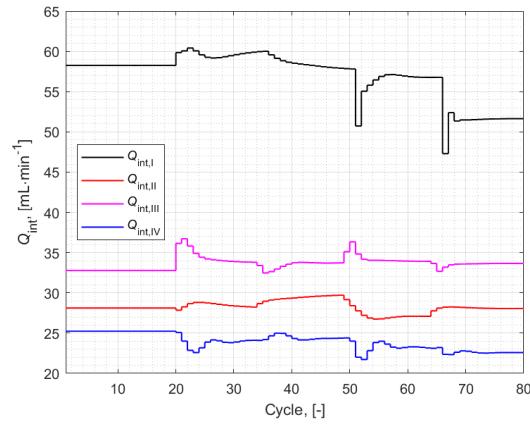
(a) $w_{\Delta m} = 0$.(b) $w_{\Delta m} = 0.03$.(c) $w_{\Delta m} = 0.05$.

Figure 4.10: Manipulated variables – internal flowrates, for control based on the exact internal concentration profiles. Values of the tuning parameters – $w_{Pu} = 8$, $N_{ph} = 2$, $N_{ch} = 2$.

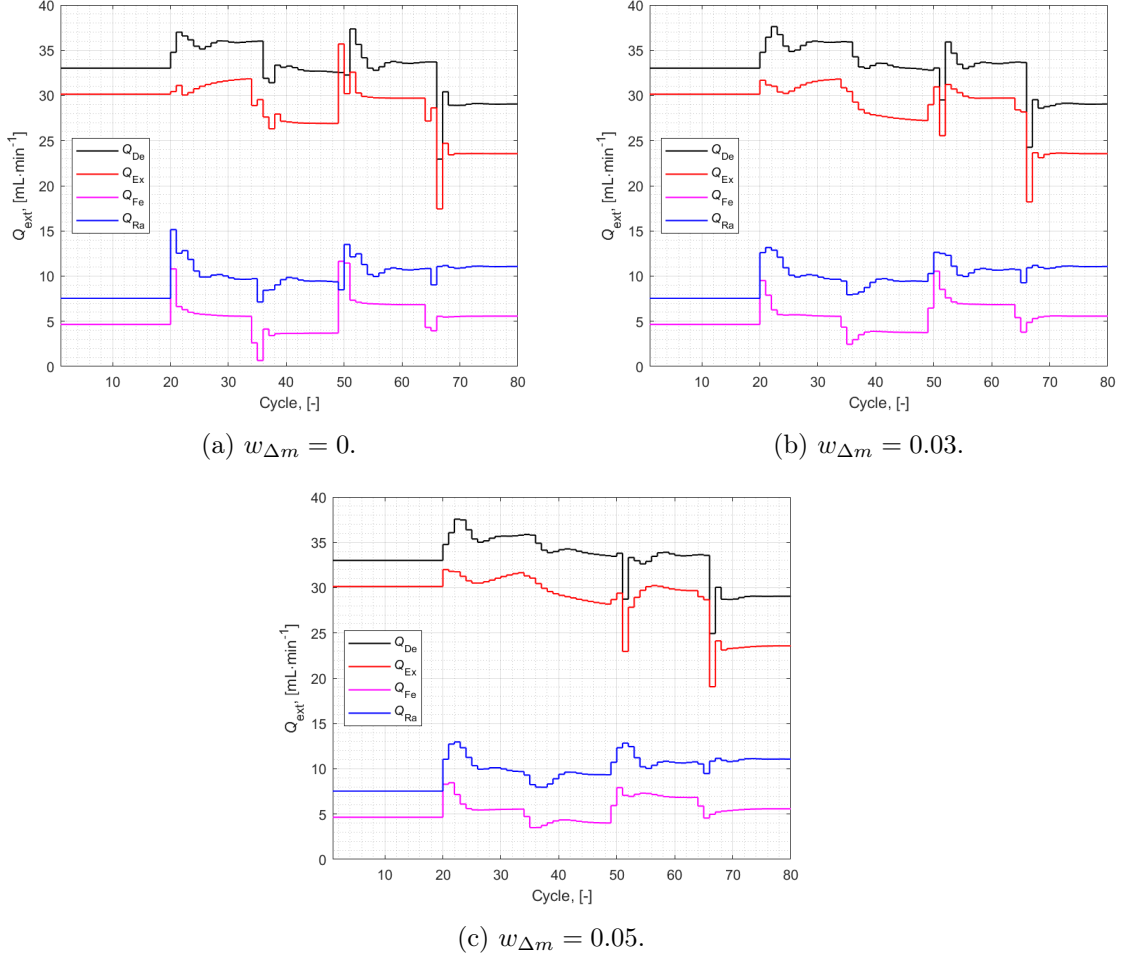


Figure 4.11: Manipulated variables – external flowrates, for control based on the exact internal concentration profiles. Values of the tuning parameters – $w_{Pu} = 8$, $N_{ph} = 2$, $N_{ch} = 2$.

As can be seen in Figure 4.8a (i.e., $w_{\Delta m} = 0$ – no penalisation of the rate of change of the manipulated variables), NMPC achieves model inversion and perfect tracking of the product purity references. Indeed, the purity dynamics are similar to those of a first-order discrete-time system, and model inversion is made possible by the fact that there is no plant-model mismatch in the considered NMPC. This phenomenon is only observed for the cases when $N_{ch} = N_{ph}$ and $N_{ph} \geq 2$. For all other cases when $N_{ch} < N_{ph}$, such perfect tracking of the references is not observed. Although such perfect behaviour could seem attractive, it is not desired in practice, because model inversion can lead to instability (in the case of non-minimum-phase behaviour) and the manipulated variables might experience too much activity (Figure 4.10a and 4.11a, especially when the product purities references are changed from 99 to 90 %, 50th cycle). That is why $w_{\Delta m} > 0$ must be considered. For $w_{\Delta m} = 0.03$, the dynamics become slower, and for $w_{\Delta m} = 0.05$, the system is too slow

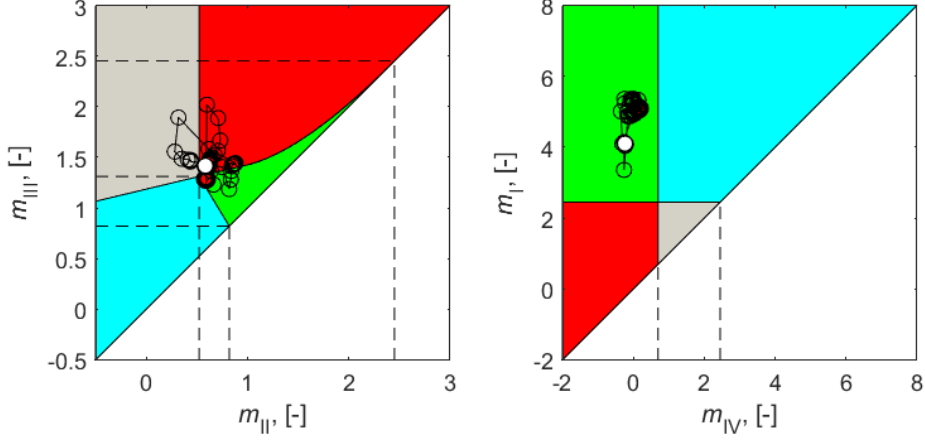


Figure 4.12: Changes of the operating point (m values) in the triangle diagram for control based on the exact internal concentration profiles for the case when $w_{\Delta m} = 0.03$ (Figures 4.8b-4.11b). Values of the other tuning parameters – $w_{Pu} = 8$, $N_{ph} = 2$, $N_{ch} = 2$. Red circle – initial operating point (1st cycle), white circle – last operating point.

as the desired extract purity of 99 % can only be reached in more than 15 cycles. Figure 4.12 shows how the operating point changes for each cycle in the $m_{II} - m_{III}$ and $m_{IV} - m_I$ planes for the case when $w_{\Delta m} = 0.03$.

4.4 Tuning of the MPC

To investigate the influence of the individual tuning parameters of the model predictive controller on the performance of the control system, the following metrics are used – sum of the root-squared control error (RS), control efforts (CE), and the average computation time (CPU) per cycle needed for the optimisation procedure to determine the control action. RS and CE are defined as follows

$$RS = \sum_k \left(\sqrt{(Pu_{Ex,k}^{ref} - Pu_{Ex,k}^{meas})^2} + \sqrt{(Pu_{Ra,k}^{ref} - Pu_{Ra,k}^{meas})^2} \right). \quad (4.10)$$

$$CE = \sum_k \sum_{p=II}^{III} \sqrt{\Delta m_{p,k}^2}. \quad (4.11)$$

RS and CE metrics are focused only on the separation zones of the SMB process because, if there is no breakthrough in the two regeneration zones, which are taken care of by the two PID controllers, the purity of the two products is determined solely by the conditions in zones II and III. Another point worth noting is that the maximum computation time usually occurs in the first cycle after the control loops are closed and, less often, when there is a reference change, such as in the 50th cycle, when the references of both purities change from 99 % to 90 %. In these cases, the maximum CPU time differs from the average and may be up to 40 % more. The results of the conducted simulation studies are summarised in Table 4.5.

The choice of $w_{\Delta m}$ was already discussed in Chapter 4.3, with recommended values for it in the range 0.01-0.03. When determining the value of w_{Pu} , it must be taken into account that concentration shocks occur in zone III, which makes the control of raffinate purity more challenging than that of the extract one. When $w_{Pu} = 1$, there is a static error for Pu_{Ra} under total separation conditions. When $w_{Pu} > 1$, higher priority is given to controlling the purity of the raffinate over that of the extract, and the static error disappears. However, for values of $w_{Pu} \leq 5$, the raffinate purity dynamics is slow; for example, at $w_{Pu} = 2$, some of the raffinate purity references fail to be reached within the given number of cycles between two consecutive changes of Pu_{Ra}^{ref} . For $7 \leq w_{Pu} \leq 15$, very good control quality is achieved. In this range, there is little change in RS and CE; as w_{Pu} increases, the RS slowly decreases, accompanied by a slow increase in CE and w_{Pu} is recommended to be within this range. For $w_{Pu} > 15$, too much priority is given to raffinate purity, and although the RS continues to decrease, this overall comes at the expense of extract purity; furthermore, the CE increases, and this also applies to the computation time.

Investigations show that long prediction horizons lead to smooth but slow control. Furthermore, the controller foresees changes in the references too early and begins to adjust both purities well before the change actually occurs, resulting in some operations outside the specifications, which may not be permitted. Therefore, values for N_{ph} in the range of 2-5 are recommended. However, when $N_{ph} = 5$, the computation time is about three times longer than for $N_{ph} = 2$. When $N_{ph} = 1$, the extract purity is not controlled. Increasing the control horizon, N_{ch} , always leads to a decrease in RS, but at the same time to an increase in computation time, since with each increase in this horizon by one cycle,

the number of optimisation variables increases by two. In this thesis, $N_{\text{ph}} = N_{\text{ch}} = 2$ is preferred because of the computation time of approximately 17 seconds per cycle.

Table 4.4: Performance of the different study cases

Experiment	Tuning parameters				RS	CE	CPU time, [s]	Remark
	w_{Pu}	$w_{\Delta m}$	N_{ph}	N_{ch}				
E1	8	0	2	2	3.5807	7.716	13.41	Static error for Pu_{Ra}
E2	8	0.03	2	2	63.4461	3.432	16.83	
E3	8	0.05	2	2	84.5704	3.168	18.47	
	1	0.03	2	2	96.0872	3.438	25.30	
	2	0.03	2	2	78.4887	3.306	16.21	
	5	0.03	2	2	66.5754	3.360	16.58	
	20	0.03	2	2	59.8044	3.882	17.85	
	50	0.03	2	2	58.2672	4.290	19.42	
	100	0.03	2	2	57.6572	4.506	19.98	
	8	0.03	1	1	313.3082	3.396	4.37	Uncontrolled Pu_{Ex}
	8	0.03	2	1	72.9865	3.060	10.52	
	8	0.03	3	3	39.7407	3.467	36.33	
	8	0.03	4	3	39.0470	3.506	49.27	
	8	0.03	4	4	38.4236	3.416	59.70	
	8	0.03	5	1	98.0087	1.872	30.61	
	8	0.03	5	2	47.6044	3.576	49.79	
	8	0.03	5	3	41.4617	3.564	63.35	
	8	0.03	5	5	39.5646	3.390	87.88	
	8	0.03	8	2	70.9674	3.654	85.87	
	8	0.03	8	5	42.3340	3.570	169.00	

4.5 Control with active input constraints

When the physical limitations of the actual SMB chromatographic process are incorporated into its model, online optimising control, such as MPC, takes these limitations into account when calculating the control action, before they become active. In this way, MPC functions proactively, unlike a PID control scheme, which functions reactively. That is why MPC generally performs better than PID in constrained control.

To investigate how the semi-centralised control scheme based on MPC within the

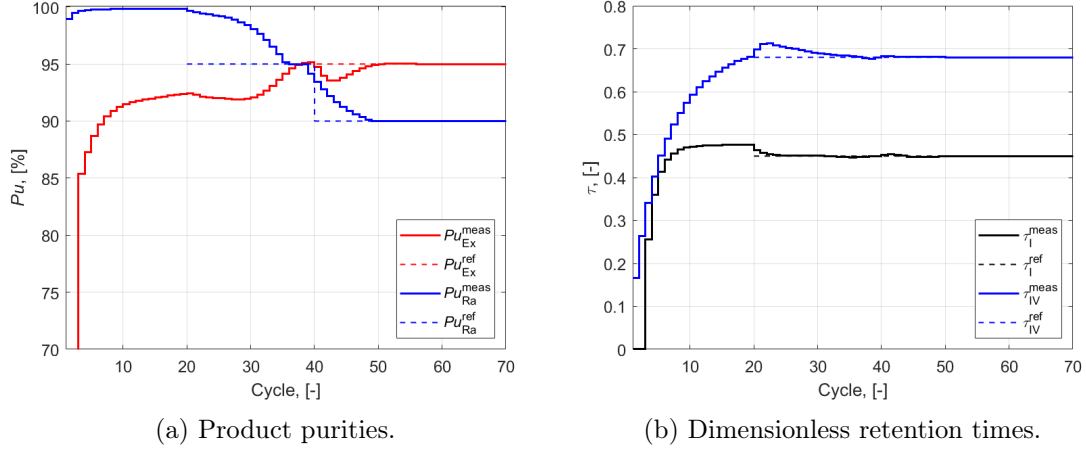


Figure 4.13: Process variables for the control with active constraint of $Q_{\text{int,III}}$. Values of the tuning parameters – $w_{P_u} = 8$, $w_{\Delta m} = 0.03$, $N_{\text{ph}} = 2$, $N_{\text{ch}} = 2$.

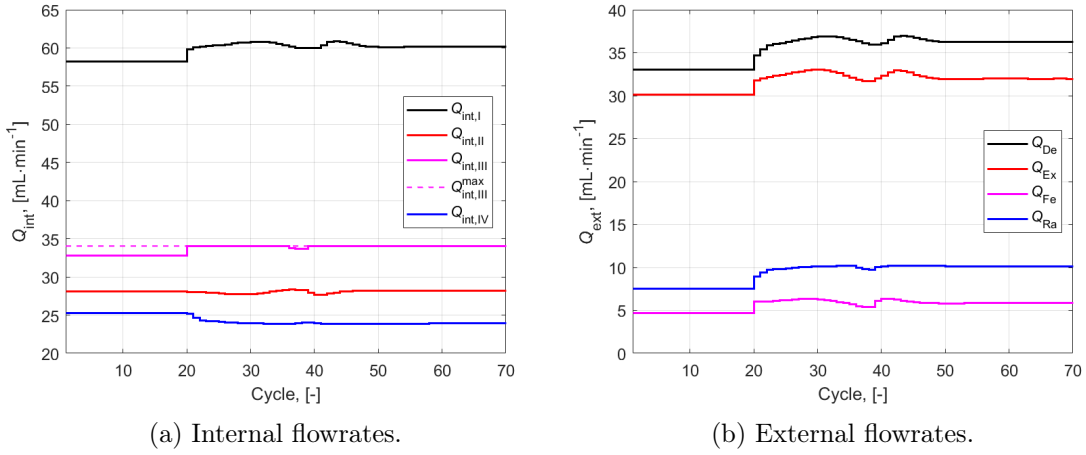


Figure 4.14: Manipulated variables for the control with active constraint of $Q_{\text{int,III}}$. Values of the tuning parameters – $w_{P_u} = 8$, $w_{\Delta m} = 0.03$, $N_{\text{ph}} = 2$, $N_{\text{ch}} = 2$.

two separation zones handles active input constraints, it is tested using the same test case shown in Figures 3.8 and 3.9. A preliminary study conducted with $Q_{\text{int,III}}^{\text{max}} = 35.5 \text{ mL} \cdot \text{min}^{-1}$ revealed that this constraint was not strict enough, as the computed manipulated variables from the proposed control system were less than this constraint and as a result it did not become active. Therefore, for the current investigation $Q_{\text{int,III}}^{\text{max}}$ is decreased and is set $Q_{\text{int,III}}^{\text{max}} = 34 \text{ mL} \cdot \text{min}^{-1}$. The results are presented in Figures 4.13 and 4.14.

In this case again, the constraint is active twice due to actuator saturation. During the first saturation, the references are reached, albeit significantly slower (15 cycles) than the case without active constraints (cf. Figure 4.8b). However, a major difference compared to the classical control scheme presented in Chapter 3.3 (cf. Figure 3.8a) is the fact that the new reference of raffinate purity of 90 % is also reached, despite the second saturation of the actuator. This demonstrates that the MPC-based control scheme is a suitable choice

to control the purity of the two end products of the SMB process when constraints are imposed on the manipulated variables, which is important from a practical point of view.

4.6 Disturbance rejection control of a measured disturbance

The SMB process is influenced by various external and internal factors that affect its performance. The appearance and change of these factors act as disturbances that the control system must handle. Along with reference tracking, disturbance rejection is the other primary task of any control system. Disturbances can be measurable or unmeasurable. When a disturbance is measured, the information about it can be utilised, thereby facilitating the work of the control system. The most common measurable disturbance acting upon SMB processes is the concentration of the feed stream. In the vast majority of cases, the feed stream is a product of some upstream process or chain of processes, and therefore its concentration is measured and/or controlled.

The present study investigates the effect of changes in feed concentration, assuming it is measured and known. In this study, once the SMB process has reached CSS, $C_{i,\text{Fe}}$ was perturbed with a significant change of +50 % in the concentrations of both components, from 5 to 7.5 g · L⁻¹, in the 50th cycle (Figure 4.15). The results are presented in Figures 4.16-4.18.

The change in the two feed concentrations occurs in the 50th cycle, after the control effect for that cycle has already been calculated using the old values of $C_{i,\text{Fe}}$. That leads to a plant-model mismatch until the end of this cycle, as seen in the raffinate purity, which drops to 88.3 % for this cycle in Figure 4.16a. After the end of the cycle, the predictive

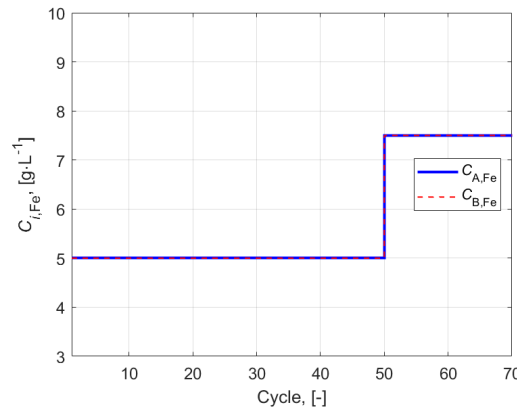


Figure 4.15: Change of the feed concentrations for the disturbance rejection control of a measured disturbance.

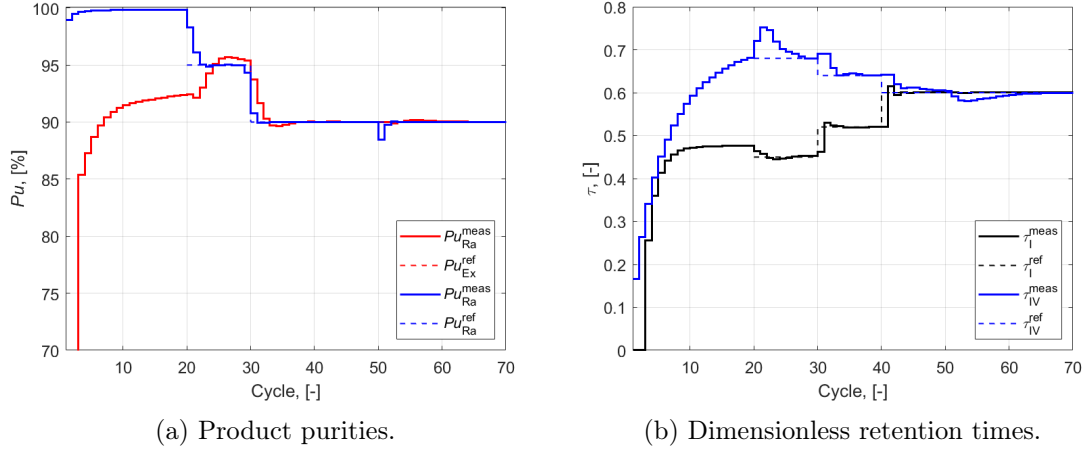


Figure 4.16: Process variables for the disturbance rejection control. Change of $C_{i,Fe}$ as a measured disturbance. Values of the tuning parameters – $w_{P_u} = 8$, $w_{\Delta m} = 0.03$, $N_{ph} = 2$, $N_{ch} = 2$.

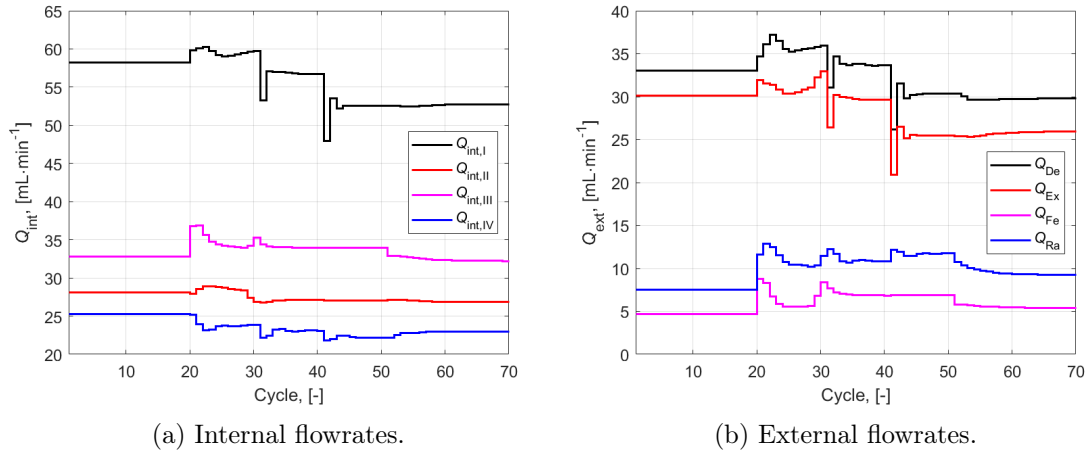


Figure 4.17: Manipulated variables for the disturbance rejection control. Change of $C_{i,Fe}$ as a measured disturbance. Values of the tuning parameters – $w_{P_u} = 8$, $w_{\Delta m} = 0.03$, $N_{ph} = 2$, $N_{ch} = 2$.

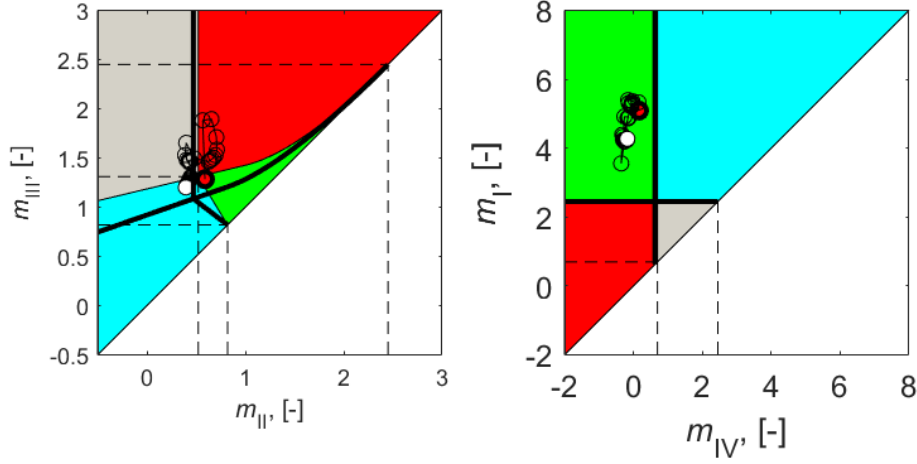


Figure 4.18: Triangle diagrams with the change of the operating point of the SMB plant for the disturbance rejection control with the change of $C_{i,Fe}$ as a measured disturbance. Red circle – initial operating point; white circle - operating point at the last cycle. Coloured areas – initial separation and regeneration regions defined by the thin curves; regions defined by the bold black curves are after the change of the C_{Fe} , i.e. for the cycles 50-70. Values of the tuning parameters – $w_{Pu} = 8$, $w_{\Delta m} = 0.03$, $N_{ph} = 2$, $N_{ch} = 2$.

model is updated with the new values of $C_{i,Fe}$ and the mismatch is eliminated, resulting in a new control action for the 51st cycle, which handles this disturbance despite its large change of +50 %. Although the change in $C_{i,Fe}$ is large, it has a negligible effect on the dimensionless retention times in Figure 4.16b. This can be explained by examining the triangular diagram in Figure 4.18. While the $m_{II} - m_{III}$ diagram changes drastically, as shown by the bold black curves, the change in the $m_{IV} - m_I$ diagram is very small, which leads to almost no change in the conditions in zones I and IV.

The problem of disturbance rejection for unmeasurable disturbances will be discussed in Chapter 6.

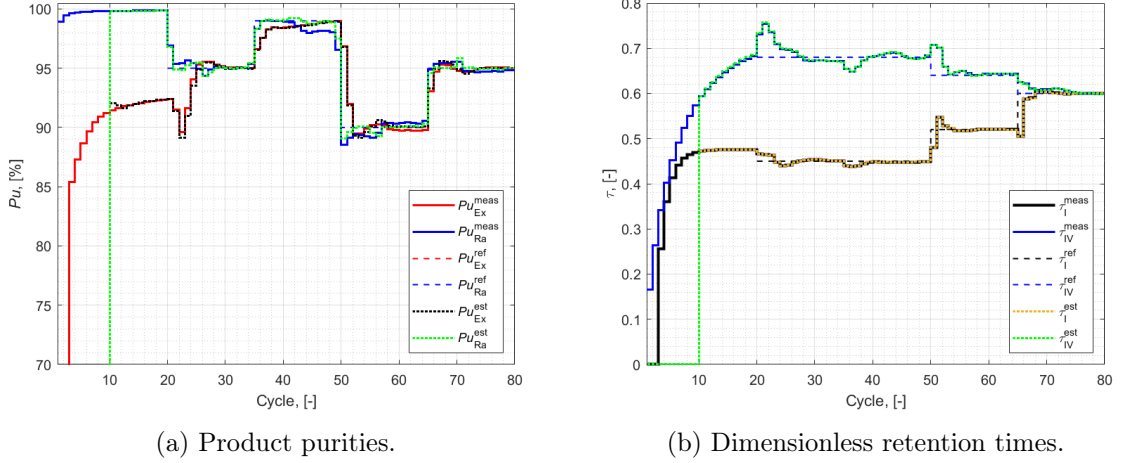


Figure 4.19: Process variables for control based on MHE processing noiseless measurements. Values of the tuning parameters – $w_{P_u} = 8$, $w_{\Delta m} = 0.01$, $N_{ph} = 2$, $N_{ch} = 2$, $N_{eh} = 6$.

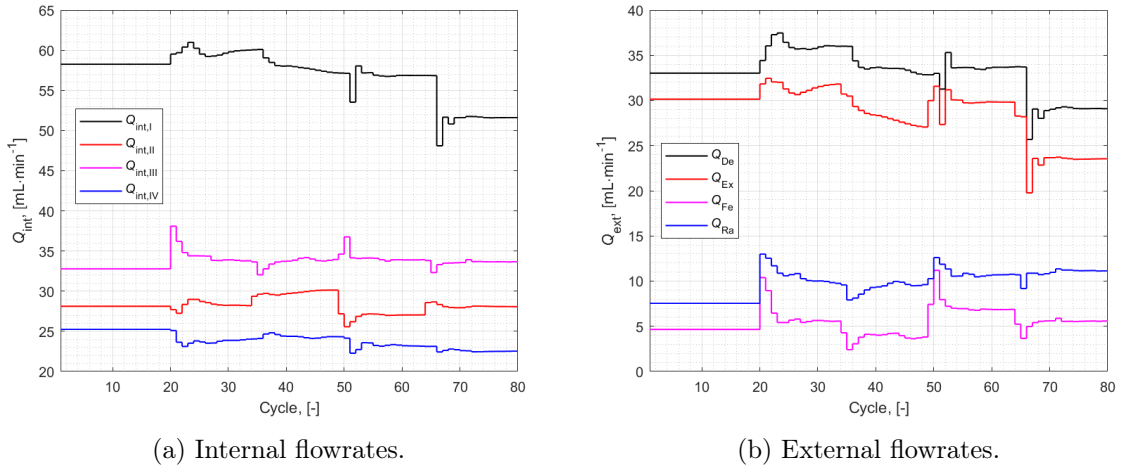


Figure 4.20: Manipulated variables for control based on MHE processing noiseless measurements. Values of the tuning parameters – $w_{P_u} = 8$, $w_{\Delta m} = 0.01$, $N_{ph} = 2$, $N_{ch} = 2$, $N_{eh} = 6$.

4.7 Control based on the moving horizon estimator processing noise-free measurements

The moving horizon estimator estimates the internal concentration profiles at the end of each cycle based on the available scarce, but noise-free, measurement information. The estimation horizon is 6 cycles, and the estimator is switched on when there are as many past cycles as the estimation horizon; in this case, the 10th cycle. The control loops are closed in the 20th cycle. Due to the error in the estimated internal concentration profiles, the closed-loop dynamics are slower than in the ideal case, which is why $w_{\Delta m} = 0.01$, while the other parameters do not change. The simulation results are presented in Figures

4.19-4.20. As can be expected, the performance of the control is highly dependent on the quality of the estimation. In this connection, performance is good in cycles 25-35 and 55-80 and deteriorates somehow in cycles 40-50 for the raffinate purity. The deviation of raffinate purity can be explained by the interaction between the MHE and MPC. However, even in the worst situation, the difference between the raffinate purity and its reference is less than 1 %.

4.8 Control with MHE and measurement noise

Measurements are now corrupted with the same additive noise as that in Chapter **4.1**. The measured signal of the dimensionless retention times is filtered with a moving average filter whose window length is 6 cycles in the past and a filtered control error is computed which is only used in the derivative part of the PID controllers.

$$e_{p,r-1}^{\text{fil}} = \tau_{p,r-1}^{\text{ref}} - \tau_{p,r-1}^{\text{fil}}, \quad p = \text{I, IV}, \quad (4.8)$$

$$m_{p,r} = K_{P,p}e_{p,r-1} + K_{I,p} \sum_{k=1}^{r-1} e_{p,k} + K_{D,p}(e_{p,r-1}^{\text{fil}} - e_{p,r-2}^{\text{fil}}), \quad p = \text{I, IV}, \quad (4.9)$$

where τ_p^{fil} and e_p^{fil} are the filtered dimensionless retention time and the filtered control error, respectively.

In order to alleviate the effect of noise, it is either possible to increase the estimation horizon length of MHE or to speed up NMPC. The second option is selected in the following, which is achieved by decreasing the weight factor $w_{\Delta m}$ and slightly increasing w_{P_u} . The PID controllers are also retuned. The control parameters are summarised in Table **4.5** and the simulation results are presented in Figures **4.21-4.22**.

As can be seen from the extract purity (Figure **4.21a**) MHE needs more cycles to converge after it is turned on. However, once the control loops are closed, the estimator converges in 6 cycles and closely follows the process state variables. MPC can control the purities successfully, even in the worst case, at the 48th cycle, where the control error is less than 1.4 % for the raffinate purity.

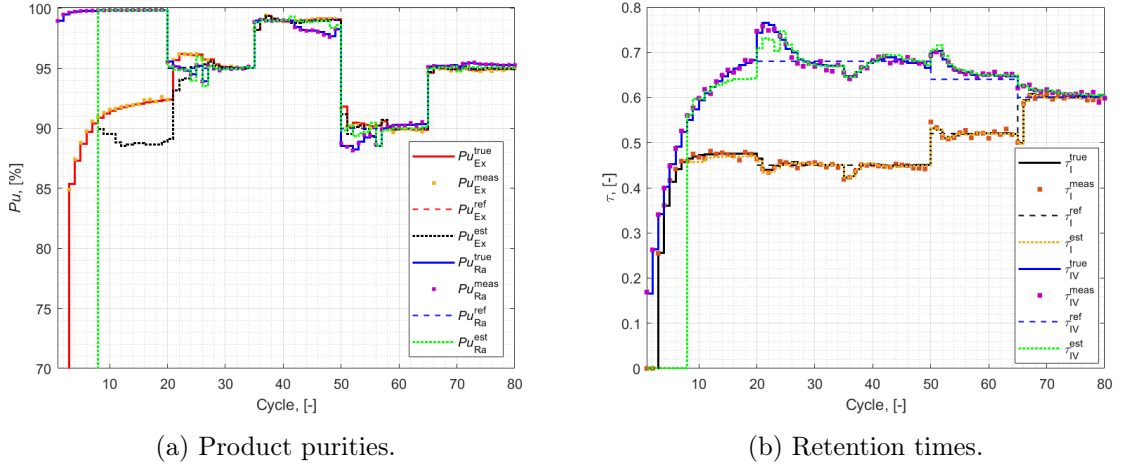


Figure 4.21: Process variables for control based on MHE processing measurements with additive measurement noise. Values of the tuning parameters – $w_{P_u} = 10$, $w_{\Delta m} = 0.001$, $N_{ph} = 2$, $N_{ch} = 2$, $N_{eh} = 6$.

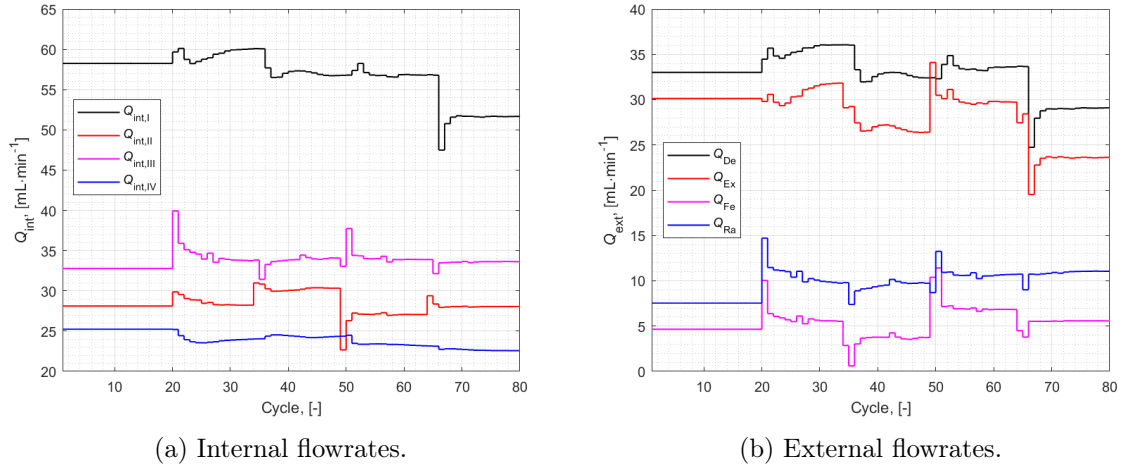


Figure 4.22: Manipulated variables for control based on MHE processing measurements with additive measurement noise. Values of the tuning parameters – $w_{P_u} = 10$, $w_{\Delta m} = 0.001$, $N_{ph} = 2$, $N_{ch} = 2$, $N_{eh} = 6$.

Table 4.5: Control parameters for the semi-centralised control scheme based on MHE processing measurements with additive measurement noise.

Quantity	Value	Quantity	Value
$K_{P,I}$, [-]	-5.63	$K_{P,IV}$, [-]	1.24
$K_{I,I}$, [-]	-7.25	$K_{I,IV}$, [-]	0.76
$K_{D,I}$, [-]	-2.23	$K_{D,IV}$, [-]	0.59
N_{ph} , [-]	2	N_{ch} , [-]	2
w_{P_u} , [-]	10	$w_{\Delta m}$, [-]	0.001
Q_{int}^{min} , [mL.min ⁻¹]	1	Q_{int}^{max} , [mL.min ⁻¹]	100

Chapter 5

Conservation element/solution element (CE/SE) method and its application for simulation of chromatographic processes

Chapter 5, including Appendix C, was published in parts in Processes, (2020), 8(10), Chernev, V. P., Vande Wouwer, A.; Kienle, A. Efficient Simulation of Chromatographic Processes Using the Conservation Element/Solution Element Method. p. 1316³.

As can be seen from the results from the previous chapter, in order to resolve the steep concentration profiles by the mixing cell method, it is necessary to spatially discretise the chromatographic columns with a very large number of equilibrium stages, in the order of thousands. Although this is not a big issue if we only want to simulate the SMB plant to study its behaviour under different conditions by varying some simulation parameters, it becomes impractical from a computational point of view for optimisation tasks, especially for online optimising control and estimation. Reducing the cell number to 100 is a compromise solution to the problem. On the one hand, the computational time on a standard desktop PC is reduced to such an extent that it becomes possible to perform the optimisation procedure within minutes; on the other hand, the concentration fronts are resolved sufficiently well in a number of cases. However, when we aim for sharp separations and

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products with very high purity, the adequate resolution of the steep concentration fronts that are typical for the nonlinear chromatography is of crucial importance.

That is why another numerical method for solving the governing PDAE system was considered in the present work, namely the space-time conservation element/solution element method (CE/SE).

5.1 CE/SE method for the numerical solution of PDAE systems

The space-time conservation element/solution element method was initially used in the field of computational fluid dynamics. The most distinctive feature of the CE/SE method developed by Chang (Chang, 1993, Chang, 1995) is that time and space are treated in the same manner through the so-called conservation element and solution element. It is based on the integral formulation of the conservation laws and can resolve steep concentration fronts, which are typical for nonlinear chromatography. It is an explicit time-marching method for the solution of the present class of PDEs. In such a way, the values of the quantities of interest or the so-called state variables (liquid-phase C and solid-phase q concentrations in chromatography) at the current time instant and their spatial derivative only depend on their values at the previous time instant (Figure 5.1a).

The model in Eq. (2.1) with $D_{ax} \rightarrow 0$ can be presented in the following vector form

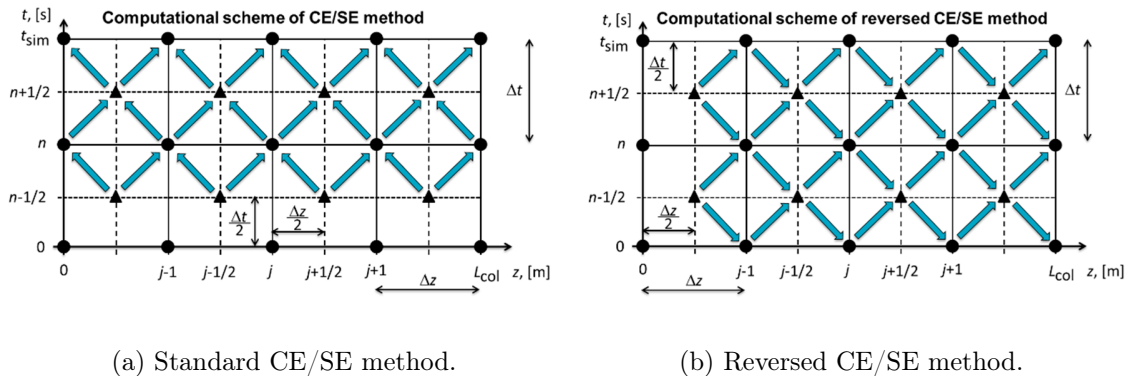


Figure 5.1: Computational schemes of the space-time conservation element/solution element (CE/SE) method.

(the indices are omitted for simplicity):

$$\mathbf{u} = \begin{bmatrix} C \\ q \end{bmatrix} \quad (5.1)$$

$$\mathbf{f} = \begin{bmatrix} vC \\ 0 \end{bmatrix} \quad (5.2)$$

$$\mathbf{p}_{\text{CE/SE}} = \begin{bmatrix} -\frac{1-\varepsilon}{\varepsilon} k_m(q^*(C) - q) \\ k_m(q^*(C) - q) \end{bmatrix} \quad (5.3)$$

where $\mathbf{u}$ is a vector of state variables, $\mathbf{f}$ is a vector of fluxes, and $\mathbf{p}_{\text{CE/SE}}$ is a vector of source terms. Eq. (2.1) can then be written in the following form:

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}(\mathbf{u})}{\partial t} = \mathbf{p}_{\text{CE/SE}}, \quad (5.4)$$

Eq. (5.4) is the PDE formulation, which is solved using the CE/SE method. For a detailed derivation of the CE/SE method, the reader is referred to (Chang and Wang, 2002, Lim et al., 2004, Lim and Jørgensen, 2004, Yao et al., 2015). Here, only the final equations of the fully discretised scheme in space and time are presented. The state variable $\mathbf{u}$ at the λ -th spatial point and n -th time instant is

$$\mathbf{u}_\lambda^n = \frac{1}{2} \left(\mathbf{u}_{\lambda-1/2}^{n-1/2} + \mathbf{u}_{\lambda+1/2}^{n-1/2} + \mathbf{s}_{\lambda-1/2}^{n-1/2} - \mathbf{s}_{\lambda+1/2}^{n-1/2} + \frac{\Delta t}{2} \left(\mathbf{p}_{\text{CE/SE}, \lambda-1/2}^{n-1/2} + \mathbf{p}_{\text{CE/SE}, \lambda+1/2}^{n-1/2} \right) \right),$$

$$\lambda = 1, 2, \dots, N_z; n = 1, 2, \dots, N_t, \quad (5.5)$$

where $\mathbf{s}_\lambda^n$ is

$$\mathbf{s}_\lambda^n = \frac{\Delta z}{4} \mathbf{u}_{z, \lambda}^n + \frac{\Delta t}{\Delta z} \mathbf{f}_\lambda^n + \frac{\Delta t^2}{4\Delta z} \mathbf{f}_{t, \lambda}^n. \quad (5.6)$$

In the last equation, $\mathbf{u}_z$ and $\mathbf{f}_t$ are the discrete analogues of the derivatives $\frac{\partial \mathbf{u}}{\partial z}$ and $\frac{\partial \mathbf{f}}{\partial t}$. The current value of $\mathbf{u}_\lambda^n$ is calculated from the already available values of $\mathbf{u}_{\lambda\pm 1/2}^{n-1/2}$, $\mathbf{u}_{z, \lambda\pm 1/2}^{n-1/2}$, $\mathbf{f}_{\lambda\pm 1/2}^{n-1/2}$, $\mathbf{f}_{t, \lambda\pm 1/2}^{n-1/2}$, and $\mathbf{p}_{\text{CE/SE}, \lambda\pm 1/2}^{n-1/2}$ at the previous time instant. The different versions of the CE/SE method only differ in the way the spatial derivative $\mathbf{u}_z$ of the state variables

$\mathbf{u}$ is calculated. In the current work, the version of (Yao et al., 2015) is used.

$$\mathbf{u}_{z,\lambda}^n = W_0(\mathbf{u}_{z-, \lambda}^n, \mathbf{u}_{z+, \lambda}^n, a) \quad (5.7)$$

The definition of the function W_0 is

$$\begin{cases} W_0(0, 0, a) = 0 \\ W_0(x_-, x_+, a) = \frac{|x_+|^a x_- + |x_-|^a x_+}{|x_+|^a + |x_-|^a}, (|x_+| + |x_-| > 0), \end{cases} \quad (5.8)$$

with $a \geq 0$. In this study, $a = 1$. $\mathbf{u}_{z-, \lambda}^n$ and $\mathbf{u}_{z+, \lambda}^n$ are the values of $\mathbf{u}_{z, \lambda}^n$ calculated from left and right of the point with coordinates (λ, n) , respectively, and are given by

$$\mathbf{u}_{z\pm, \lambda}^n = \pm \frac{\tilde{\mathbf{u}}_{\lambda\pm 1/2}^n - \mathbf{u}_{\lambda}^n}{\frac{\Delta z}{2}}, \quad (5.9)$$

where

$$\tilde{\mathbf{u}}_{\lambda\pm 1/2}^n = \mathbf{u}_{\lambda\pm 1/2}^{n-1/2} + \frac{\Delta t}{2} \mathbf{u}_{t, \lambda\pm 1/2}^{n-1/2}, \quad (5.10)$$

$\mathbf{u}_t$ is the numerical analogue of $\frac{\partial \mathbf{u}}{\partial t}$ and is

$$\mathbf{u}_{t, \lambda}^n = -\mathbf{f}_{z, \lambda}^n + \mathbf{p}_{\text{CE/SE}, \lambda}^n \quad (5.11)$$

Since CE/SE method is a fully discrete explicit scheme, the Courant-Friedrichs-Lewy condition has to be fulfilled to guarantee numerical stability. This condition gives the relationship between the spatial and time step sizes through the Courant-Friedrichs-Lewy (CFL) number

$$\text{CFL} = v \frac{\Delta t}{\Delta z} \leq 1. \quad (5.12)$$

Here, the interstitial velocity, v , of the liquid phase is also an upper bound for the propagation velocity of the concentration fronts. In practice, we specify Δz or Δt and the CFL number and then calculate the other unspecified step Δt or Δz from Eq. (5.12).

5.2 Simulation of batch and SMB chromatographic processes

Several examples are considered to compare the CE/SE method and the cell model (MOL) for the simulation of chromatographic processes in terms of computational effi-

ciency. All simulations in Chapter 5 were carried out on a desktop PC with an Intel Core i7 6700 CPU (3.4 GHz) and 8 GB RAM, using MathWorks MATLAB R2017a under Microsoft Windows 8.1.

5.2.1 Single column with the ideal equilibrium model

First, the focus is on the equilibrium model without axial dispersion, which can be used to simulate highly efficient columns. In practice, column efficiency is specified in terms of the number of theoretical stages N corresponding to the number of grid points of the first-order finite volume MOL formulation. For $N \rightarrow \infty$ and a step input, discontinuities travel through the column during the loading of an empty bed for the linear and the Langmuir isotherms considered in this paper. For a linear isotherm, these are contact discontinuities, and, for the Langmuir isotherm, these are self-sharpening shock waves (Rhee et al., 1989).

In practice, the number of theoretical plates can be high, but it will be a finite number. In the remainder, N is fixed, and therefore the number of grid points in the MOL formulation is fixed as well. The number of grid points for the CE/SE method is adapted accordingly to meet the given column efficiency. For comparison, the analytical solution for $N \rightarrow \infty$ is also given.

The investigated case scenario involves loading an empty bed (single column) with a binary system described by Langmuir isotherms. Table 5.1 summarises all the simulation parameters. The feed concentrations of the two components serve as boundary conditions for the PDE system in Eq. (2.5).

As mentioned, the formulation of the CE/SE method is given by Eq. (5.4) while the ideal chromatographic model is of the type:

$$\frac{\partial \mathbf{f}(\mathbf{u})}{\partial t} + \frac{\partial \mathbf{u}}{\partial z} = 0, \quad (5.13)$$

Table 5.1: Simulation parameters and reversed CE/SE method parameters for the single-column binary process with Langmuir isotherms described by the ideal equilibrium model.

Quantity	Value	Quantity	Value	Quantity	Value
L_{col} , [m]	1.0	H_A , [-]	2	H_B , [-]	4
v , [$\text{m} \cdot \text{s}^{-1}$]	0.1	b_A , [$\text{L} \cdot \text{g}^{-1}$]	2	b_B , [$\text{L} \cdot \text{g}^{-1}$]	4
ε , [-]	0.8	$C_{A,\text{Fe}}$, [$\text{g} \cdot \text{L}^{-1}$]	0.9	$C_{B,\text{Fe}}$, [$\text{g} \cdot \text{L}^{-1}$]	0.8
t_{sim} , [s]	10	N_t , [-]	501	$\overline{\text{CFL}}$, [-]	0.4

For nonlinear isotherms, only in special cases can the model in Eq. (5.13) be directly converted to the type in Eq. (5.4). A detailed discussion for Langmuir isotherms is given in Appendix C. A possible alternative is to interchange the spatial and the time coordinates, i.e., instead of propagating in time, the solution procedure propagates in space (Figure 5.1b). This formulation of the CE/SE method is called the reversed CE/SE method, and the CFL number is defined as:

$$\overline{\text{CFL}} = \frac{1}{v} \frac{\Delta z}{\Delta t}. \quad (5.14)$$

The algorithm is as follows:

1. Specify the simulation time t_{sim} ;
2. Specify the number of time steps N_t , i.e., the number of conservation elements;
3. Calculate the time step size Δt ;
4. Specify $\overline{\text{CFL}}$ ($0 < \overline{\text{CFL}} \leq 1$);
5. Calculate the spatial step size Δz from Equation (5.14);
6. Continue with the reversed CE/SE method.

For the solution obtained via MOL, the time integration was performed using the built-in MATLAB ODE solver **ode45**, which uses the Runge–Kutta method (MATLAB, 2017). The double computational time was found using **ode23**, which is also a Runge–Kutta method, but of lower order. As an alternative for stiff systems, the **ode15s** solver was also used, but it was three times slower than ode45. This indicates that the present system is nonstiff.

The concentration profiles along the column calculated by the two numerical methods are shown in Figure 5.2a at three time instants: 0, 5, and 10 seconds. Additionally, the analytical solution of the system (2.2) obtained by the method of characteristics for $N \rightarrow \infty$ is shown (Guiochon and Lin, 2003). It can be seen that both MOL and reversed CE/SE methods resolve the concentration shocks with the same number of points. For MOL, the number of points refers to the number of finite volumes in space; for the reversed CE/SE method, the number of points refers to the number of elements in time. The corresponding number of elements in space follows from the given number of elements in time and the given $\overline{\text{CFL}}$ number from Eq. (5.14). The computation time of the MOL simulation is 7.85 seconds, which is the reference time for all of the simulations in the current work. The reversed CE/SE method is much more efficient (approximately 34 times) as shown in

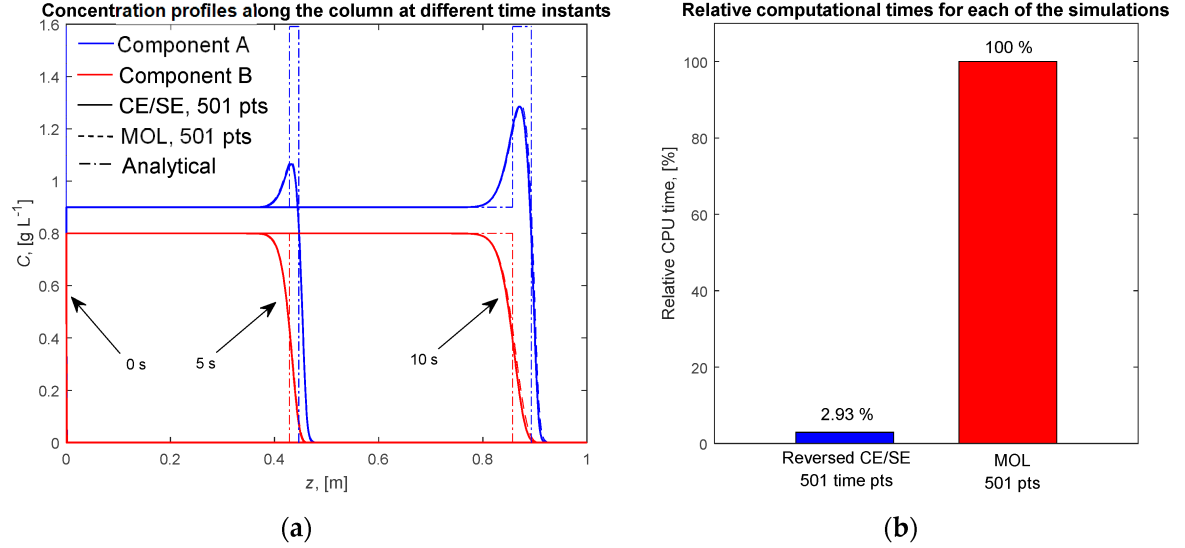


Figure 5.2: Single-column binary chromatographic process with Langmuir adsorption isotherms – ideal equilibrium model. (a) Concentration profiles along the column calculated using the two numerical methods and the analytical solution. (b) Comparison of the computational times for each of the numerical methods.

Figure 5.2b, even with the chosen $\overline{\text{CFL}}$ number.

5.2.2 Single column with the LDF Model

Unfortunately, the reversed formulation of the CE/SE method can be only used for single-column simulations because the feed concentrations, i.e., the boundary conditions are known a priori at all time steps. However, this is not true for SMB processes and control problems. That is why, for systems with nonlinear Langmuir isotherms, the mass-transfer model in Eq. (2.1) is used. The LDF chromatographic model corresponds to Eq. (5.4), and the original formulation of the CE/SE method can be used for simulations. The LDF model has a simpler mathematical structure, but the number of equations is doubled.

The parameters for this example are the same as those for the example presented in Table 5.1. Additional parameters are the mass transfer coefficients. Here, the same value is assumed for both components. The values of $k_{m,i}$ and the CE/SE method parameters are given in Table 5.2. The values of the mass transfer coefficients are chosen to be high enough such that the physical mass transfer is negligible.

Table 5.2: Simulation parameters and CE/SE method parameters for the single-column binary process with Langmuir isotherms described by the linear driving force model.

Quantity	Value	Quantity	Value
$k_{m,A}$, [s ⁻¹]	10	$k_{m,B}$, [s ⁻¹]	10
N_z , [-]	101	CFL, [-]	0.4

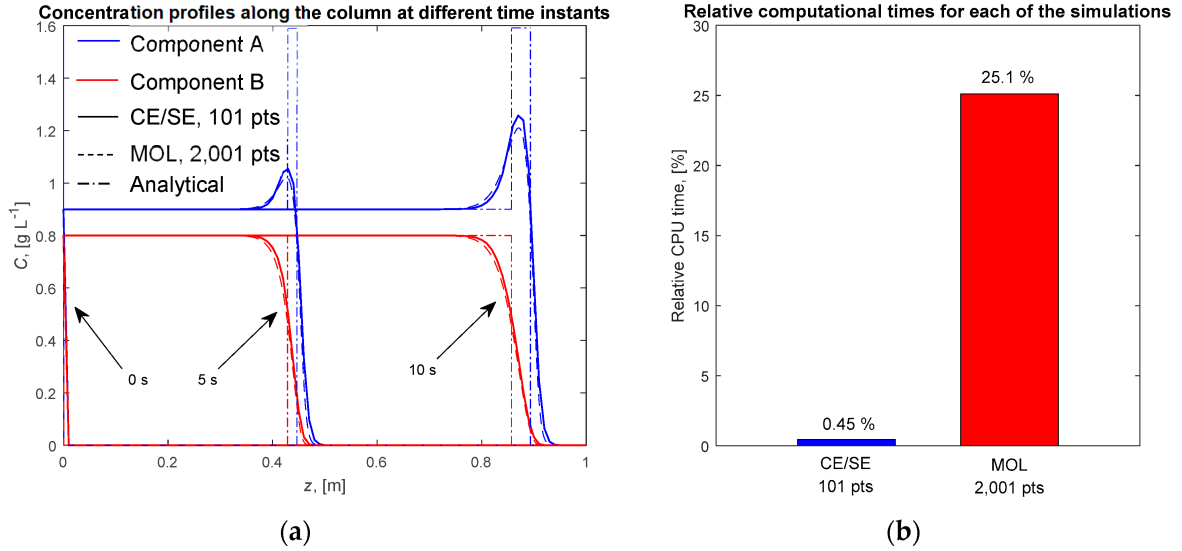


Figure 5.3: Single-column binary chromatographic process with Langmuir adsorption isotherms – LDF model. (a) Concentration profiles along the column calculated by two numerical methods and the analytical solution. (b) Comparison of the computational times for each of the numerical methods.

The results of the CE/SE and MOL simulations are shown in Figure 5.3a together with the analytical solution for $N \rightarrow \infty$ and $k_{m,i} \rightarrow \infty$. It can be seen that the original CE/SE method formulation could resolve the concentration fronts very well with a small number of spatial points, i.e., 101 points, whereas MOL needed more than 2000 spatial points to achieve a similar accuracy. Thus, the computational times of the two methods differed significantly (Figure 5.3b), and the CE/SE method is 55 times faster than MOL. Here, again, the ODE solver used for MOL is **ode45**. In particular, **ode45** was about 280 times faster than **ode15s**, indicating that the LDF model was considerably less stiff than the equilibrium one.

5.2.3 Binary SMB Process with the LDF Model

The dynamical model of the SMB processes includes the PDAE system in Eq. (2.1) to describe each chromatographic column, as well as the nodal material balances in Eqs. (2.6), (2.8), (2.10), and (2.12), which define the boundary conditions for the system in Eq. (2.1).

The process configuration for the binary SMB process is shown in Figure 1.3. In this simulation, each zone has only one chromatographic column. The simulation parameters and the operating conditions are presented in Table 5.3. The operating point, i.e., the m values, are close to the optimal operating point for this system according to the triangle theory (Mazzotti et al., 1997).

After several column switchings, a cyclic steady state (CSS) is reached. In this par-

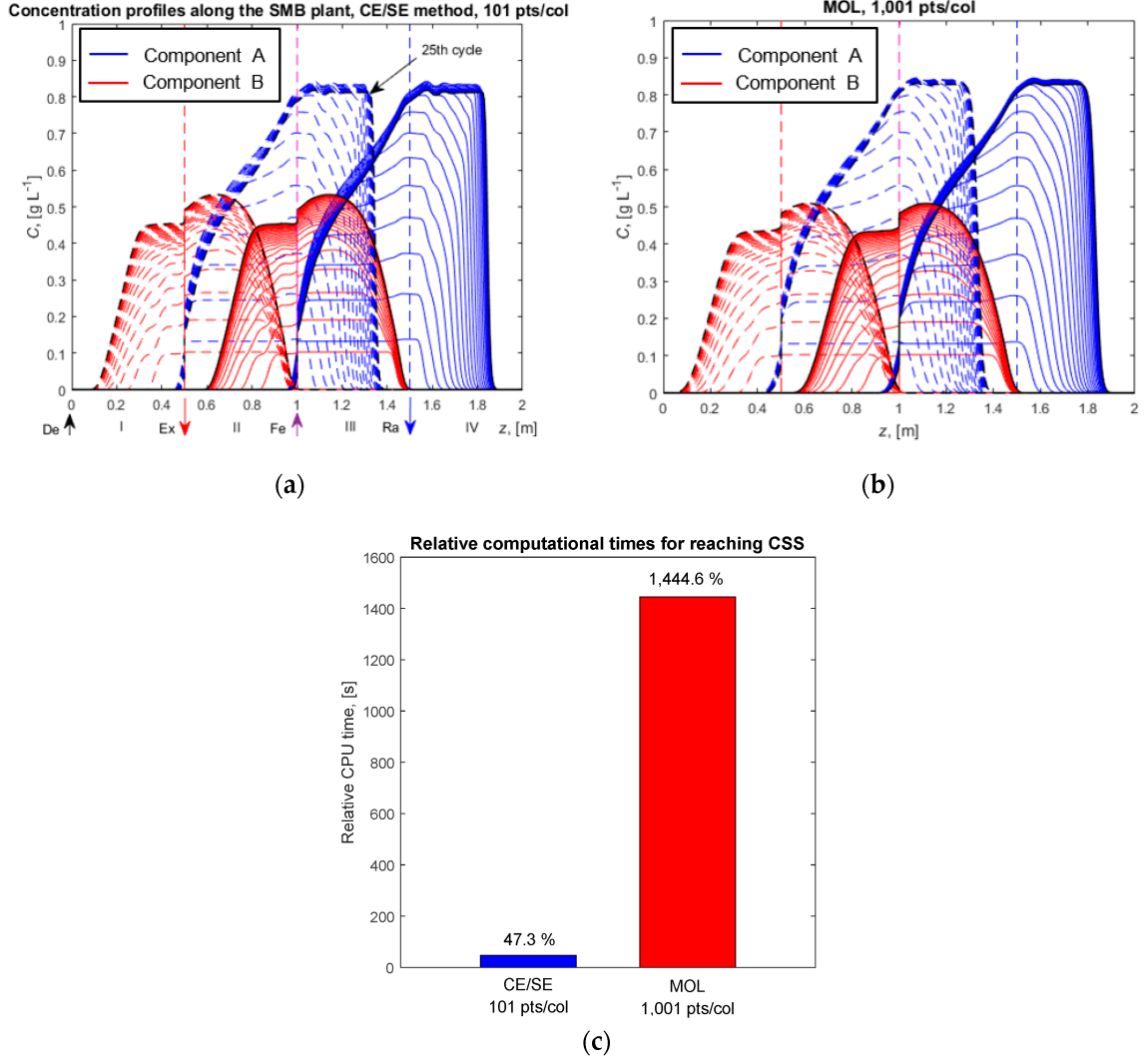


Figure 5.4: Binary SMB chromatographic process with Langmuir adsorption isotherms – LDF model. (a) Internal concentration profiles along the SMB plant calculated using the CE/SE method. (b) Internal concentration profiles along the SMB plant calculated using MOL. (c) Comparison of the computational times for each of the methods. Dashed curves in (a) and (b) are the profiles at the beginning of each cycle, whereas solid curves are the profiles at the end.

particular example, this cyclic steady state was reached after 25 cycles. Figure 5.4a,b shows the concentration profiles of the two components inside the SMB plant during this startup.

Table 5.3: Simulation parameters for the binary SMB process with Langmuir isotherms described by the linear driving force model.

Quantity	Value	Quantity	Value	Quantity	Value	Quantity	Value
L_{col} , [m]	0.5	m_I , [-]	5	H_A , [-]	2	H_B , [-]	4
D_{col} , [m]	0.02	m_{II} , [-]	1.8	b_A , [L·g ⁻¹]	0.2	b_B , [L·g ⁻¹]	0.4
ε , [-]	0.8	m_{III} , [-]	2.8	$k_{m,A}$, [s ⁻¹]	10	$k_{m,B}$, [s ⁻¹]	10
t_{sw} , [s]	40	m_{IV} , [-]	1.3	$C_{A,Fe}$, [g·L ⁻¹]	0.9	$C_{B,Fe}$, [g·L ⁻¹]	0.7

The dashed curves are the profiles at the beginning of every cycle just after the column switching, while the solid curves correspond to the profiles at the end of the cycle. With the values chosen for the isotherm parameters H_i and b_i , the component A (blue) has a lower affinity to the solid phase and flows out of the raffinate port, while the component B (red) has a higher affinity to the solid phase and flows out from the extract port.

For the simulation with the CE/SE method, the value of the CFL number was selected as 0.4. To satisfy the CFL condition in each zone of the SMB, the value of the time step Δt size was calculated from the highest liquid velocity in the system, i.e., in zone I.

$$\Delta t = \text{CFL} \frac{\Delta z}{\max(v_p)}, \quad p = \text{I, II, } \dots, N_{\text{col}}, \quad (5.15)$$

The number of spatial points was 101 per column, and the CSS was reached at 47% of the relative computational time. To achieve similar accuracy with the MOL, 10 times more spatial discretisation points per column were needed. This leads to the much slower performance of the MOL simulation (cf. Figure 5.4c).

For MOL in Figure 5.4b,c, **ode45** was used. Again, the performance of **ode23** was similar (1002% compared to 1444.6% for **ode45**). In contrast, **ode15s** needed 580-fold more computational time than the simulation with **ode45**.

5.2.4 Ternary centre-cut eight-zone SMB process with linear isotherms and the ideal equilibrium model

Ternary SMB processes, called centre-cut separations, play an important role in biotechnology and pharmaceutical industries for the isolation of desired key components that have medium affinity to the solid phase in comparison to the other fractions in the mixture. There are several configurations for ternary centre-cut separation – cascade of two SMB units, eight-zone SMB, Japan Organo process, etc. (Agrawal and Kawajiri, 2012). Eight-zone SMB processes can be used with raffinate or with extract recycle (Da Silva and Seidel-Morgenstern, 2016). In the present work, an eight-zone SMB process with raffinate recycle and both linear and Langmuir isotherms was investigated.

Since, for systems with linear isotherms, the ideal equilibrium model Eq. (2.2) can be converted directly to form Eq. (5.4), it was used as a first example. The process configuration is shown in Figure 5.5a. Again, each zone had one column. The nodal

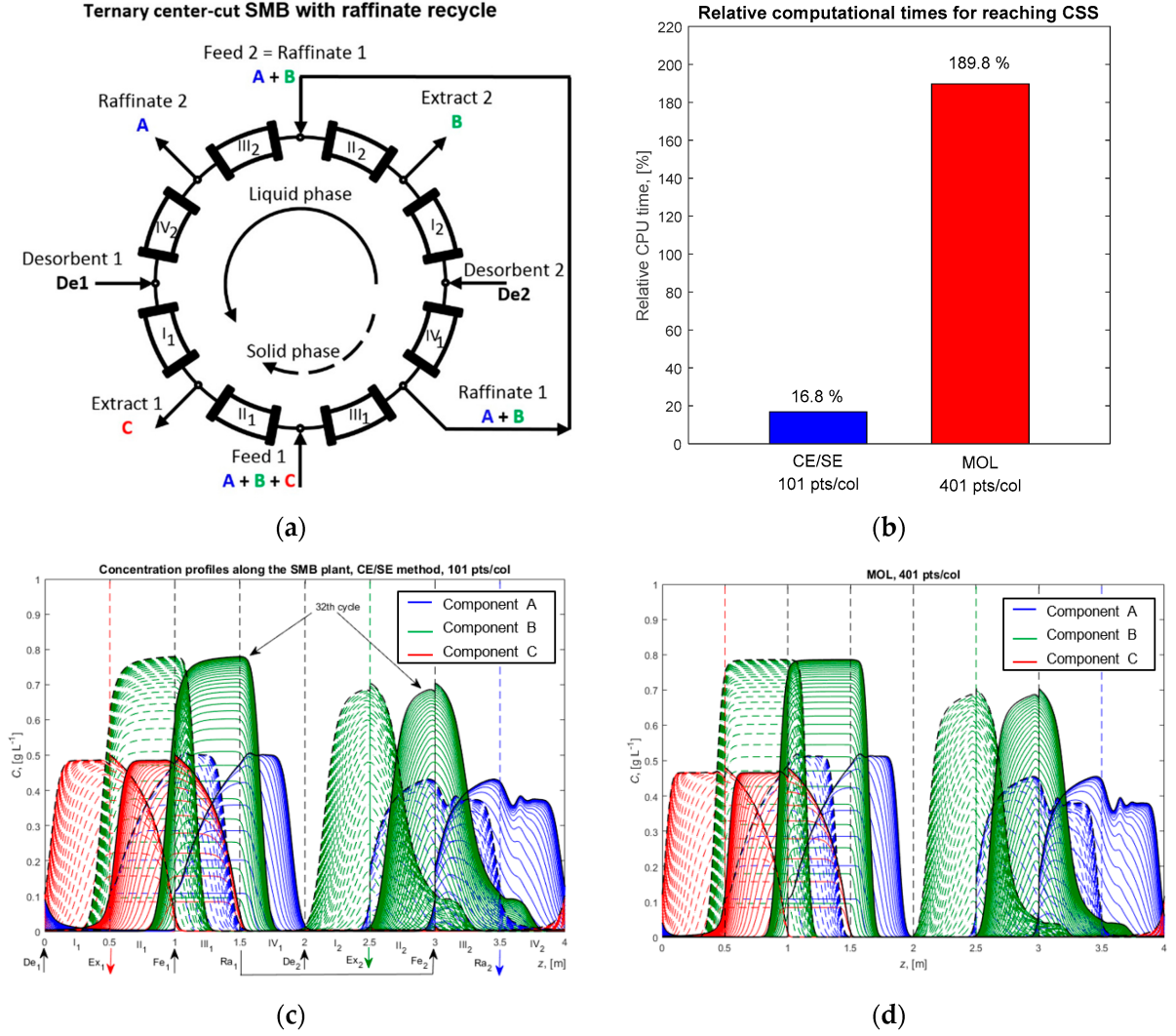


Figure 5.5: Ternary centre-cut eight-zone SMB chromatographic process with raffinate recycle with linear adsorption isotherms – ideal equilibrium model. (a) Process configuration. (b) Comparison of the computational times for each of the methods. (c) Internal concentration profiles along the SMB plant calculated using the CE/SE method. (d) Internal concentration profiles along the SMB plant calculated using MOL. Dashed curves in (c) and (d) are the profiles at the beginning of each cycle, whereas solid curves are at the end.

material balances for the eight-zone SMB with raffinate recycle are as follows:

- First desorbent node

$$Q_{De,1} = Q_{int,I,1} - Q_{int,IV,2}, \quad (5.16)$$

$$Q_{De,1} C_{i,De,1}(t) = Q_{int,I,1} C_{i,I,1}(0, t) - Q_{int,IV,2} C_{i,IV,2}(L_{col}, t), \quad i = A, B, \dots, N_{comp}. \quad (5.17)$$

When only clean desorbent is fed to the SMB plant $C_{i,De,1}(t) = 0$ for each of the components i .

- First extract node

$$Q_{\text{Ex},1} = Q_{\text{int},\text{I},1} - Q_{\text{int},\text{II},1}, \quad (5.18)$$

$$C_{i,\text{Ex},1}(t) = C_{i,\text{I},1}(L_{\text{col}}, t) = C_{i,\text{II},1}(0, t), \quad i = \text{A}, \text{B}, \dots, N_{\text{comp}}. \quad (5.19)$$

- First feed node

$$Q_{\text{Fe},1} = Q_{\text{int},\text{III},1} - Q_{\text{int},\text{II},1}, \quad (5.20)$$

$$Q_{\text{Fe},1} C_{i,\text{Fe},1}(t) = Q_{\text{int},\text{III},1} C_{i,\text{III},1}(0, t) - Q_{\text{int},\text{II},1} C_{i,\text{II},1}(L_{\text{col}}, t), \quad i = \text{A}, \text{B}, \dots, N_{\text{comp}}. \quad (5.21)$$

- First raffinate node

$$Q_{\text{Ra},1} = Q_{\text{int},\text{III},1} - Q_{\text{int},\text{IV},1}, \quad (5.22)$$

$$C_{i,\text{Ra},1}(t) = C_{i,\text{III},1}(L_{\text{col}}, t) = C_{i,\text{IV},1}(0, t), \quad i = \text{A}, \text{B}, \dots, N_{\text{comp}}. \quad (5.23)$$

- Second desorbent node

$$Q_{\text{De},2} = Q_{\text{int},\text{I},2} - Q_{\text{int},\text{IV},1}, \quad (5.24)$$

$$Q_{\text{De},2} C_{i,\text{De},2}(t) = Q_{\text{int},\text{I},2} C_{i,\text{I},2}(0, t) - Q_{\text{int},\text{IV},1} C_{i,\text{IV},1}(L_{\text{col}}, t), \quad i = \text{A}, \text{B}, \dots, N_{\text{comp}}. \quad (5.25)$$

- Second extract node

$$Q_{\text{Ex},2} = Q_{\text{int},\text{I},2} - Q_{\text{int},\text{II},2}, \quad (5.26)$$

$$C_{i,\text{Ex},2}(t) = C_{i,\text{I},2}(L_{\text{col}}, t) = C_{i,\text{II},2}(0, t), \quad i = \text{A}, \text{B}, \dots, N_{\text{comp}}. \quad (5.27)$$

- Second feed node

$$Q_{\text{Fe},2} = Q_{\text{Ra},1}, \quad (5.28)$$

$$C_{i,\text{Fe},2}(t) = C_{i,\text{Ra},1}(t), \quad i = \text{A}, \text{B}, \dots, N_{\text{comp}}. \quad (5.29)$$

- Second raffinate node

$$Q_{\text{Ra},2} = Q_{\text{int},\text{III},2} - Q_{\text{int},\text{IV},2}, \quad (5.30)$$

$$C_{i,\text{Ra},2}(t) = C_{i,\text{III},2}(L_{\text{col}}, t) = C_{i,\text{IV},2}(0, t), \quad i = \text{A}, \text{B}, \dots, N_{\text{comp}}. \quad (5.31)$$

Table 5.4 summarises the simulation parameters and the operating conditions, which were taken from (Kessler and Seidel-Morgenstern, 2006). Component C (red) had the highest affinity to the solid phase and flowed out through the first extract port together

Table 5.4: Simulation parameters for the ternary centre-cut eight-zone SMB process with linear isotherms described by the ideal equilibrium model.

Quantity	Value	Quantity	Value	Quantity	Value	Quantity	Value
L_{col} , [m]	0.5	D_{col} , [m]	0.02	ε , [-]	0.75	t_{sw} , [s]	60
$m_{\text{I},1}$, [-]	2.55	$m_{\text{II},1}$, [-]	1.57	$m_{\text{III},1}$, [-]	2.19	$m_{\text{IV},1}$, [-]	0.86
$m_{\text{I},2}$, [-]	1.82	$m_{\text{II},2}$, [-]	1.22	$m_{\text{III},2}$, [-]	2.55	$m_{\text{IV},2}$, [-]	1.01
H_{A} , [-]	1.1	H_{B} , [-]	1.7	H_{C} , [-]	2.5		
$C_{\text{A,Fe}}$, [g·L ⁻¹]	0.9	$C_{\text{B,Fe}}$, [g·L ⁻¹]	0.8	$C_{\text{C,Fe}}$, [g·L ⁻¹]	0.7		

with some amount of the middle component B (green). The outlet mixture from the first raffinate port, which contained the components A (blue, the component with the lowest affinity to the solid phase) and B, was fed to the second subunit, where it was separated. Component B flowed out from the second extract port, while component A flowed out from the second raffinate port. Simulation results show that CSS was reached after 32 cycles, and the concentration profiles are shown in Figure 5.5c,d. For the CE/SE method simulation, the CFL number was 0.8, and the number of spatial discretisation points per column was 101. For the MOL, 401 such points were needed per column, and the mass matrix for systems with linear isotherms is constant and, therefore, was calculated once at the beginning of the simulation, leading to a very fast computational time. As a result, the CE/SE method was only 11-fold faster than MOL (Figure 5.5b).

5.2.5 Ternary centre-cut eight-zone SMB process with Langmuir isotherms and the LDF model

Lastly, a study of a ternary system with Langmuir isotherms is performed. The process configuration is the same as in Figure 5.5a, i.e., ternary eight-zone centre-cut SMB with raffinate recycle. The nodal material balances are those in Eqs. (5.16)–(5.31). Physical parameters of the chromatographic columns are presented in Table 5.4 (first row), while the operating conditions and the parameters of the participating components are listed in Table 5.5. The operating conditions are calculated with the methodology presented in (Nicolaos et al., 2001) for a true moving bed process with little adjustments to take the SMB process into account; to the best of our knowledge, this was the first simulation of ternary centre-cut eight-zone SMB with Langmuir isotherms in the nonlinear concentration range. Calculations for the nonlinear case are based on the LDF model.

Table 5.5: Simulation parameters for the ternary centre-cut eight-zone SMB process with Langmuir isotherms described by the LDF model.

Quantity	Value	Quantity	Value	Quantity	Value	Quantity	Value
$m_{I,1}$, [-]	2.55	$m_{II,1}$, [-]	1.893	$m_{III,1}$, [-]	1.90	$m_{IV,1}$, [-]	0.915
$m_{I,2}$, [-]	2.10	$m_{II,2}$, [-]	0.928	$m_{III,2}$, [-]	1.99	$m_{IV,2}$, [-]	0.85
H_A , [-]	1	H_B , [-]	2	H_C , [-]	2.5	t_{sw} , [s]	40
b_A , [L·g ⁻¹]	1	b_B , [L·g ⁻¹]	2	b_C , [L·g ⁻¹]	2.5		
$k_{m,A}$, [s ⁻¹]	10	$k_{m,B}$, [s ⁻¹]	10	$k_{m,C}$, [s ⁻¹]	10		
$C_{A,Fe}$, [g·L ⁻¹]	0.5	$C_{B,Fe}$, [g·L ⁻¹]	5.0	$C_{C,Fe}$, [g·L ⁻¹]	1.5		

The concentration profiles along the SMB plant are shown in Figure 5.6a,b. The target component B is obtained as a pure component from the extract port of the second SMB subunit. The cyclic steady state is reached after around 60 cycles. The CFL number for the CE/SE method is 0.4, and the number of spatial discretisation points is 101 per column. On the other hand, MOL required 10 times as many spatial points per column to achieve an accuracy similar to that of CE/SE. This results in enormous computational time for the MOL method (approximately 2.5 hours in absolute time) compared to the CE/SE simulation (Figure 5.6c), which is 350 times greater than that of CE/SE.

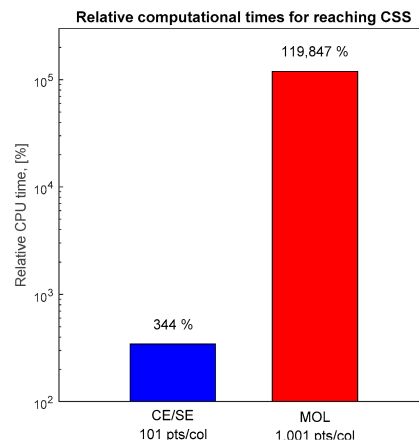
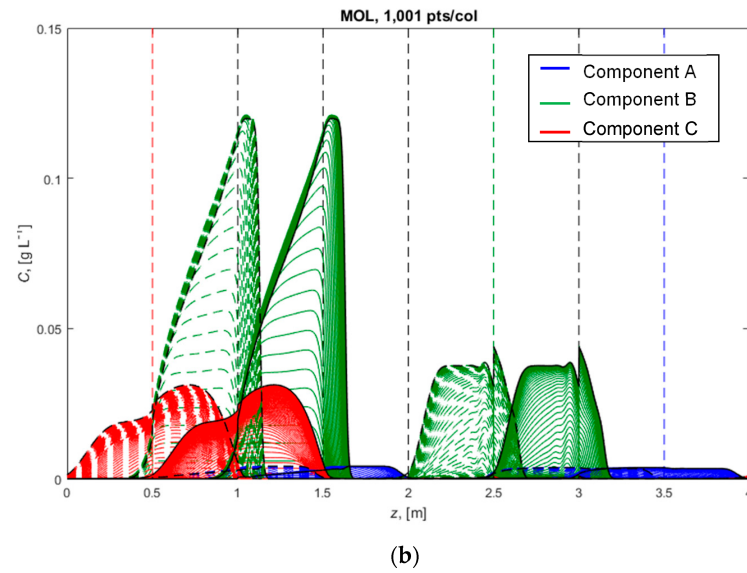
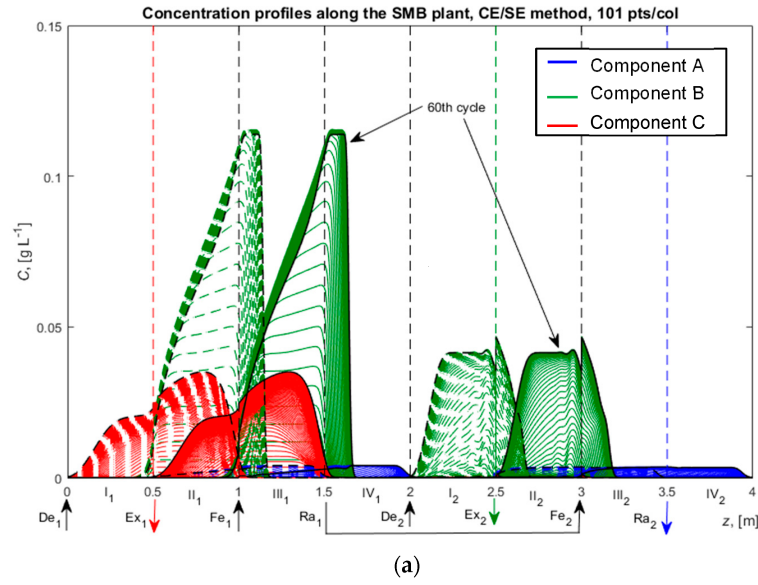


Figure 5.6: Ternary centre-cut eight-zone SMB chromatographic process with raffinate recycle with Langmuir adsorption isotherms – LDF model. (a) Internal concentration profiles along the SMB plant calculated using CE/SE method. (b) Internal concentration profiles along the SMB plant calculated using MOL. (c) Comparison of the computational times for each of the methods. Dashed curves in (a) and (b) are the profiles at the beginning of each cycle, whereas solid curves are the profiles at the end.

Chapter 6

Application of conservation element/solution element method for control of SMB chromatographic processes

6.1 MPC control within the linear range of the adsorption isotherms based on the exact internal concentration pro- files

Chapter 6 was published in parts in the IEEE Xplore[®] online database.

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The investigated SMB system consists of four chromatographic columns in a 1-1-1-1 configuration and is used for the separation of a racemic mixture of bicalutamides. One of

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the isomers, (R)-bicalutamide, is an active pharmaceutical ingredient for the treatment of prostate cancer, while the other isomer, (S)-bicalutamide, is a byproduct. The adsorption and column parameters are taken from (Lee et al., 2020b) and are summarised in Table 6.1 along with CE/SE method parameters. According to the values of the adsorption Henry coefficients, the more retained compound in the solid phase is the (R)-bicalutamide and thus, it shall flow-out from the extract outlet under complete separation conditions, while the (S)-bicalutamide is less retained in the solid and it shall flow out from the raffinate outlet under the same conditions. The composition of the feed mixture is in the low concentration region ($0.5 \text{ g}\cdot\text{L}^{-1}$ for both of the components), i.e., linear isotherms (2.19) are used to describe the adsorption equilibrium between the two phases. The initial operating point is chosen to be close to the optimal operating point for this system according to the triangle theory (Migliorini et al., 1998) and the switching time t_{sw} is chosen to be large enough to allow all time-consuming analytical measurements to take place during one cycle. The initial values for the internal flowrates $Q_{\text{int, init}}$ are computed from the expression of the m values (Eq. (2.32)).

The SMB process and instrumentation diagram is shown in Figure 6.1. The purity of the two products is measured by an online analytical analyser, namely HPLC and the movement of the concentration waves within the columns is detected by two UV sensors

Table 6.1: Simulation and CE/SE parameters of the binary SMB chromatographic plant with linear adsorption isotherms.

Quantity	Value	Quantity	Value
$H_{(\text{S})}$, [-]	0.82	$H_{(\text{R})}$, [-]	2.45
L_{col} , [m]	0.1467	D_{col} , [m]	0.025
ε , [-]	0.783	t_{sw} , [s]	178
CFL, [-]	0.8	N_z , [-]	201
m_{init} , [-]	$[6.091, 0.857, 2.379, 0.335]^T$		

Table 6.2: Control parameters for model predictive control of the binary SMB process with linear adsorption isotherms and simulated with the CE/SE method.

Quantity	Value	Quantity	Value
N_{ph} , [-]	2	N_{ch} , [-]	1
w_{Pu} , [-]	7	w_{τ} , [-]	1.5
$w_{\Delta Q}$, [-]	1.5	Footpoint threshold, $[\text{g}\cdot\text{L}^{-1}]$	0.01
$Q_{\text{int}}^{\text{min}}$, $[\text{mL}\cdot\text{min}^{-1}]$	1	$Q_{\text{int}}^{\text{max}}$, $[\text{mL}\cdot\text{min}^{-1}]$	70

$i = \textcolor{red}{(R)}, \textcolor{blue}{(S)}$ -bicalutamides – components
 $p = \text{I, II, III, IV}$ – columns

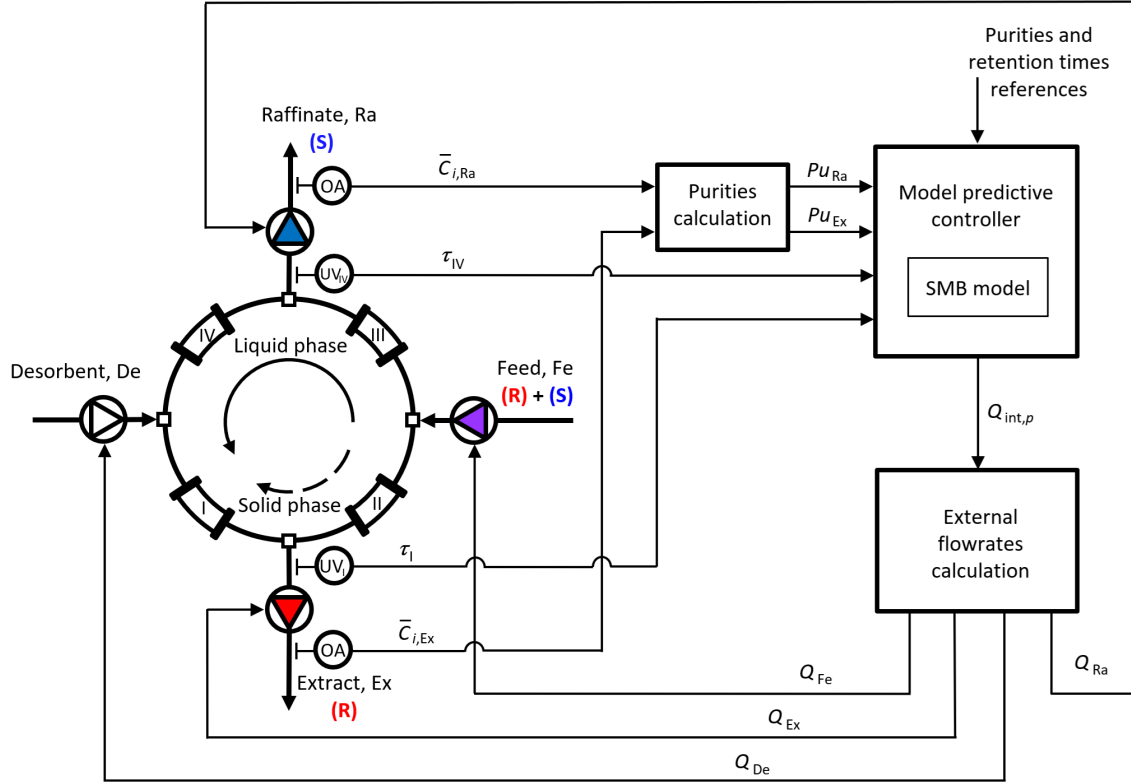


Figure 6.1: SMB process configuration with model predictive control for the case with linear adsorption isotherms and simulation with the CE/SE method.

placed at each outlet port. The movement of concentration waves is characterised by dimensionless retention times τ , whose values are received from the UV sensors when the wave footpoint crosses the sensor. The threshold value of the footpoints should be as low as possible to give representative information about the position of the concentration waves, but at the same time, it should be higher than the measurement noise. The manipulated variables are the four internal flowrates Q_{int} , which are determined by the pumps on each of the inlet and outlet ports. The applied strategy is cycle-to-cycle control, which means that the control action is introduced to the SMB plant at the beginning of each cycle and it is kept constant until the end of this cycle. In the next cycle, this procedure is repeated and so on.

NMPC minimises in real time the following cost function:

$$\begin{aligned}
\min_{Q_{\text{int},p}} J_{\text{MPC}} = & w_{Pu} \sum_{k=r}^{r+N_{\text{ph}}-1} \left((Pu_{\text{Ex},k}^{\text{ref}} - \hat{P}u_{\text{Ex},k})^2 + (Pu_{\text{Ra},k}^{\text{ref}} - \hat{P}u_{\text{Ra},k})^2 \right) \\
& + w_{\tau} \sum_{k=r}^{r+N_{\text{ph}}-1} \left((\tau_{\text{I},k}^{\text{ref}} - \hat{\tau}_{\text{I},k})^2 + (\tau_{\text{IV},k}^{\text{ref}} - \hat{\tau}_{\text{IV},k})^2 \right) \\
& + w_{\Delta Q} \sum_{k=r}^{r+N_{\text{ch}}-1} \sum_{h=1}^{N_{\text{col}}} \Delta Q_{\text{int},h,k}^2, \quad p = \text{I, II, ..., } N_{\text{col}},
\end{aligned} \tag{6.1a}$$

$$\begin{aligned}
\text{s.t. } Q_{\text{De},g} &= Q_{\text{int},\text{I},g} - Q_{\text{int},\text{IV},g} \geq 0, \\
Q_{\text{Ex},g} &= Q_{\text{int},\text{I},g} - Q_{\text{int},\text{II},g} \geq 0, \\
Q_{\text{Fe},g} &= Q_{\text{int},\text{III},g} - Q_{\text{int},\text{II},g} \geq 0, \\
Q_{\text{Ra},g} &= Q_{\text{int},\text{III},g} - Q_{\text{int},\text{IV},g} \geq 0, \\
&\forall g = 1, 2, \dots, N_{\text{ch}}.
\end{aligned} \tag{6.1b}$$

In addition to the inequality constraints in Eqs. (6.1b) imposed onto the control variables Q_{int} , the latter are also bounded during the optimisation procedure between $Q_{\text{int}}^{\min}$ and $Q_{\text{int}}^{\max}$. All control parameters are summarised in Table 6.2. The sampling period for the MPC is equal to the cycle duration t_{sw} of the SMB plant. In this way, the controller predicts exactly the model outputs N_{ph} cycles ahead. This makes the implementation and the tuning of MPC easier (at least the choice of suitable horizon lengths, $N_{\text{ch}} \leq N_{\text{ph}}$).

The optimisation method used for the minimisation of the optimisation problem in Eqs. (6.1a)-(6.1b) is the active set method, which is implemented in the MATLAB Optimization Toolbox function **fmincon** (MATLAB, 2022b). Both the SMB plant emulator and the NMPC predictor are therefore described by a system of eight PDEs, which is solved numerically using the CE/SE method with a CFL number of 0.8 and a spatial discretisation of 200 conservation elements per column (which leads to 201 grid points per column). All computations in Chapter 6.1 are performed in MathWorks MATLAB 2022b under Microsoft Windows 10 on a laptop computer with an AMD Ryzen 7 2700U central processing unit (2.2 GHz) and 8 GB RAM.

The investigation scenario is as follows: (i) startup of the process with completely empty columns in open-loop mode with the initial operating point obtained from the triangle theory for this system (red circle in Figure 6.2); (ii) before the start of the 10th cycle the control loops are closed with references for the purities for complete separation (98

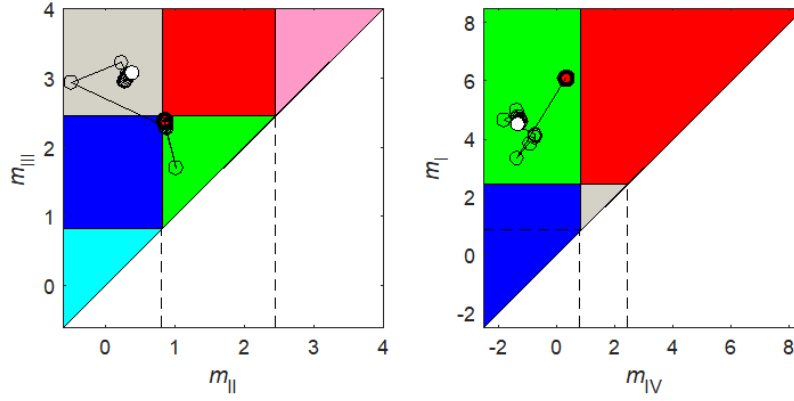


Figure 6.2: Triangle diagram with the change of the operating point for the model predictive control of SMB process with linear adsorptions simulated with the CE/SE method. Red circle – initial operating point (1st cycle), white circle – operating point at the 70th cycle.

%) and 0.6 for the dimensionless retention times, thus guaranteeing complete regeneration of both solid and liquid phases in the first and the fourth columns, respectively. The latter references do not change until the end of the simulation; (iii) before the 30th cycle the purity references are lowered for incomplete separation (80 %) and they also do not change until the end of the simulation; (iv) before the 50th cycle perturbation of the feed composition is introduced in the form of step change of the feed concentration of the (S)-bicalutamide which remains unchanged until the end of the simulation. Despite the fact that in reality the values of $C_{i,\text{Fe}}$ are usually known because the feed stream of the SMB process is a product of upstream processes in the current case study, they are considered not measured. i.e. their change acts as an unmeasured disturbance to the control system. The simulation scenario is summarised in Table 6.3

The tuning parameters of the controller are the lengths of the prediction, N_{ph} , and control, N_{ch} , horizons and the weights, w , of the different terms of the cost function.

Table 6.3: Reference and disturbance parameters for the model predictive control of the binary SMB process with linear adsorption isotherms and simulated with the CE/SE method.

Cycles, k	Pu^{ref} , [%]		τ^{ref} , [-]		$C_{i,\text{Fe}}$, [g·L ⁻¹]	
	Extract	Raffinate	I	IV	(S)	(R)
1 - 9	Open loop				0.5	0.5
10 - 29	98	98	0.6	0.6	0.5	0.5
30 - 49	80	80	0.6	0.6	0.5	0.5
50 - 70	80	80	0.6	0.6	0.6	0.5

Different prediction horizon lengths were explored, varying from 1 to 10 cycles. A long prediction horizon leads to the fact that the controller sees early the change of the references and acts accordingly, but at the cost of significant computational time, which is important in the context of real-time control. The optimisation procedure (plus a possible parameter estimation procedure) should be faster than the online analytical analysis of the outlet samples; otherwise, the outcome of the optimisation will be suboptimal control variables. Simulations show that a prediction horizon of 2 cycles gives good predictive capabilities of the controller, combined with reasonable computational load. Long control horizons lead to smooth control but again with a higher computational price, and that is why a control horizon of one cycle is chosen. Weights are chosen by trial and error. Initially, the weights of the purity and retention time terms were equal, but this led to a suboptimal solution for the first cycle after the control loops were closed (10th cycle). Since the initial guess for the following cycle is the outcome from the previous one, this leads to a sequence of suboptimal cycles. This can be explained by the fact that there is a big change of about 50 % for τ_I and τ_{IV} from the 9th to 10th cycle. However, giving priority to the purities over the retention times through w_{Pu} and w_τ prevents sticking in a suboptimal point.

Time evolution of the process variables is shown in Figure 6.3. The corresponding time evolution of the manipulated variables is shown in Figure 6.4. After the startup, both the purities and the retention times are converging to their cyclic steady state until the 10th cycle, when the control loops are closed. Because τ_I and τ_{IV} are far from their reference values, large control actions are necessary in order to move them towards these newly introduced references. As can be seen, this is especially true for $Q_{int,I}$, which in turn results in a very big change of the extract flowrate Q_{Ex} . As soon as the first two cycles after the closing of the loops (in which most of the control efforts are concentrated) are over, the SMB plant is led fast by the controller to its new CSS in the complete separation region. Typical internal concentration profiles from this CSS are shown in Figure 6.5 for the 20th cycle. The concentration profiles in this figure are considered at the end of the respective cycles just before switching the columns in a direction opposite to the liquid flow.

After the 29th cycle, the purity references for both components are lowered to 80 % or to an incomplete separation state. This operating regime may be required in practice when product specifications are not so tight or high throughput is required from the SMB plant.

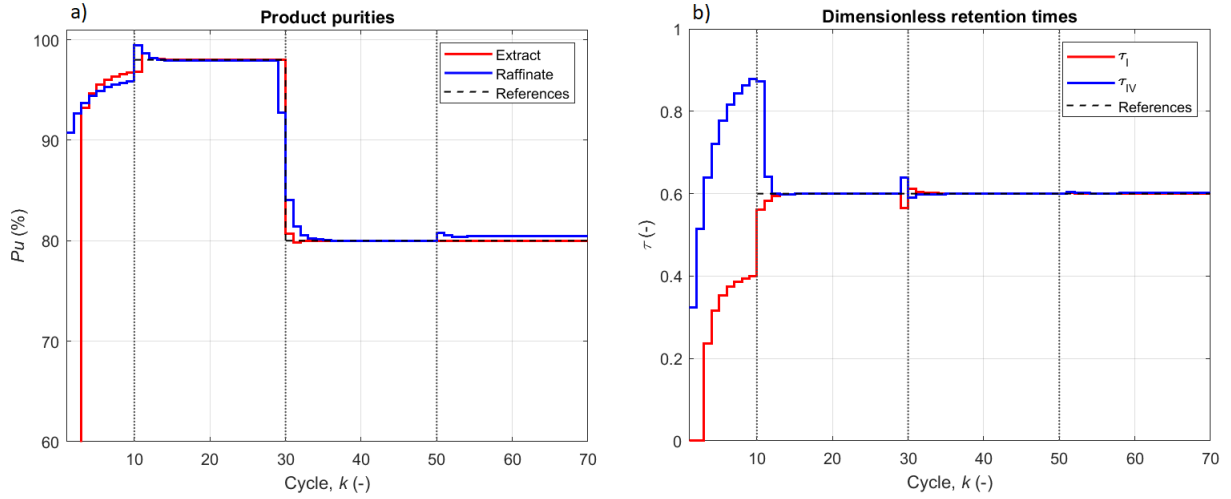


Figure 6.3: Process variables temporal change for the model predictive control of SMB process with linear adsorption isotherms simulated with the CE/SE method.

(a) Purities of the two outlet streams. (b) Dimensionless retention times.

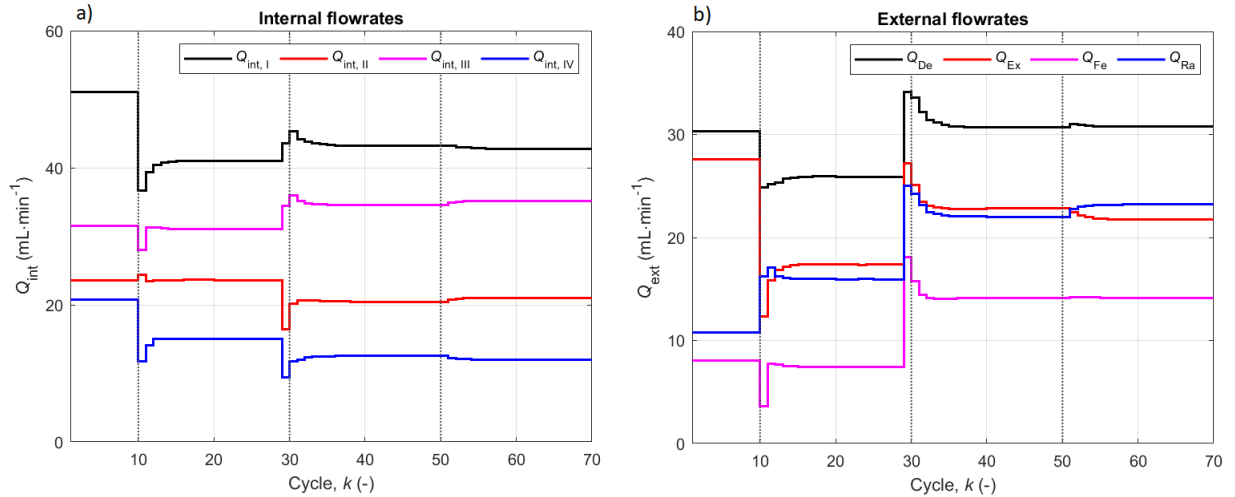


Figure 6.4: Manipulated variables temporal change for the model predictive control of SMB process with linear adsorption isotherms simulated with the CE/SE method.

(a) Internal flowrates. (b) External flowrates.

This change of almost 20 % is very big from a practical point of view but as can be seen, the controller is able to drive the process outputs to the desired values only after 4 cycles. However, this comes at the price of large control efforts, especially for $Q_{int,II}$ and $Q_{int,IV}$, which in turn are transferred to all of the external flowrates. For example, the change of $Q_{int,II}$ for this transient from complete to incomplete separation is -30 %. If so, big rates of change of the manipulated variables are not permitted, then the weight factor $w_{\Delta Q}$, which penalises such behaviour, could be increased. Another possibility is to add constraints for ΔQ_{int} into the optimisation model or to increase the length N_{ch} of the control horizon. It can be seen in Figure 6.3 and Figure 6.4 that the controller sees the change of the references and starts to act on this change early, before the change itself, depending on the length

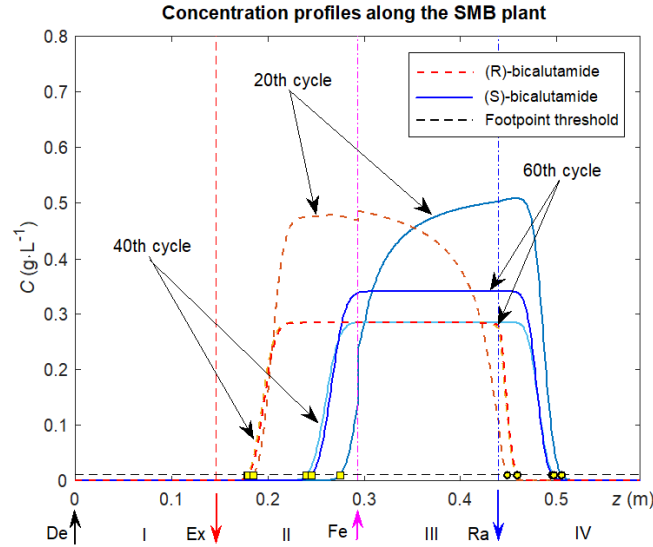


Figure 6.5: Internal concentration profiles along the SMB plant at the end of several cycles at which cyclic steady state is reached for the model predictive control of SMB process with linear adsorption isotherms simulated with the CE/SE method. Yellow circle and square marks are the wave footpoints. Circles – front footpoints; squares – rear footpoints.

N_{ph} of the prediction horizon. This is one of the key characteristics of MPC compared to standard PID control. To examine the abilities of the controller to reject perturbation from the 50th cycle, the value of the feed concentration of the (S)-bicalutamide is changed by a step of +20 % to $C_{(\text{S}),\text{Fe}} = 0.6 \text{ g}\cdot\text{L}^{-1}$. It is assumed that this perturbation is unmeasured and accordingly, the value of $C_{(\text{S}),\text{Fe}}$ is not updated in the NMPC predictor. Thus, the disturbance acts as plant-model mismatch for the controller. As a result, the predicted output variable from the controller for the k -th cycle $\hat{y}_k$ and the measured one y_k^{meas} are different. A simple technique is applied to alleviate the mismatch by compensating the reference with the observed difference:

$$y_k^{\text{ref}'} := y_k^{\text{ref}} + (y_k^{\text{meas}} - \hat{y}_k). \quad (6.2)$$

The controller starts to act on the perturbation with this modified reference $y_k^{\text{ref}'}$ by decreasing Q_{Ex} and increasing Q_{Ra} but this is not enough and as a result, there is an offset of the raffinate purity from its reference at the new CSS to 80.5 %. One might expect π_{IV} to be offset too but as it can be seen in Figure 5 for the 60th cycle only the height of (S)-bicalutamide concentration profile is higher, while the position of its footpoints does not change in comparison to the cycles (40th cycle) before the change of the feed concentration. This behaviour can be expected only for low feed concentrations in the linear range of the isotherms, which is the current case. For operations in the nonlinear range, such

favourable behaviour cannot be expected. Another simulation was performed in which the NMPC predictor was updated with the new value of the perturbed feed concentration five cycles after its change. Then, as expected, the controller moves the raffinate purity back to its reference value at 80 %. This clearly indicates that the predictive controller could take advantage of a parameter estimator to reject the unmeasured disturbances and to deal with plant-model mismatch and the measurement noise and errors.

As one can expect, the computational efforts are not distributed evenly from cycle to cycle. The highest computational times are for the cycles around which a big change of the operating conditions occurs, namely 10, 11, 29, and 30, but even then, the longest optimisation procedure takes only 18 seconds, which is far less than t_{sw} . This confirms the assumption that the CE/SE method could be a suitable numerical solver for the NMPC predictor.

6.2 Mixed PID-MPC control within the nonlinear range of the adsorption isotherms

6.2.1 Open-loop validation of the moving horizon estimator

In Chapter 4.1, we saw that the proposed MHE can estimate the internal concentration profiles in both open-loop and closed-loop modes when the governing PDAE system is solved using the mixing cell model. In the current chapter, we shall see whether the estimator is able to do the same when the system in Eq. (2.1) is solved by the CE/SE method under open-loop conditions. First, the noiseless case is investigated and then the case with added noise is considered.

All simulations in Chapter 6.2 are performed with Mathworks MATLAB R2022b on a desktop PC with Intel Core i9-12900K (3.20 GHz) CPU and 32 GB RAM under the Microsoft Windows 11 Pro operating system. The optimisation procedure is performed with the built-in MATLAB Optimization Toolbox function for constrained NLP optimisation – **fmincon** with the active set method and with the default values for the stopping criteria (MATLAB, 2022b).

The ability of the CE/SE method to resolve steep concentration fronts typical for nonlinear chromatography was demonstrated in Chapter 5, using rather conservative values for the number of conservation/solution elements and the CFL number. That is why, in

the current study, to improve computational time while maintaining a reasonable front resolution, the number of conservation/solution elements is 80, i.e. 81 spatial grid points, while the CFL number is 0.2. The investigated SMB process is the same as that in Chapters 3 and 4 and its parameters are given in Table 3.1. The simulation scenario is also the same and the operating conditions for the open-loop MHE validation are given in Table 4.1 and shown in Figure 4.2. In the current CE/SE investigation, the length of the estimation horizon N_{est} is again 6 cycles. The resolution quality of the concentration profiles is strongly influenced by the numerical diffusion inherent in the numerical solution of the PDAE chromatographic system in Eq. (2.1). The use of numerical methods with lower numerical diffusion results in steeper concentration profiles and vice versa. The comparison of the CE/SE method and the mixing cell model shows that the internal concentration profiles are much steeper and the fronts are better resolved by the CE/SE method for the chosen parameters (number of conservation/solution elements and CFL number by the CE/SE method and number of equilibrium stages for the cell model) than by the mixing cell model. Figure 6.6 displays the internal concentration profiles at the end of the 67th cycle, immediately before the column switching. The same profiles for the mixing cell model are shown in Figure 4.6. This improved front resolution, in turn, leads to steeper outlet profiles in both product outlets (Figure 6.7) and ultimately to different quantitative behaviours of the measured variables, as shown in the average outlet concentrations in Figure 6.8 and the retention times in Figure 6.9b (for reference with the mixing cell model, cf. Figure 4.3b). That is why the product purities differ considerably for the two methods at the same operating points. As shown in Figure 6.9a, the extract purity is significantly higher than that for the mixing cell model (cf. Figure 4.4) at all operating points. However, the raffinate purity is lower only for the second operating point because the other three operating points lie relatively far from the separation boundaries on the $m_{\text{II}} - m_{\text{III}}$ diagram, where the effect of the numerical diffusion is not that pronounced.

These steeper internal concentration profiles due to the CE/SE method solution are better approximated by the sigmoid functions than by the smooth fronts of the mixing cell model, especially in zones I and III (Figure 6.6), which are the most important zones in terms of estimation because all measurement information is obtained from the measurement devices which are mounted on the two outlet ports at the end of these two zones. As shown in Figure 6.7, the quality of the estimates is very good, as the estimated and true

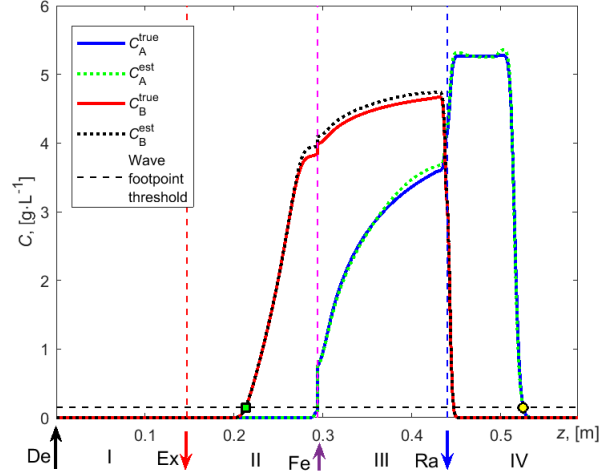


Figure 6.6: Comparison between the true and estimated liquid phase internal concentration profiles at the end of the 67th cycle for the case with noiseless measurements for SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. $N_{eh} = 6$. Green square – footprint of the desorption wave of the more retained component B, yellow circle – footprint of the adsorption wave of the less retained component A.

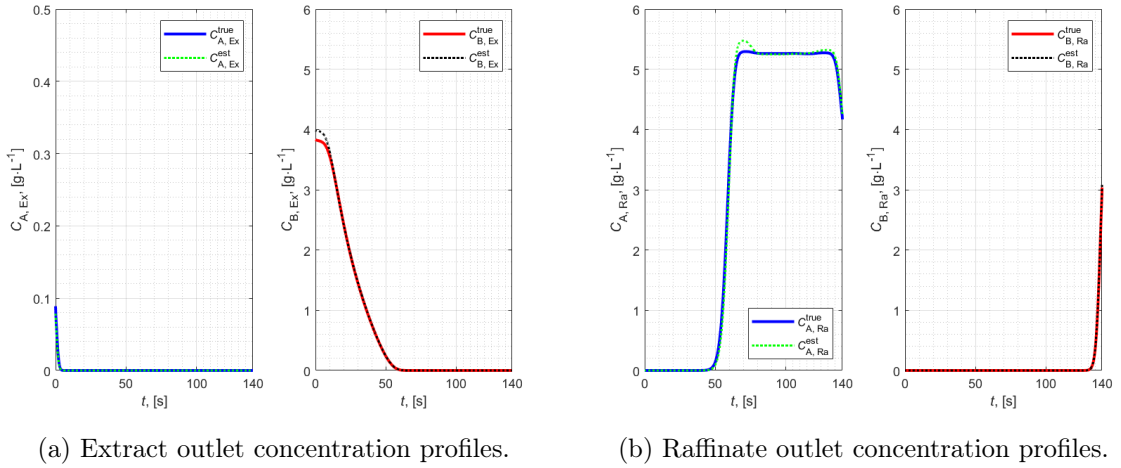


Figure 6.7: Comparison between the true and estimated outlet concentration profiles for the 67th cycle for the case with noiseless measurements for SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. $N_{eh} = 6$.

plant outlet profiles practically coincide. In particular, these two zones are also the most important ones for product purity control, as the outlet products of the SMB plant are formed within them. Thus, having high-quality estimates in these two zones is of utmost importance when the fronts are so steep and, for this reason, are very sensitive to the control actions and the perturbations.

Figures [6.10](#) and [6.11](#) show the simulation results for the case with added constant relative measurement noise with a magnitude of 1 %. One can see that the MHE is able to estimate both measured and process variables reasonably well, even in this demanding case with the shortest possible length of the estimation horizon of 6 cycles, which is of

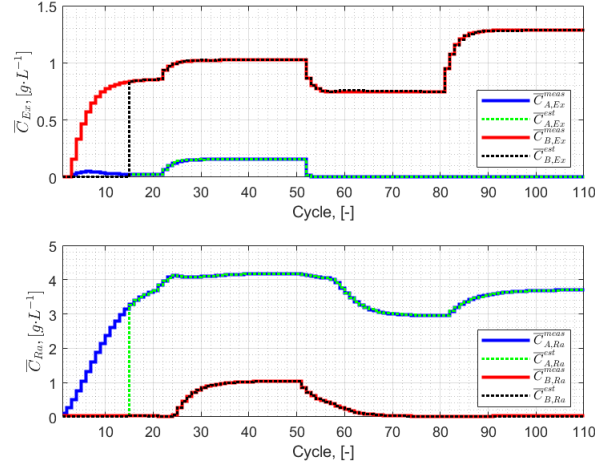


Figure 6.8: Average outlet concentrations for the case with noiseless measurements for SMB process with Langmuir adsorption isotherms simulated with the CE/SE method.

Upper diagram – extract, lower diagram – raffinate. $N_{eh} = 6$.

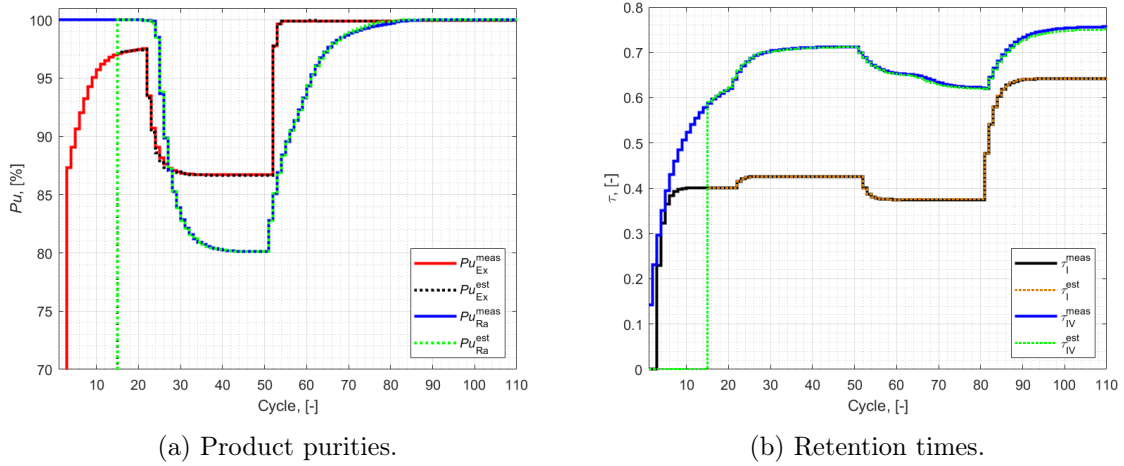


Figure 6.9: Process variables for the case with noiseless measurement for SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. $N_{eh} = 6$.

great practical importance.

6.2.2 Control based on the exact internal concentration profiles

To investigate the applicability of the CE/SE method for the control of systems within the nonlinear range of the Langmuir isotherms, the hypothetical case where exact internal concentration profiles are available to the controller is first considered. The studied system and simulation scenarios are identical to those for the mixing cell method presented in Chapter 4. As seen in Figure 6.6, the fronts of the internal concentration profiles are considerably steeper than those with the mixing cell method, even with fewer conservation elements compared to the number of equilibrium cells of the mixing cell method and a very conservative CFL number. In zones I and III, the internal concentration fronts estimated

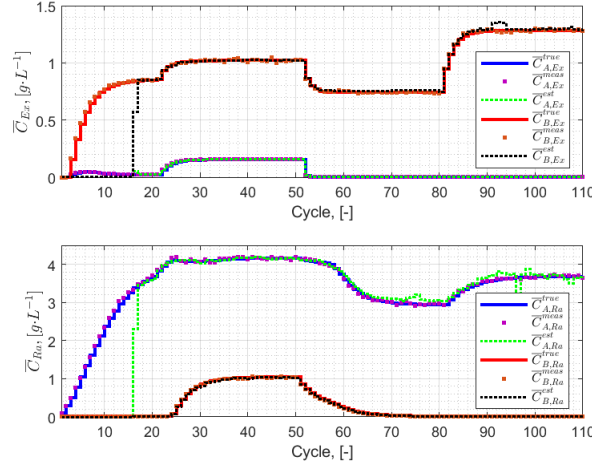


Figure 6.10: Average outlet concentrations for the case with additive measurement noise for SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. Upper diagram – extract, lower diagram – raffinate. $N_{eh} = 6$.

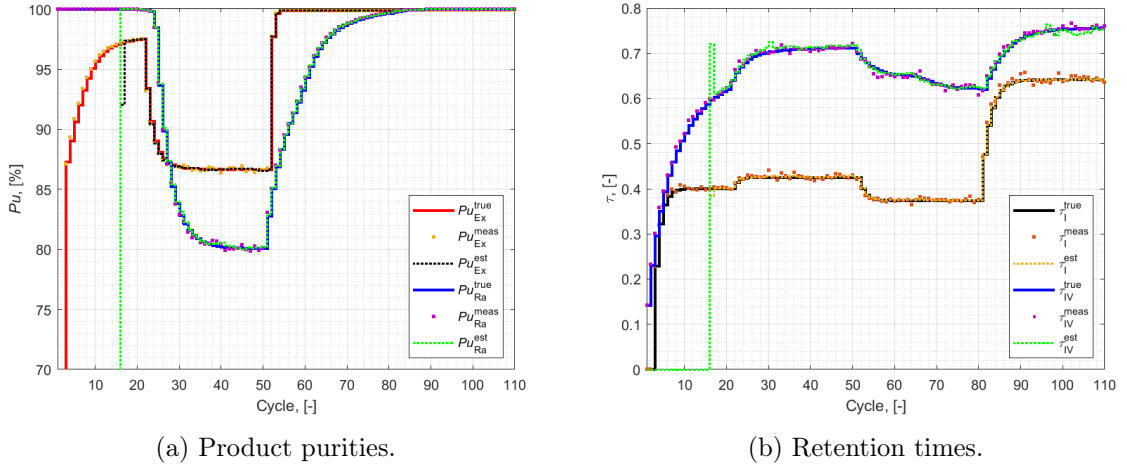


Figure 6.11: Process variables for the case with additive measurement noise for SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. $N_{eh} = 6$.

by the MHE virtually coincide with the true ones. These two zones are the most important because the outlet products are formed inside them. The MPC and the PID controllers are retuned, and the values of the controller tuning parameters for this case are in Table 6.4. The control scenario is similar to that for the control with the mixing cell method in Chapter 4.2. Here, only the simulation results for the case where $w_{\Delta m} = 0.005$ are presented in Figures 6.12-6.13. As can be seen, with these values of the tuning parameters, a very strict control can be achieved for this ideal scenario.

An interesting phenomenon is observed in the control of the retention times. When the reference of τ_1 is changed to a higher value simultaneously with an increase of the extract purity, then τ_1 goes first in the opposite direction and after that to the new reference (65th

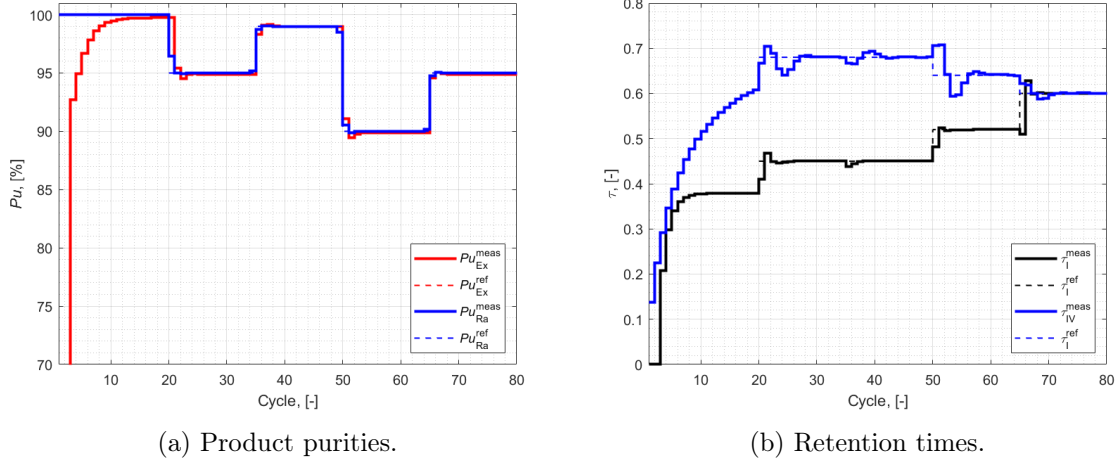


Figure 6.12: Process variables for control based on the exact internal concentration profiles for the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. Values of the tuning parameters – $w_{P_u} = 5$, $w_{\Delta m} = 0.005$, $N_{ph} = 2$, $N_{ch} = 2$.

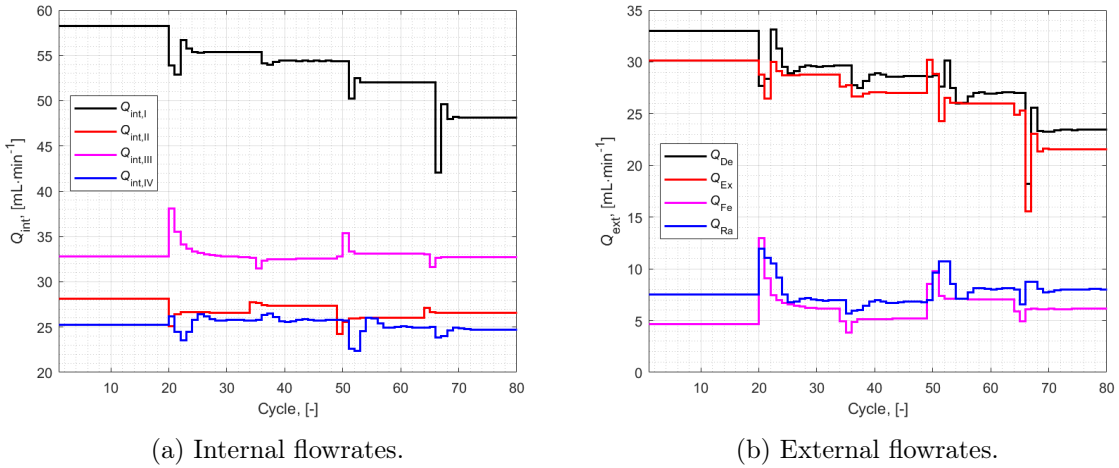


Figure 6.13: Manipulated variables for control based on the exact internal concentration profiles for the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. Values of the tuning parameters – $w_{P_u} = 5$, $w_{\Delta m} = 0.005$, $N_{ph} = 2$, $N_{ch} = 2$.

Table 6.4: Control parameters for the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms simulated with the CE/SE method.

Quantity	Value	Quantity	Value
$K_{P,I}$, [-]	-7.12	$K_{P,IV}$, [-]	3.15
$K_{I,I}$, [-]	-9.18	$K_{I,IV}$, [-]	2.23
$K_{D,I}$, [-]	-0.14	$K_{D,IV}$, [-]	1.89
N_{ph} , [-]	2	N_{ch} , [-]	2
w_{P_u} , [-]	5	$w_{\Delta m}$, [-]	0.005
Q_{int}^{min} , [mL·min ⁻¹]	1	Q_{int}^{max} , [mL·min ⁻¹]	100

cycle in Figure 6.12. Similar non-minimum phase behaviour is also observed for τ_{IV} when the reference of τ_{IV} is decreased, and at the same time, the raffinate purity is also decreased. In this case, τ_{IV} initially increases and then goes to the new lower reference (50th cycle in Figure 6.12). This has the following explanation. Since both product purities are the most important variables characterising the operations of the SMB chromatographic plant, their control is of highest priority compared to that of any other variables. In practice, this leads to the fact that first the manipulated variables $Q_{int,I}$ and $Q_{int,IV}$ for the control of τ_I and τ_{IV} are computed by the two PID controllers and after that the MPC computes the remaining manipulated variables ($Q_{int,II}$ and $Q_{int,III}$) for the purity control taking into account the already available values of $Q_{int,I}$ and $Q_{int,IV}$. For the case with the control of τ_I , to increase its value, it is necessary to decrease $Q_{int,I}$ to shift the desorption wave of the more retained component B towards the inlet of the first column. However, this decrease in internal flow rate also shifts the desorption wave of the less retained component A in the same direction, which in turn leads to a decrease in extract purity. At the same time, the extract purity reference has a new, higher value, in order to counteract the drop in the extract purity due to the change in τ_I , the predictor increases $Q_{int,II}$ resulting in the aforementioned drop of τ_I . Similar conclusions could be drawn for the τ_{IV} case. To decrease τ_{IV} it is necessary to decrease the internal flowrate in the fourth column so that the adsorption waves of both components are shifted towards the column inlet. However, this action also makes the raffinate product purer and again, to counteract that and move the raffinate to its new lower reference, the MPC increases $Q_{int,III}$, which leads to an increase of τ_{IV} . One can infer that this phenomenon shows the effects of cross-coupling in the $Q_{int,II} \rightarrow \tau_I$ and $Q_{int,III} \rightarrow \tau_{IV}$ control channels in the vicinity of the selected operating points.

6.2.3 Control based on the moving horizon estimator processing noiseless measurements

The next step is to verify whether the proposed control scheme is viable when there is limited measurement information and the internal concentration profiles are reconstructed by the moving horizon estimator with the model within it solved by the CE/SE method. The values of the manipulated variables are obtained according to Eqs. (4.4)-(4.6) and Eqs. (4.7a)-(4.7b). The values of the MPC tuning parameters are slightly changed from those in the previous subchapter in order to take into account the fact that the concen-

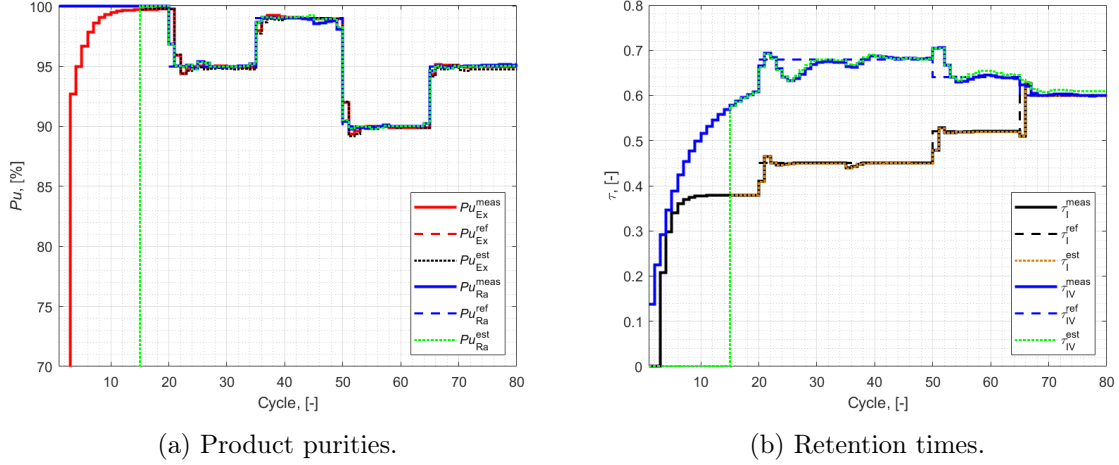


Figure 6.14: Process variables for control based on MHE processing noiseless measurements for the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. Values of the tuning parameters – $w_{P_u} = 7.5$, $w_{\Delta m} = 0.01$, $N_{ph} = 2$, $N_{ch} = 2$, $N_{eh} = 6$.

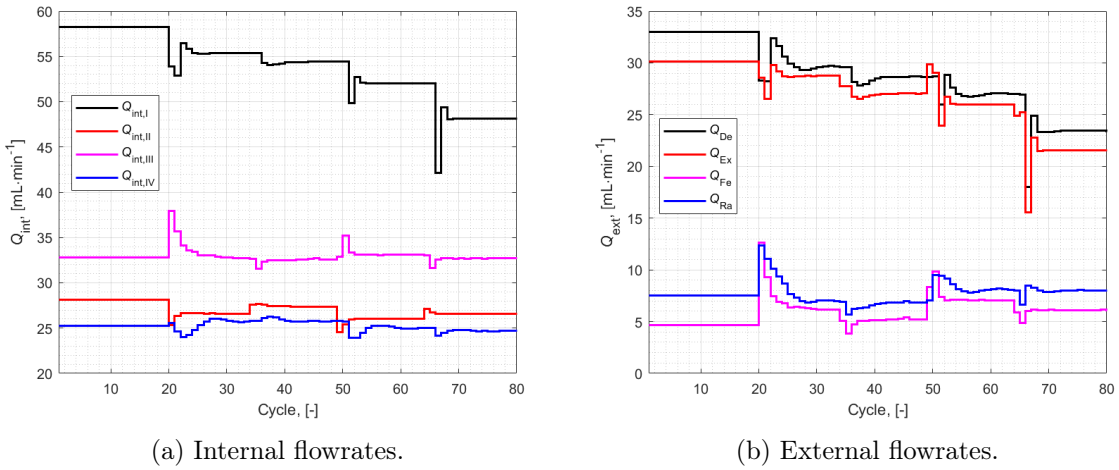


Figure 6.15: Manipulated variables for control based on MHE processing noiseless measurements for the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. Values of the tuning parameters – $w_{P_u} = 7.5$, $w_{\Delta m} = 0.01$, $N_{ph} = 2$, $N_{ch} = 2$, $N_{eh} = 6$.

tration profiles are obtained from the MHE, which processes ideal noiseless measurements from the measuring devices. The controller is made less aggressive by increasing $w_{\Delta m}$, and more attention is paid to raffinate purity by increasing w_{P_u} . The results are presented in Figures [6.14](#)–[6.15](#). Because the internal concentration profiles are estimated with high quality, especially in zones I and III, the predictor is able to control both product purities better than in the case with the mixing cell model. The most significant control error for the product purities under cyclic steady state conditions is below 0.5 % in the 45th cycle. The aforementioned behaviour of the dimensionless retention times in cycles 50 and 65 is also preserved here.

6.2.4 Control based on the moving horizon estimator processing noisy measurement

Finally, the most interesting case from a practical point of view is considered: noisy measurements with the same additive noise of 1 % as in mixing the cell model case and an MHE whose model is solved using the CE/SE method. Again, the derivative part of the PID controller uses the measured dimensionless retention times signal filtered by a moving-average filter with a window length of 6 cycles in the past (Eqs. (4.8)-(4.9)). The results are shown in Figures 6.14-6.17. The control error for both product purities is below 1 % for all cycles, even for the most demanding conditions after the controllers were turned on at the 20th cycle and the most considerable change in references after the 50th cycle, when the product purity references change from 99 to 90 %.

In the following subchapter, the problem of disturbance rejection shall be discussed.

6.2.5 Simultaneous state and parameter estimation. Estimator validation

In addition to ensuring stable operation of the SMB plant during tracking of the references, disturbance rejection is the other main objective of the control system. The task of reference tracking was presented in the previous subsection. In the present subchapter and the following one, the problem of disturbance rejection is considered. As already mentioned, the most common disturbances associated with SMB processes are changes in

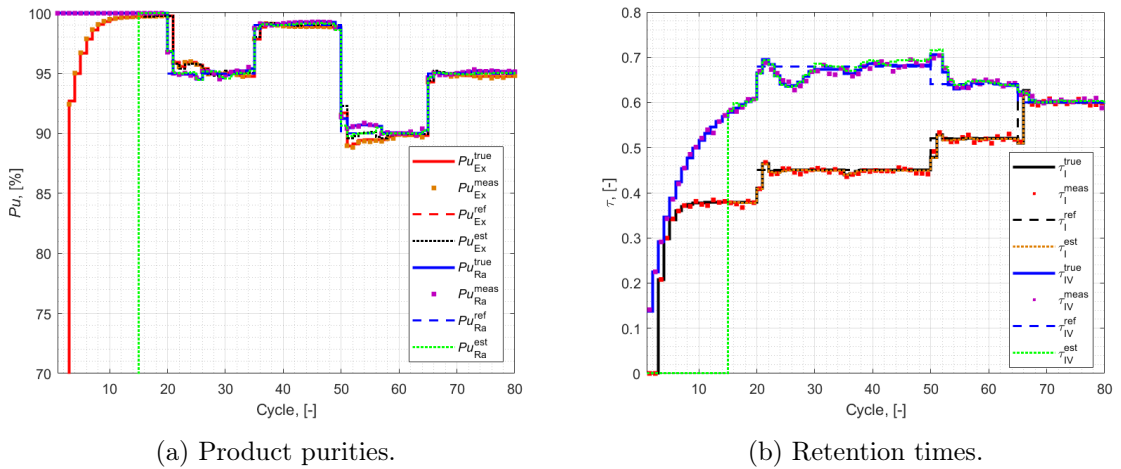


Figure 6.16: Process variables for control based on MHE processing measurements with additive measurement noise for the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. Values of the tuning parameters – $w_{P_u} = 7.5$, $w_{\Delta m} = 0.01$, $N_{ph} = 2$, $N_{ch} = 2$, $N_{eh} = 6$.

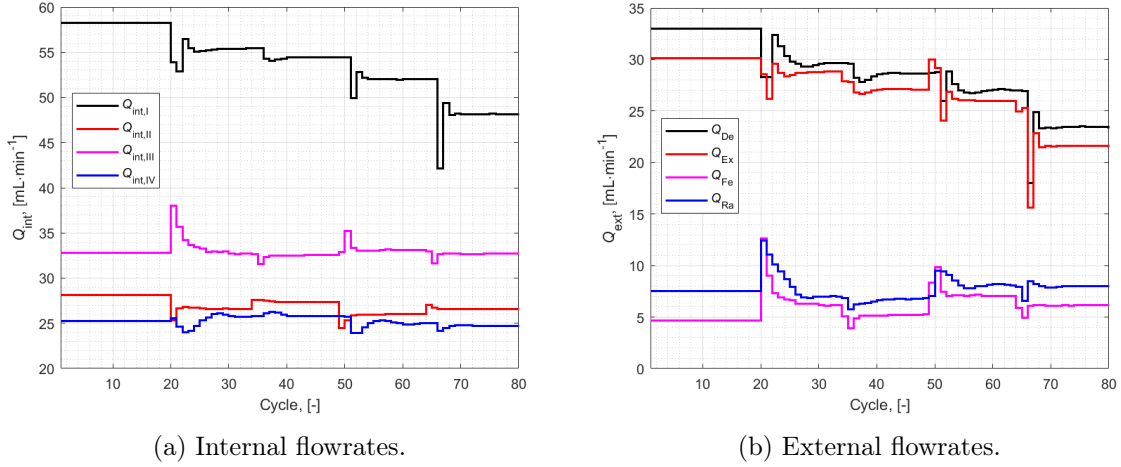


Figure 6.17: Manipulated variables for control based on MHE processing measurements with additive measurement noise for the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms simulated with the CE/SE method. Values of the tuning parameters – $w_{Pu} = 7.5$, $w_{\Delta m} = 0.01$, $N_{ph} = 2$, $N_{ch} = 2$, $N_{eh} = 6$.

the parameters of the adsorption isotherms due to adsorbent ageing or changes in ambient temperature, as well as feed concentrations. The parameters of adsorption isotherms are among the most important quantities required for the design and operation of SMB processes. Their values are obtained experimentally using either retention time or frontal analysis methods. The problem is that the values obtained by these methods are not precise, and the measurements are slow. Moreover, the values of the adsorption isotherm parameters change during the operation of the SMB plant.

The parameter whose influence will be investigated in this work is the adsorption Henry coefficient, H . In order to deal with disturbances due to changes in H , the MPC must know its value in real-time. Because of the aforementioned reasons, these values are generally not available by measurements. Therefore, they must be estimated online from the available measurements. For this purpose, the proposed MHE will be extended; therefore, in addition to the coefficients α , β , and γ , it will also estimate H .

First, let us see how the change in H affects the operating regions in the triangular diagrams. Figure 6.18 shows how the separation and regeneration regions change when the Henry coefficient of the less retained component A is changed while that of the more retained component B is kept constant. The coloured areas correspond to the initial state, and the bold black curves show the boundaries of the individual regions after the corresponding change in H_A . When H_A decreases, the vertical boundary of the complete separation region ($m_{II} - m_{III}$ diagram) shifts to the left. Thus, this region expands. Conversely, when H_A increases, the boundary moves to the right, and the total separation

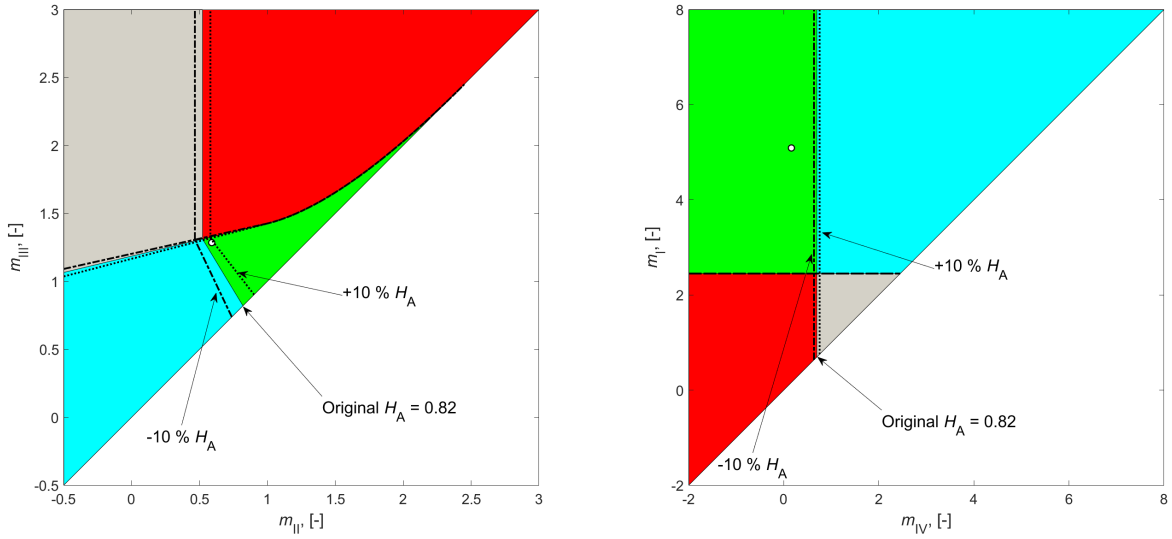


Figure 6.18: Triangle diagrams showing the change of the separation and regeneration regions as a function of the adsorption Henry coefficient of the less retained component A, H_A , for a system with Langmuir adsorption isotherms. H_B , b_A , b_B , $C_{A,Fe}$ and $C_{B,Fe}$ are kept constant. Boundaries of the separation regions at: dotted curves $+10\% H_A$; dash-dotted curve $-10\% H_A$. White circle – operating point at $m = [5.09, 0.59, 1.28, 0.16]^T$.

region shrinks accordingly. The total regeneration region ($m_{IV} - m_I$ diagram) becomes smaller when H_A decreases since the vertical boundary of this region moves to the left. Furthermore, when H_A increases, the boundary moves to the right, and the total separation region becomes bigger. The influence of H_B , while H_A is kept constant, is demonstrated in Figure 6.19. When H_B decreases, the horizontal boundary moves down, and the region of total separation ($m_{II} - m_{III}$ diagram) shrinks; when H_B increases, the boundary moves up and the region of total separation expands. The region of total regeneration ($m_{IV} - m_I$ diagram) becomes bigger when H_B decreases, and vice versa; when H_B increases, the region of total regeneration decreases because the horizontal boundary of the total regeneration region moves down or up, respectively. That is, if the values of two Henry coefficients approach each other (H_A increases and H_B decreases), then the relative selectivity, $\alpha_{rel} = \frac{H_B}{H_A}$, decreases, which in turn makes separation more difficult. This is represented graphically by the reduction of the total separation region in the $m_{II} - m_{III}$ diagram.

It is worth mentioning that if the operating point is in the vicinity of the tip of the triangle (in this case $m = [5.09, 0.59, 1.28, 0.16]^T$), even changes smaller than 10 % in any of the Henry coefficients could shift the boundaries so that the operating point ends up outside the total separation region.

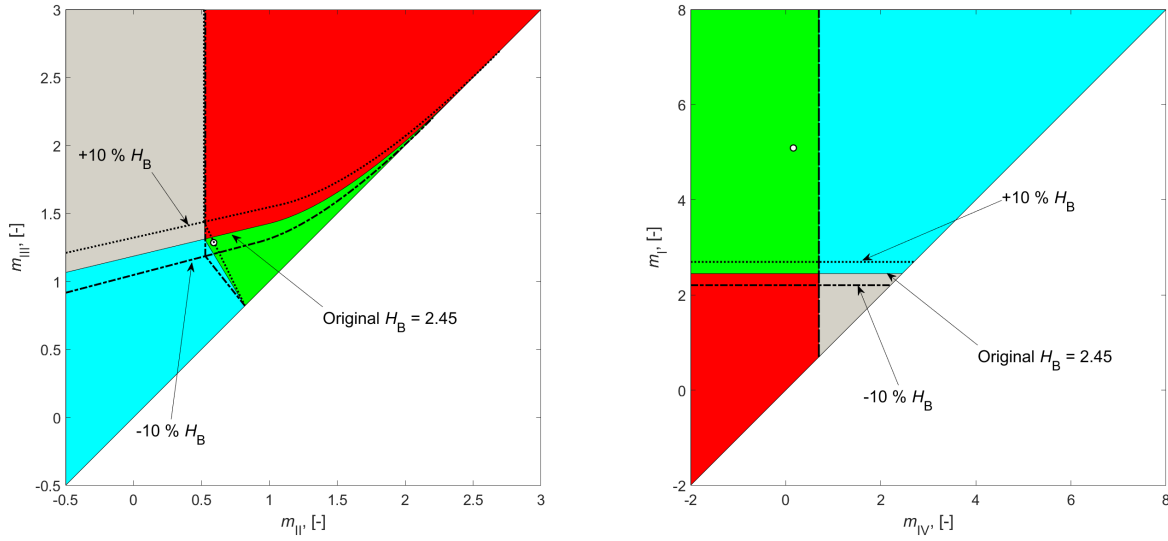


Figure 6.19: Triangle diagrams showing the change of the separation and regeneration regions as a function of the adsorption Henry coefficient of the more retained component B, H_B , for a system with Langmuir adsorption isotherms. H_A , b_A , b_B , $C_{A,Fe}$ and $C_{B,Fe}$ are kept constant. Boundaries of the separation regions at: dotted curves +10 % H_B ; dash-dotted curve -10 % H_B . White circle – operating point at $m = [5.09, 0.59, 1.28, 0.16]^T$.

To estimate the adsorption Henry coefficients, the original MHE problem is modified by adding H in addition to the coefficients α , β , and γ used to reconstruct the internal concentration profiles. To estimate the adsorption Henry coefficients, the original MHE problem is modified by adding H in addition to the coefficients α , β , and γ used to reconstruct the internal concentration profiles. Since that extension of the MHE does not change its structure, the addition of new parameters for estimation, in this case H , is admissible. Accordingly, the estimation horizon, N_{eh} , has to reflect this change in the number of estimated parameters, by increasing its length whenever necessary. There are two possible variants for the value of H as an optimisation variable in each sample of the estimation horizon. The first variant is H to assume a constant for the entire estimation horizon, that is, having only one H , and the second variant is H to vary for each sample of the estimation horizon, that is, having as many H s as the length of the estimation horizon. Both cases are considered in the open-loop validation of the MHE parameter estimator.

For the variant with constant H the MHE optimisation problem becomes

$$\min_{\alpha_{i,p}^{liq}, \alpha_{i,p}^{sol}, \beta_{i,p}, \gamma_{i,p}, H_{i,r}} J_{MHE} = \sum_{k=r-N_{eh}+1}^r (\mathbf{y}_k^{\text{meas}} - \mathbf{y}_k^{\text{est}})^T \mathbf{P}_k (\mathbf{y}_k^{\text{meas}} - \mathbf{y}_k^{\text{est}}), \quad (6.3a)$$

$$\begin{aligned}
\text{s.t. } & \alpha_{i,p}^{\text{liq, min}} \leq \alpha_{i,p}^{\text{liq}} \leq \alpha_{i,p}^{\text{liq, max}}, \\
& \alpha_{i,p}^{\text{sol, min}} \leq \alpha_{i,p}^{\text{sol}} \leq \alpha_{i,p}^{\text{sol, max}}, \\
& \beta_{i,p}^{\text{min}} \leq \beta_{i,p} \leq \beta_{i,p}^{\text{max}}, \\
& \gamma_{i,p}^{\text{min}} \leq \gamma_{i,p} \leq \gamma_{i,p}^{\text{max}}, \\
& H_{i,r}^{\text{min}} \leq H_{i,r} \leq H_{i,r}^{\text{max}}, \\
& i = \text{A, B, } \dots, N_{\text{comp}}; \quad p = \text{I, II, } \dots, N_{\text{col}}.
\end{aligned} \tag{6.3b}$$

Thus, in addition to the 32 variables (α , β and γ) of the sigmoid functions approximating the internal concentration profiles at the beginning of the estimation horizon, two adsorption Henry coefficients, H , are added, one for each of the components, thus making the total number of estimated parameters 34. In such a case, with 6 measurements per cycle, the minimum length of the estimation horizon has to be 6 cycles according to the Hautus criterion.

The simulation scenario for the open-loop validation is as follows. The SMB plant is started with an operating point near the tip of the triangle, then in the 20th cycle, the MHE is turned on, initialised with the actual values of H s. All initial values for α , β , γ , and H , as well as their lower and upper bounds, are given in Appendix B. After that, for each subsequent cycle, their initial values are the optimal ones from the previous one. In the 48th cycle, the value of H_A increases by 10 % from 0.82 to 0.902, and then in the 55th cycle, the value of H_B decreases by 10 % from 2.45 to 2.205. In real practice, due to the change of the ambient temperature or due to the ageing of the adsorbent, its adsorption properties change for all components of the separated mixture. Therefore, a case of simultaneous change of H for both components is also considered; this happens in the 65th cycle when H_A increases again by 10 % from its initial value to 0.984, and H_B returns back to its initial value of 2.45. The scenario is summarised in Table 6.5, and the respective separation zones can be seen in Figure 6.20. The simulation results are

Table 6.5: Values of the actual adsorption Henry coefficients used in the open-loop moving horizon parameter estimator validation.

Label	Cycles, k	H_A , [-]	H_B , [-]
(A)	1 - 47	0.82	2.45
(B)	48 - 54	0.902	2.45
(C)	55 - 64	0.902	2.205
(D)	65 - 75	0.984	2.45

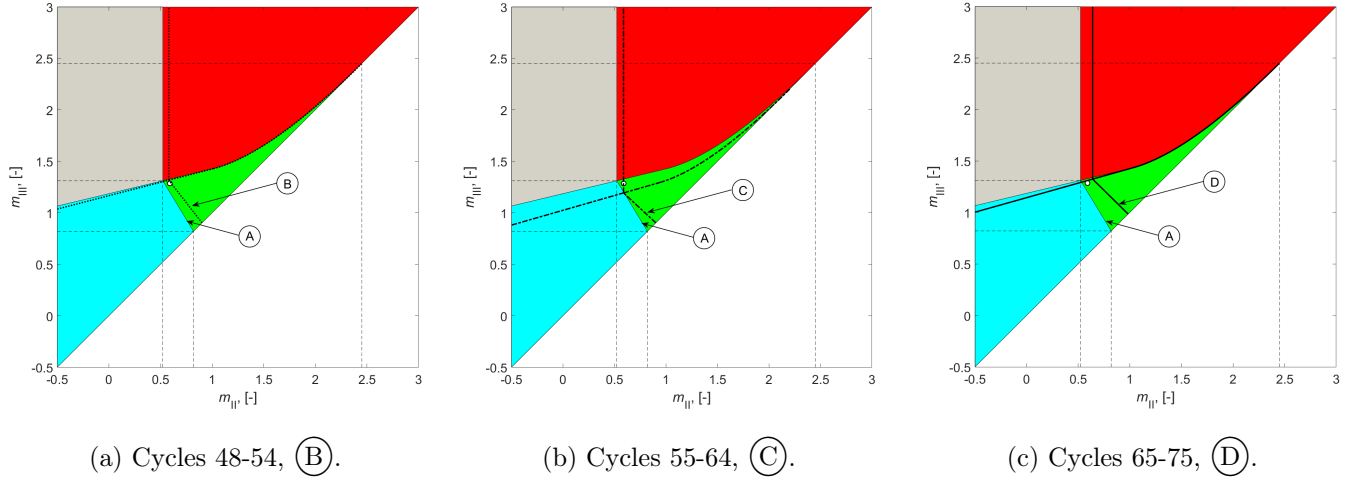


Figure 6.20: Triangle diagrams showing the separation regions ($m_{II} - m_{III}$ plane) and their boundaries for each group of cycles of the open-loop moving horizon parameter estimator validation. Coloured areas and the thin solid curves show the initial separation zones for cycles 1-47, (A); bold curves show the separation boundaries for the respective cycles. The exact values of H corresponding to each group of cycles can be seen in Table 6.5. White circle – operating point at $m = [5.09, 0.59, 1.28, 0.16]^T$.

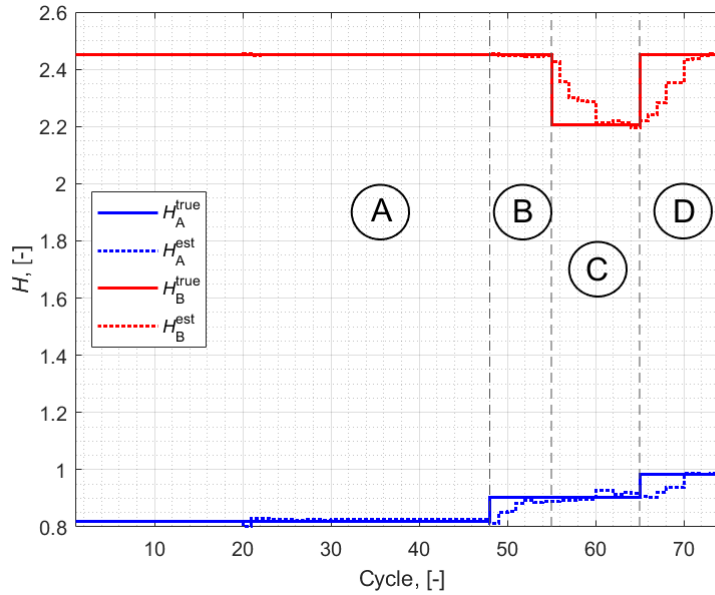


Figure 6.21: Comparison between true and estimated adsorption Henry coefficients for the variant with constant H within the estimation horizon for the open-loop validation of the extended MHE. Solid curves – true Henry coefficients, dashed curves – estimated ones. $N_{eh} = 6$.

presented in Figures 6.21-6.22

The presented results are when using the minimum evaluation horizon of 6 cycles. As can be seen, the MHE manages to track the change of both H with some delay. In the best case, this delay is expected to be N_{eh} cycles, which is also confirmed by the results in Figures 6.21-6.22. The explanation is that the estimated values of the Henry coefficients,

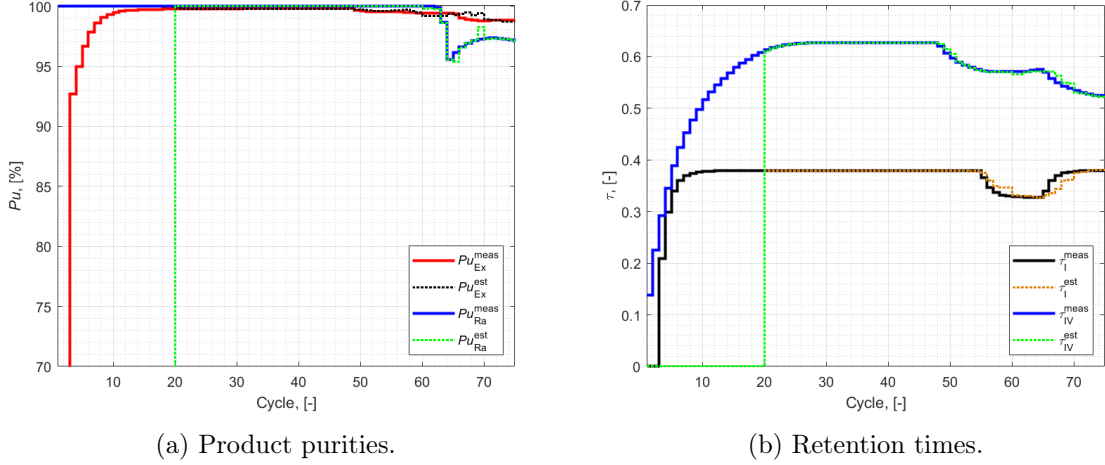


Figure 6.22: Comparison between true and estimated process variables for the variant with constant H within the estimation horizon for the open-loop validation of the extended MHE. $N_{eh} = 6$.

H^{est} , slowly creep cycle after cycle toward the new actual values of H and only when there are only measurements with the new values of H within the entire evaluation horizon, it is possible for the MHE to reach the new state. For example, when H_B is changed by -10 % in the 45th cycle, 6 cycles (inclusive from the 45th to 50th cycles) are necessary for H_B^{est} to converge to the new value of H_B of 2.205. Of course, this delay in estimating H ultimately affects the estimation of other quantities, such as product purity and retention times. The subsequent use of these estimates of H_i for control purposes by the MPC could lead to an even greater delay in the purity regulation.

The optimisation problem for the variant with varying H for each cycle within the estimation horizon has the most general form and is

$$\min_{\alpha_{i,p}^{liq}, \alpha_{i,p}^{sol}, \beta_{i,p}, \gamma_{i,p}, H_{i,g}} J_{MHE} = \sum_{k=r-N_{eh}+1}^r (\mathbf{y}_k^{meas} - \mathbf{y}_k^{est})^T \mathbf{P}_k (\mathbf{y}_k^{meas} - \mathbf{y}_k^{est}), \quad (6.4a)$$

$$\begin{aligned} \text{s.t. } & \alpha_{i,p}^{liq, \min} \leq \alpha_{i,p}^{liq} \leq \alpha_{i,p}^{liq, \max}, \\ & \alpha_{i,p}^{sol, \min} \leq \alpha_{i,p}^{sol} \leq \alpha_{i,p}^{sol, \max}, \\ & \beta_{i,p}^{\min} \leq \beta_{i,p} \leq \beta_{i,p}^{\max}, \\ & \gamma_{i,p}^{\min} \leq \gamma_{i,p} \leq \gamma_{i,p}^{\max}, \\ & H_{i,g}^{\min} \leq H_{i,g} \leq H_{i,g}^{\max}, \end{aligned} \quad (6.4b)$$

$$i = A, B, \dots, N_{comp}; \quad p = I, II, \dots, N_{col}; \quad g = r - N_{eh} + 1, r - N_{eh} + 2, \dots, r.$$

In this formulation of the problem, there are $32 + N_{comp} N_{eh}$ optimisation variables, or for binary separation, this leads to $32 + 2N_{eh}$ variables. To satisfy the Hautus criterion with

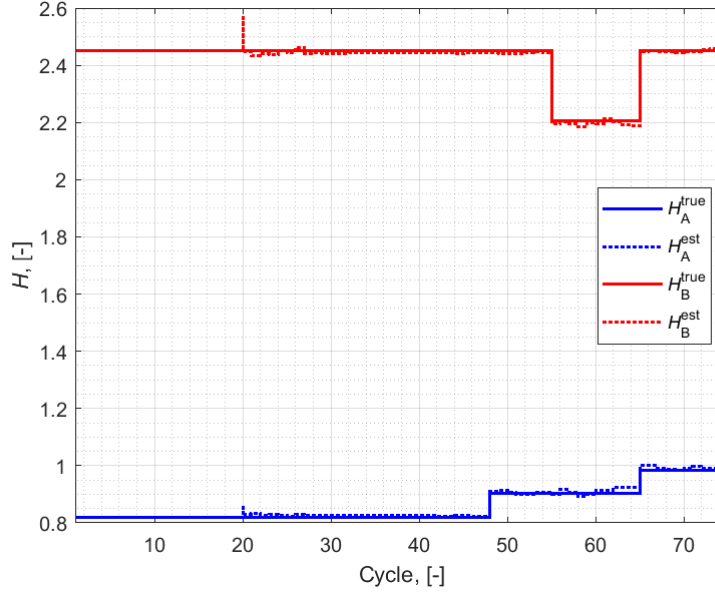


Figure 6.23: Comparison between true and estimated adsorption Henry coefficients for the variant with varying H within the estimation horizon for the open-loop validation of the extended MHE. Solid curves – true Henry coefficients, dashed curves – estimated ones. $N_{\text{eh}} = 8$.

6 measurements per cycle, the minimum length of the evaluation horizon needs to be 8 cycles because then the number of optimisation variables is $32 + 2 \cdot 8 = 48$, exactly the same as the total number of measurements for these 8 cycles.

The simulation scenario for open-loop validation of the MHE with the varying H is similar to the previous variant with the constant H with just one exception. Since the exact value of Henry coefficients is difficult to measure precisely, the initial values of H for the cycle in which the MHE is turned on are 5 % more than the actual ones to test whether the MHE is capable of finding the actual ones when initialised with uncertain information. Thus, the optimisation procedure starts with initial values of $H_{A,0}^{\text{est}} = 0.961$ and $H_{B,0}^{\text{est}} = 2.573$ instead of 0.82 and 2.45, respectively. The simulation results are presented in Figures [6.23](#)–[6.24](#).

As can be seen in Figure [6.23](#), the MHE is able to overcome the false initial information for both H . Furthermore, since the adsorption Henry coefficients can take different values for each cycle within the estimation horizon, the MHE optimisation procedure can adjust them to track sudden changes in the actual H in the same cycle in which these changes occur. The results are with $N_{\text{eh}} = 8$. The improved estimation of H also leads to a much better estimation of product purities and retention times (Figure [6.24](#)).

In summary, the MHE variant with a constant value of H requires shorter estimation

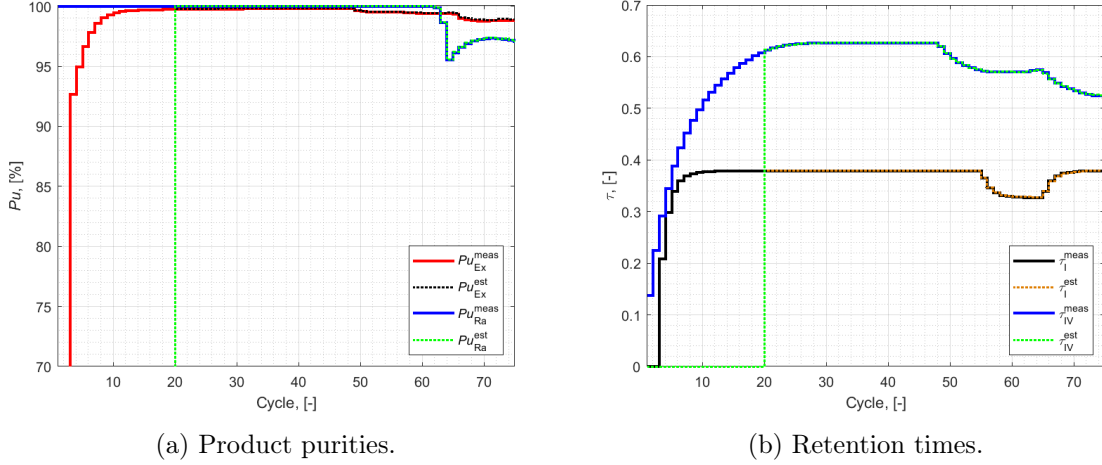


Figure 6.24: Comparison between true and estimated process variables for the variant with varying H within the estimation horizon for the open-loop validation of the extended MHE. $N_{eh} = 8$.

horizons compared to the one with a varying H , which in the most general case means an easier and faster optimisation procedure. However, it cannot respond quickly to changes in actual Henry coefficients, which is crucial for the use of the estimated H in future online optimising control. Therefore, in the following subsection, which will be devoted to the control based on the estimated H , the MHE with a varying H within the estimation horizon will be used. It is also worth mentioning that Henry coefficients can be substituted as estimation variables by any other parameter, e.g. retention factors, b , or feed concentrations, C_{Fe} , or even these parameters to be estimated together with H , thus the controller will have information about more of the possible disturbances acting on the SMB plant.

6.2.6 Control based on the moving horizon estimator including parameter estimation

The following simulation scenario is used to investigate the ability of the proposed control scheme to reject disturbances resulting from changes in the adsorption Henry coefficients. The SMB plant is started in open-loop mode with the operating point near the tip of the triangle in the complete separation region and with empty columns. In the 20th cycle, the moving horizon parameter estimator is turned on; in the 26th cycle, the control loops are closed with references of 95 % for both product purities (after which they do not change any more until the end of the simulation) and 0.45 and 0.60 for τ_I and τ_{IV} , respectively. After that, the reference of τ_I is changed twice, first to 0.53 in the 35th cycle and then to 0.6 in the 40th cycle. In this way, the references of the two retention times

are equalised to a value of 0.6 and do not change any more until the end of the simulation. After the SMB plant has already reached the desired state, the change in the actual values of the Henry coefficients begins. First, H_A changes its value by +10 % from the initial value of 0.82 to 0.902 in the 48th cycle. Then H_B changes with -10 % from its initial value of 2.45 to 2.205 in the 55th cycle. Finally, in order to study the behaviour of the control system when both Henry coefficients change simultaneously, they are changed in the 65th cycle; H_A increases by another 10 % to 0.984, and H_B returns to its initial value of 2.45. The simulation scenario is summarised in Table 6.6

Table 6.6: Change of the references and the values of the actual adsorption Henry coefficients used in the disturbance rejection control study of the semi-centralised control scheme based on the extended MHE.

Cycles, k	Pu_{Ex}^{ref} , [%]	Pu_{Ra}^{ref} , [%]	τ_I^{ref} , [-]	τ_{IV}^{ref} , [-]	H_A , [-]	H_B , [-]
1 - 25	Open loop				0.82	2.45
26 - 34	95	95	0.45	0.60	0.82	2.45
35 - 39	95	95	0.53	0.60	0.82	2.45
40 - 47	95	95	0.60	0.60	0.82	2.45
48 - 54	95	95	0.60	0.60	0.902	2.45
55 - 64	95	95	0.60	0.60	0.902	2.205
65 - 75	95	95	0.60	0.60	0.984	2.45

As discussed in the previous subsection, the MHE variant with varying values of H within the evaluation horizon detects changes in Henry coefficients significantly faster compared to the variant with a constant H , which is why the former is used in the proposed control scheme. The optimisation problem is the same as in the case with the reference tracking control (Eqs. (4.4)-(4.6) and Eqs. (4.7a)-(4.7b)), and the MPC is retuned in order to counteract more quickly on the disturbances by slightly increasing w_{Pu} and decreasing $w_{\Delta m}$. The value of the control parameters is given in Table 6.7.

Table 6.7: Control parameters for the disturbance rejection study of the semi-centralised control scheme based on the extended MHE.

Quantity	Value	Quantity	Value
$K_{P,I}$, [-]	-7.12	$K_{P,IV}$, [-]	3.15
$K_{I,I}$, [-]	-9.18	$K_{I,IV}$, [-]	2.23
$K_{D,I}$, [-]	-0.14	$K_{D,IV}$, [-]	1.89
N_{ph} , [-]	2	N_{ch} , [-]	2
w_{Pu} , [-]	9	$w_{\Delta m}$, [-]	0.008
Q_{int}^{min} , [mL·min ⁻¹]	1	Q_{int}^{max} , [mL·min ⁻¹]	100

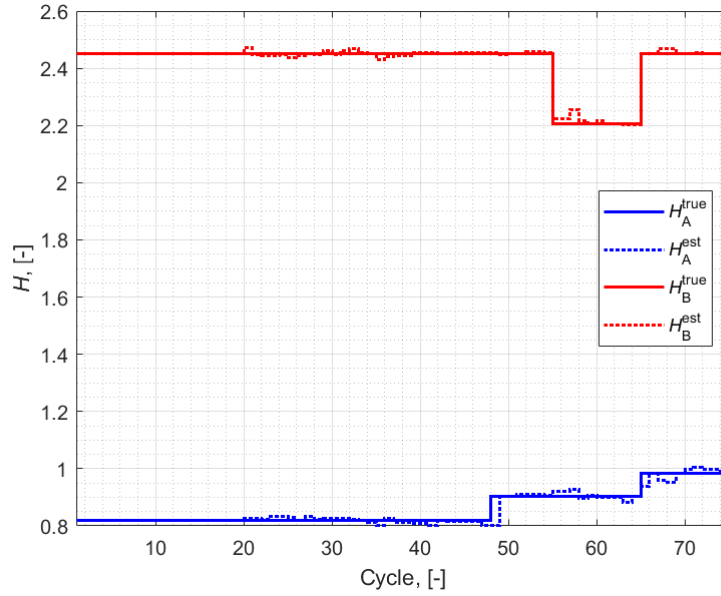
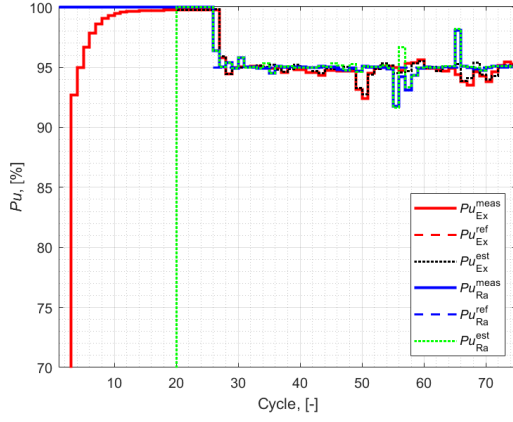
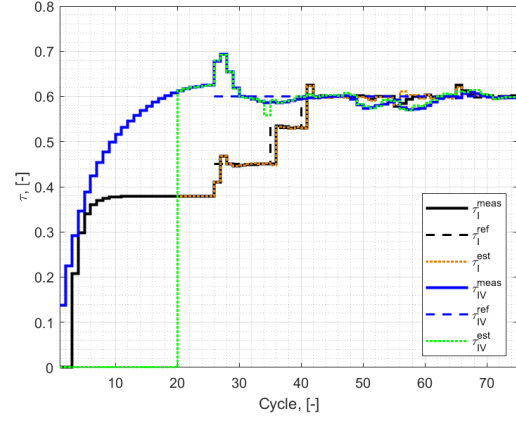


Figure 6.25: Adsorption Henry coefficients evolution for the disturbance rejection control study with the extended MHE in the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms. Solid curves – true Henry coefficients, dashed curves – estimated ones by the MHE and used by the MPC. $N_{eh} = 8$.

The simulation results are shown in Figures 6.25-6.28. As can be seen in Figure 6.26a, the change in H_A (less retained component) triggers a change in the extract purity, and the change in H_B (more retained component) triggers a change in the raffinate purity. Of course, the simultaneous change of both Henry coefficients leads to a change in both product purities. It is also worth noting that at this operating point, the 10 % change of H_B has a more substantial effect on the product purities than the identical change of H_A . Regardless of the case, the NMPC is able to overcome all of these disturbances of the Henry coefficients in just a few cycles with very low control activity (Figure 6.27) due to the fast estimation of H_i . The two PID controllers also successfully maintain the two

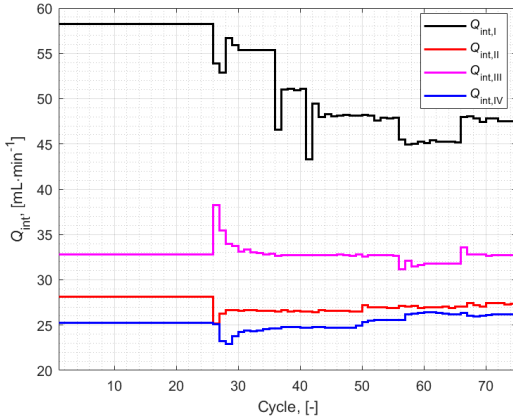


(a) Product purities.

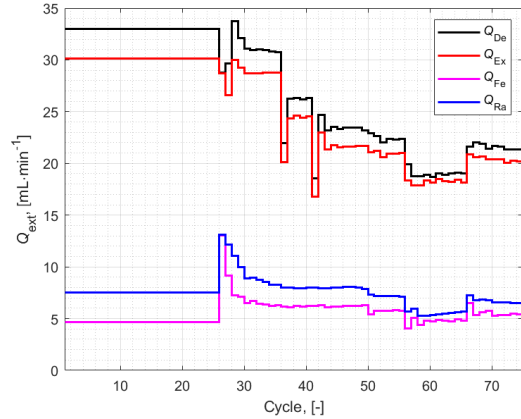


(b) Retention times.

Figure 6.26: Process variables for the disturbance rejection control study with the extended MHE in the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms. Values of the tuning parameters – $w_{P_u} = 9$, $w_{\Delta m} = 0.008$, $N_{ph} = 2$, $N_{ch} = 2$, $N_{eh} = 8$.



(a) Internal flowrates.



(b) External flowrates.

Figure 6.27: Manipulated variables for the disturbance rejection control study with the extended MHE in the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms. Values of the tuning parameters – $w_{P_u} = 9$, $w_{\Delta m} = 0.008$, $N_{ph} = 2$, $N_{ch} = 2$, $N_{est} = 8$.

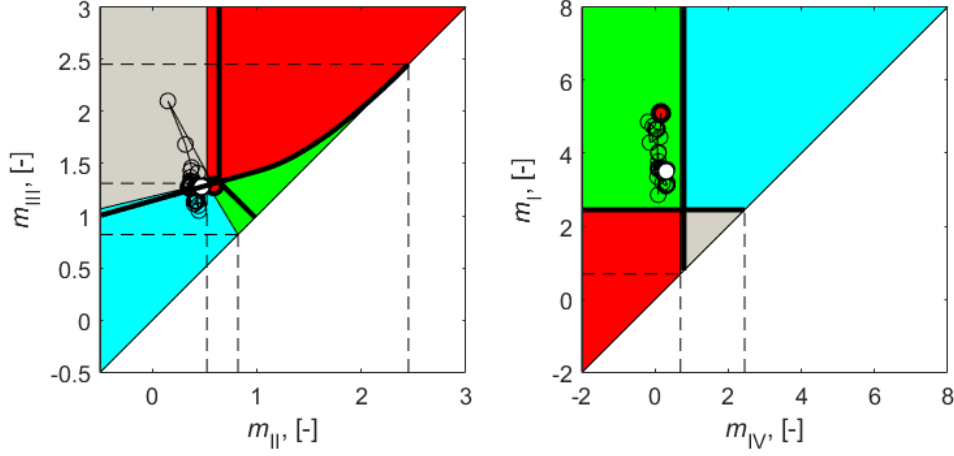


Figure 6.28: Triangle diagrams with the change of the operating point of the SMB plant for the disturbance rejection control study with the extended MHE in the semi-centralised control scheme of SMB process with Langmuir adsorption isotherms. Red circle – initial operating point; white circle - operating point at the last 75th cycle. Coloured areas – initial separation and regeneration regions defined by the thin curves; regions defined by the bold black curves are after the last change of the Henry coefficients, i.e. for the cycles 65-75.

retention times (Figure 6.26b), and again τ_{IV} is the more challenging of the two retention times. One should keep in mind that such severe step changes of the Henry coefficients in the real practice of the SMB plant are highly unlikely, regardless of the reason for these changes: temperature changes, temperature fluctuations, or ageing of the adsorbent. The expected changes are gradual and have magnitudes smaller than the investigated ones.

Unfortunately, no set of tuning parameters for the case with additive measurement noise has been found so far when simultaneous state and parameter estimation, and control are considered. The mutual interaction between the controller and estimator and the high sensitivity of the considered process with steep concentration fronts leads to the fact that the MHE optimisation procedure cannot find a feasible solution to the estimation problem. A possible way to overcome this issue is to measure the adsorption Henry coefficients precisely before operating the SMB plant and use these values as initial estimates of H_i for the MHE optimisation, and to use a more advanced optimisation method.

Chapter 7

Conclusions and perspectives

7.1 Concluding remarks

This thesis aims to investigate in-silico different control schemes for the binary SMB chromatographic process, focusing primarily on the nonlinear region of Langmuir adsorption isotherms, both for complete and incomplete separation. For this purpose, a modular simulation tool was initially written in MATLAB, which allows in silico investigation of various chromatographic processes ranging from the simple batch process to the binary four-zone SMB and the ternary centre-cut eight-zone SMB, for systems with linear and Langmuir adsorption isotherms. The numerical algorithms used in this simulation tool are the mixing cell model and the CE/SE method. It is possible to perform open-loop simulations for the purposes of model design and offline optimisation of chromatographic processes, and to add closed-loop control as well.

In a previous study conducted by Paul Suvarov in Mons and Magdeburg, a control scheme based on single-loop PI controllers for feedback and a parameter estimator for forward action was proposed. This simple control scheme proved quite powerful for controlling SMB processes in the linear region of adsorption isotherms when solutes are highly diluted. However, when working with systems featuring nonlinear adsorption isotherms (such as Langmuir or bi-Langmuir), at high feed concentrations in the nonlinear region of the isotherms, the proposed control scheme struggled to control the process as desired. Cross-coupling in the control channels cannot be ignored any longer. Therefore, the starting step in the current study was to check whether the use of centralised control with simple decoupling would improve the performance of this control scheme. For this purpose, a con-

trol scheme with classical PID controllers and two static decouplers was considered. The utilised numerical method was the mixing cell model. The decouplers were included in the control loops of the separation zones of the SMB plant, specifically in the $Q_{\text{int,II}} \rightarrow Pu_{\text{Ra}}$ and $Q_{\text{int,III}} \rightarrow Pu_{\text{Ex}}$ channels. The results of this MIMO control showed that the control system performs well only in the vicinity of the operating point for which the controllers and the two decouplers were tuned beforehand. When it is necessary to operate away from the initial operating point, both increased settling time and the appearance of oscillations were observed, i.e. the strong nonlinearities of the process are crucial for the reduced performance of the control system. This issue can be overcome by using a gain-scheduling control approach or by introducing adaptive or self-tuning mechanisms to adjust the parameters of the control system. However, this would negate the main advantage of this approach, namely its simplicity. Therefore, as a next step, a control scheme based on model predictive control utilising limited measurement information was considered.

Then, a semi-centralised system combining online optimisation control and linear controllers was considered for the control of the SMB process with Langmuir adsorption isotherms in the nonlinear region. Precisely, two single loops with PID controllers for controlling the dimensionless retention times in the two regeneration zones I and IV and centralised control in the two separation zones II and III with a nonlinear MPC for controlling the purity of the two products. Since product purities have a higher priority than retention times, the control action of the PID controllers is first computed, i.e. $Q_{\text{int,I}}$ and $Q_{\text{int,IV}}$, and then, taking into account these values, the MPC controller performs the online optimisation procedure to find the other two control actions $Q_{\text{int,II}}$ and $Q_{\text{int,III}}$ for the current cycle. The applied cost function is the classic one for MPC control: the difference between the reference and the measured value of the product purity is minimised, plus a term penalising the rate of change of the control actions, and it is subject to inequality constraints from the external material balance of the SMB process. These inequality constraints include the values of $Q_{\text{int,I}}$ and $Q_{\text{int,IV}}$ calculated by the PID controllers. Thus, the MPC controller has only four tuning parameters: the two horizons and the two weight factors in the cost function. Finding appropriate values for these tuning parameters is a time-consuming procedure, starting with values from the case of linear adsorption isotherms.

The internal concentration profiles at the beginning of the current cycle are required

as input information for the predictor model. In practice, these profiles cannot be obtained with the scarce measurement information. Therefore, as a first step toward the practical implementation of the semi-centralised control scheme, a hypothetical case study was considered in which an array of measuring devices along the length of the SMB plant provides these internal concentration profiles without measurement noise. The scenario studied included open-loop start-up of the SMB plant, followed by closed-loop reference tracking control for total and reduced-purity separations, with a step change in the purity references of 10 %. The results showed that the performance of this control scheme for reference tracking is excellent. The next step was to add an estimator to the control scheme to reconstruct the required internal concentration profiles from the available measurement information in real time. The applied estimator is an online moving horizon estimator. The sampling interval for both MHE and MPC is the same and is one cycle of the SMB plant. For the initial approximation of the internal concentration profiles in the online optimisation procedure of the MHE, sigmoidal approximation functions of logistic type were considered, with their parameters being minimised. The results of the in-silico studies with MHE processing, both ideal and noisy measurements, show very good performance of this control scheme (PID + MHE/MPC). In all studies, the numerical method used was the mixing cell model with 101 cells per chromatographic column. With this number of equilibrium cells, the bottleneck in terms of required computational time was the MHE optimisation routine. For comparison, MPC optimisation took about 20 seconds on average, while MHE optimisation took about 17 minutes, which is significantly longer than the 140-second duration of a single SMB cycle. Moreover, due to the numerical diffusion of this method, a large number of equilibrium cells (at least 800 per column) is required to resolve the steep concentration fronts and shocks typical for nonlinear chromatography. Therefore, an alternative numerical method was sought out to solve the SMB chromatographic model, enabling a significant reduction in the computational time of the MHE and MPC online optimisation routines. One such method is the CE/SE method.

The CE/SE method is an explicit, fully discretised time-marching scheme for solving PDEs based on an integral formulation of conservation laws. Therefore, it can resolve steep concentration fronts very efficiently. To evaluate the efficiency of this method compared to the mixing cell model in terms of the computational time required to achieve the same degree of resolution of the concentration profiles, comprehensive simulation studies

were conducted on various chromatographic processes ranging from simple batch chromatographic process to ternary 8-zone centre-cut SMB processes for systems with linear and Langmuir adsorption isotherms and formulated by the ideal equilibrium model and the LDF model. The results showed that the CE/SE model was faster than the mixing cell model by 30 times (for a binary SMB process described by the LDF model with Langmuir isotherms) to 348 times (for a ternary centre-cut SMB process described by the LDF model with Langmuir isotherms). That fact made the CE/SE method an ideal candidate for a numerical method for solving the chromatographic PDAE model in the online optimisation routines of MHE and MPC.

To verify the applicability of the CE/SE method for control purposes, this method was first used to control a binary SMB process with linear adsorption isotherms. The control scheme is based on MPC without MHE, i.e., it is assumed that the internal concentration profiles are available to the controller. The simulation scenario includes both reference tracking and disturbance rejection control. The results showed that this control scheme performs excellently in the reference tracking case, but, as expected, it was not able to completely overcome disturbances because the predictor model was not updated with information about them.

The application of the CE/SE method to the previously presented semi-centralised control scheme for binary SMBs with Langmuir isotherms in the nonlinear region shows that this approach is highly effective in reference tracking control and outperforms the same control scheme using the mixing cell model as a numerical tool. That applies to each of the cases: control based on exact internal concentration profiles, and reconstructed profiles by the MHE processing, both ideal and noisy measurements. The CPU time for the MPC optimisation procedure was about 3 seconds, while that for the MHE was less than 140 seconds, yielding very well-resolved steep concentration profiles.

Changes in adsorption Henry coefficients H in the adsorption isotherms due to temperature fluctuations or adsorbent ageing are among the most common disturbances in the operation of SMB plants. Unfortunately, they are very challenging to measure online. Therefore, information about them is crucial to the control system's performance. Finally, to verify the disturbance rejection capability of the control scheme, the ability to evaluate H has also been added to the MHE. Two possible ways of evaluating H have been discussed: a constant value over the entire estimation horizon or a variable value per cycle.

The open-loop verification of this modified MHE showed that the scheme with a variable H for each cycle converged faster to the actual value of H . Therefore, this approach was considered in the closed-loop semi-centralised control. The investigated scenario includes both a separate step 10%-change in H for each participating component and a simultaneous change for both components. Again, the two cases of ideal and noisy measurements were considered. In the case of ideal measurements, the control system rejected the disturbance and was able to maintain the product purities at their reference values. Unfortunately, in the presence of measurement noise, the MHE optimisation procedure had difficulty finding a feasible solution to the underlying estimation optimisation problem. A possible reason for that is the coupled controller and estimator dynamics and the high sensitivity of the considered process.

7.2 Perspectives

The first step in the follow-up work is to overcome the existing problem of simultaneous state and parameter estimation and control when noisy measurements are considered. A possible solution to this problem is to use more advanced algorithms for nonlinear constrained optimisation, such as Ipopt (Andersson et al., 2018).

Some of the possible directions for further research based on the presented semi-centralised control system (MHE + MPC/PID) are: adding more parameters for estimation by the MHE, control of the ternary centre-cut eight-zone SMB process, use of artificial neural networks (ANN) as a surrogate model for both MHE and MPC, and, of course, experimental validation of the control system.

In the Langmuir isotherms, in addition to adsorption Henry coefficient, H , the retention factor, b , also varies with changes in operating conditions or with adsorbent ageing. Therefore, real-time information about it is crucial for process control, which is difficult to obtain by any means other than online estimation. Another common disturbance in the operation of SMB processes is the inlet concentration, C_{Fe} . Since separation and purification are downstream processes, the feed to the SMB is the product of upstream processes. Therefore, C_{Fe} is, in most practical cases, subject to control or measurement. However, when this is not the case, it is also possible to estimate this value using MHE.

The use of a surrogate model to replace computationally intensive numerical methods is a particularly tempting option. In Pishkari et al., 2023 and Pishkari, 2025, a surrogate

model based on a simple ANN is used for offline optimisation of the productivity of a binary SMB process under cyclic steady-state conditions. The network was trained on a set of numerically generated input-output data. Unfortunately, this simple approach cannot work for control purposes, as online optimisation is inherently dynamic. This problem can be overcome by using a new class of ANNs called physics-informed neural networks (PINNs) (Raissi et al., 2019), which explicitly account for process dynamics by calculating the differential terms in the PDEs of the modelled processes. Once trained, such a PINN network will have improved computational performance. The CE/SE method has great potential as a tool for generating training data, thanks to its high computational efficiency. Subraveti et al., 2022 used a PINN to simulate a batch chromatographic process. We also have preliminary results from simulations of batch processes using the Python-based DeepXDE code library (Lu et al., 2021). The main challenge remains the transition from batch to SMB processes.

Appendix

Appendix A Transfer functions for the PID control of SMB processes

Linear plant transfer functions for the PID control scheme of the SMB process with Langmuir isotherms in the nonlinear range, as shown in the control diagram in Figure 3.2, are obtained via offline identification. When the SMB plant had reached a cyclic steady state, it was perturbed by a step change in each of the m values separately. Then, after reaching the new cyclic steady state, the transfer functions for each control channel were determined from the process variable step responses. Step responses of zones II and III are based on integral concentration values for each cycle. Retention times are defined per cycle.

For example, for the transfer function $G_{11}(s)$ the SMB plant is perturbed by a -20 % change in m_I , leading to a -11.7 % change in $Q_{\text{int},I}$ as shown in Figure A.1, and the resulting step response of τ_I is given in Figure A.2.

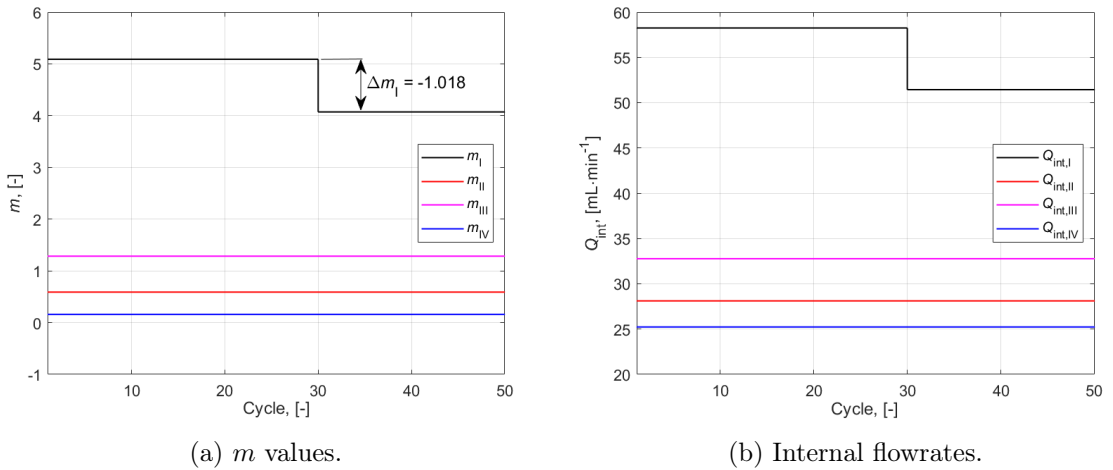


Figure A.1: Open-loop change of m_I with -20 % and the corresponding internal flowrates for the identification of transfer function $G_{11}(s)$.

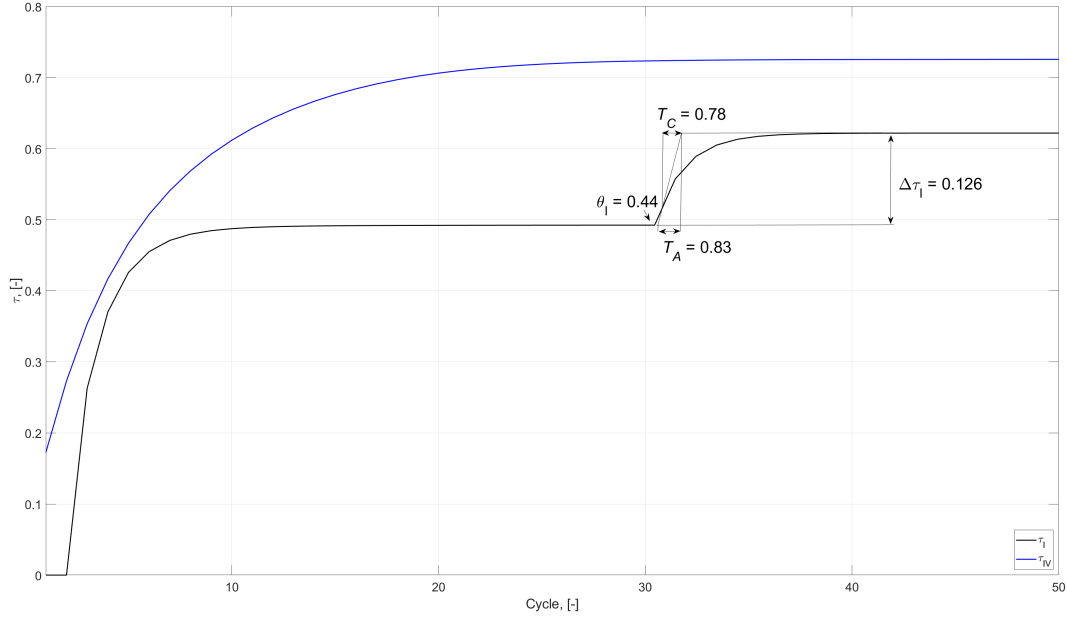


Figure A.2: τ_I step response to a change in m_I and the identification procedure.

τ_I changes from 0.496 to 0.622 or $\Delta\tau_I = 0.126$. The reason for the negative response is that the decrease in m_I , and from it, the decrease in the internal flowrate within this zone of the SMB plant leads to the movement of the desorption wave towards the beginning of that zone and the increase of the residence time of the wave which is expressed by the increase in dimensionless retention time τ_I . The gain of this channel is

$$K_{11} = \frac{\Delta\tau_I}{\Delta m_I} = \frac{0.126}{-1.018} = -0.124. \quad (\text{A.1})$$

To determine the time constants, the well-known graphical method of Oldenbourg-Sartorius is used. In the step response of τ_I , the two times T_A and T_C are measured. Then, with the ratio

$$\frac{T_C}{T_A} = \frac{0.78}{0.83} = 0.94, \quad (\text{A.2})$$

from a nomogram, the ratios T_1/T_A and T_2/T_A are found.

$$\begin{cases} \frac{T_1}{T_A} = 0.910, \\ \frac{T_2}{T_A} = 0.024. \end{cases} \quad (\text{A.3})$$

Finally, for T_1 and T_2 we have

$$\begin{cases} T_1 = 0.910 T_A = 0.910 \cdot 0.83 = 0.755, \\ T_2 = 0.024 T_A = 0.024 \cdot 0.83 = 0.020. \end{cases} \quad (\text{A.4})$$

The delay is also measured from Figure [A.2](#) and for the transfer function $G_{11}(s)$ we have

$$G_{11}(s) = -\frac{0.124e^{-0.44s}}{(0.755s + 1)(0.020s + 1)} = -\frac{0.124e^{-0.44s}}{0.015s^2 + 0.775s + 1}. \quad (\text{A.5})$$

The rest of the plant transfer functions are found in a similar manner, and these are as follows

$$G_{22}(s) = \frac{1.37e^{-0.16s}}{(0.734s + 1)(0.637s + 1)}, \quad (\text{A.6})$$

$$G_{33}(s) = -\frac{5.67e^{-0.05s}}{(1.03s + 1)(0.318s + 1)}, \quad (\text{A.7})$$

$$G_{44}(s) = \frac{0.581e^{-1.06s}}{(2.295s + 1)(0.828s + 1)}, \quad (\text{A.8})$$

$$G_{32}(s) = -\frac{0.11e^{-0.09s}}{(4.521s + 1)(1.391s + 1)}, \quad (\text{A.9})$$

$$G_{23}(s) = -\frac{0.15e^{-1.12s}}{(5.149s + 1)(0.387s + 1)}, \quad (\text{A.10})$$

As discussed in Chapter 3, the regeneration zones are decoupled under the assumption of total regeneration, so that all other transfer functions are set to zero. For the tuning of the PID controllers, the SIMC rules (Skogestad, [2003](#)) are used. For the first PID, we have:

$$K_{P,I} = \frac{0.5}{K_{11}} \frac{T_1 + T_2}{\theta_1} = \frac{0.5}{-0.124} \frac{0.755 + 0.020}{0.44} = -7.12, \quad (\text{A.11})$$

$$T_{I,I} = T_1 + T_2 = 0.755 + 0.020 = 0.775, \quad (\text{A.12})$$

$$T_{D,I} = \frac{T_2}{1 + \frac{T_2}{T_1}} = \frac{0.020}{1 + \frac{0.020}{0.755}} = 0.019, \quad (\text{A.13})$$

$$K_{I,I} = \frac{K_{P,I}}{T_{I,I}} = -\frac{7.12}{0.775} = -9.18, \quad (\text{A.14})$$

$$K_{D,I} = K_{P,I} T_{D,I} = -7.12 \cdot 0.019 = -0.14. \quad (\text{A.15})$$

The tuning parameters for the other controllers are obtained in the same manner and are given in Table [3.2](#). Decouplers are given in Chapter [3](#).

Appendix B Lower and upper bounds and initial guesses for the α , β and γ coefficients and the adsorption Henry coefficients, H , used in the MHE optimisations

Table B.1: Lower bounds for the α , β and γ coefficients used in the MHE optimisations for the less retained component A in Chapters 4 and 6.

Column, p	$\alpha_{A,p}^{\text{liq, min}}, [\text{g}\cdot\text{L}^{-1}]$	$\alpha_{A,p}^{\text{sol, min}}, [\text{g}\cdot\text{L}^{-1}]$	$\beta_{A,p}^{\text{min}}, [\text{m}^{-1}]$	$\gamma_{A,p}^{\text{min}}, [\text{m}]$
I	$-30C_{A,\text{Fe}} = -150$	$-6C_{A,\text{Fe}} = -30$	-2,000	$-4L_{\text{col}} = -0.5868$
II	$-6C_{A,\text{Fe}} = -30$	$-6C_{A,\text{Fe}} = -30$	-1,000	$-3L_{\text{col}} = -0.4401$
III	$-30C_{A,\text{Fe}} = -150$	$-30C_{A,\text{Fe}} = -150$	-1,000	$-2L_{\text{col}} = -0.2934$
IV	$-30C_{A,\text{Fe}} = -150$	$-30C_{A,\text{Fe}} = -150$	-1,000	$-L_{\text{col}} = -0.1467$

Table B.2: Lower bounds for the α , β and γ coefficients used in the MHE optimisations for the more retained component B in Chapters 4 and 6.

Column, p	$\alpha_{B,p}^{\text{liq, min}}, [\text{g}\cdot\text{L}^{-1}]$	$\alpha_{B,p}^{\text{sol, min}}, [\text{g}\cdot\text{L}^{-1}]$	$\beta_{B,p}^{\text{min}}, [\text{m}^{-1}]$	$\gamma_{B,p}^{\text{min}}, [\text{m}]$
I	$-6C_{B,\text{Fe}} = -30$	$-30C_{B,\text{Fe}} = -150$	-2,000	$-4L_{\text{col}} = -0.5868$
II	$-6C_{B,\text{Fe}} = -30$	$-30C_{B,\text{Fe}} = -150$	-1,000	$-3L_{\text{col}} = -0.4401$
III	$-6C_{B,\text{Fe}} = -30$	$-30C_{B,\text{Fe}} = -150$	-1,000	$-2L_{\text{col}} = -0.2934$
IV	$-6C_{B,\text{Fe}} = -30$	$-6C_{B,\text{Fe}} = -30$	-1,000	$-L_{\text{col}} = -0.1467$

Table B.3: Upper bounds for the α , β and γ coefficients used in the MHE optimisations for the less retained component A in Chapters 4 and 6.

Column, p	$\alpha_{A,p}^{\text{liq, max}}, [\text{g}\cdot\text{L}^{-1}]$	$\alpha_{A,p}^{\text{sol, max}}, [\text{g}\cdot\text{L}^{-1}]$	$\beta_{A,p}^{\text{max}}, [\text{m}^{-1}]$	$\gamma_{A,p}^{\text{max}}, [\text{m}]$
I	$30C_{A,\text{Fe}} = 150$	$30C_{A,\text{Fe}} = 150$	1,000	$5L_{\text{col}} = 0.7335$
II	$30C_{A,\text{Fe}} = 150$	$30C_{A,\text{Fe}} = 150$	1,000	$6L_{\text{col}} = 0.8802$
III	$30C_{A,\text{Fe}} = 150$	$30C_{A,\text{Fe}} = 150$	5,000	$7L_{\text{col}} = 1.0269$
IV	$30C_{A,\text{Fe}} = 150$	$30C_{A,\text{Fe}} = 150$	1,000	$8L_{\text{col}} = 1.1736$

Table B.4: Upper bounds for the α , β and γ coefficients used in the MHE optimisations for the more retained component B in Chapters 4 and 6.

Column, p	$\alpha_{B,p}^{\text{liq, max}}, [\text{g}\cdot\text{L}^{-1}]$	$\alpha_{B,p}^{\text{sol, max}}, [\text{g}\cdot\text{L}^{-1}]$	$\beta_{B,p}^{\text{max}}, [\text{m}^{-1}]$	$\gamma_{B,p}^{\text{max}}, [\text{m}]$
I	$30C_{B,\text{Fe}} = 150$	$10C_{B,\text{Fe}} = 50$	1,000	$5L_{\text{col}} = 0.7335$
II	$30C_{B,\text{Fe}} = 150$	$10C_{B,\text{Fe}} = 50$	2,000	$15L_{\text{col}} = 2.2005$
III	$10C_{B,\text{Fe}} = 50$	$10C_{B,\text{Fe}} = 50$	1,000	$7L_{\text{col}} = 1.0269$
IV	$10C_{B,\text{Fe}} = 50$	$10C_{B,\text{Fe}} = 50$	2,000	$8L_{\text{col}} = 1.1736$

Table B.5: Initial guesses for the α , β and γ coefficients used in the MHE optimisations for the less retained component A in Chapters 4 and 6.

Column, p	$\alpha_{A,p}^{\text{liq},0}$, [g·L ⁻¹]	$\alpha_{A,p}^{\text{sol},0}$, [g·L ⁻¹]	$\beta_{A,p}^0$, [m ⁻¹]	$\gamma_{A,p}^0$, [m]
I	0.1	0.1	-50	$\frac{L_{\text{col}}}{2} = 0.0734$
II	$C_{A,\text{Fe}} = 5$	$C_{A,\text{Fe}} = 5$	-50	$\frac{L_{\text{col}} + 2L_{\text{col}}}{2} = 0.2201$
III	$C_{A,\text{Fe}} = 5$	$C_{A,\text{Fe}} = 5$	200	$\frac{2L_{\text{col}} + 3L_{\text{col}}}{2} = 0.3668$
IV	0.0001	0.0001	0.1	$\frac{3L_{\text{col}} + 4L_{\text{col}}}{2} = 0.5135$

Table B.6: Initial guesses for the α , β and γ coefficients used in the MHE optimisations for the more retained component B in Chapters 4 and 6.

Column, p	$\alpha_{B,p}^{\text{liq},0}$, [g·L ⁻¹]	$\alpha_{B,p}^{\text{sol},0}$, [g·L ⁻¹]	$\beta_{B,p}^0$, [m ⁻¹]	$\gamma_{B,p}^0$, [m]
I	$C_{B,\text{Fe}} = 5$	$C_{B,\text{Fe}} = 5$	-50	$\frac{L_{\text{col}}}{2} = 0.0734$
II	$C_{B,\text{Fe}} = 5$	$C_{B,\text{Fe}} = 5$	100	$\frac{L_{\text{col}} + 2L_{\text{col}}}{2} = 0.2201$
III	0.1	0.1	50	$\frac{2L_{\text{col}} + 3L_{\text{col}}}{2} = 0.3668$
IV	0.0001	0.0001	0.1	$\frac{3L_{\text{col}} + 4L_{\text{col}}}{2} = 0.5135$

Table B.7: Lower and upper bounds for the adsorption Henry coefficients, H , used in the MHE optimisations in Chapter 6.

H_A^{min} , [-]	H_A^{max} , [-]	H_B^{min} , [-]	H_B^{max} , [-]
0.5	1.2	1.8	2.8

Appendix C Direct conversion of the ideal equilibrium model to the form given by Eq. (5.4)

As already mentioned above, the CE/SE method applies to PDEs of the type shown in Eq. (5.4). In contrast to this, the equilibrium model of chromatography is given by

$$\frac{\partial \mathbf{f}(\mathbf{u})}{\partial t} + \frac{\partial \mathbf{u}}{\partial z} = 0, \quad (\text{C.1})$$

with $\mathbf{u} = \mathbf{C}$ and $\mathbf{f}(\mathbf{u}) = \varepsilon \mathbf{C} + (1 - \varepsilon) \mathbf{q}(\mathbf{C})$. In the case of a linear isotherm, shown in Eq. (2.19), its derivative is constant, and Eq. (C.1) can be easily rearranged to the form in Eq. (5.4). For nonlinear isotherms, an inversion of the function is required and the corresponding system of PDEs can then be solved for $\mathbf{f}$ instead of $\mathbf{u}$ according to

$$\frac{\partial \mathbf{f}}{\partial t} + \frac{\partial \mathbf{u}(\mathbf{f})}{\partial z} = 0, \quad (\text{C.2})$$

which is again equivalent to the form given by Eq. (5.4). For the single-component Langmuir isotherm, inversion leads to the solution of a single quadratic equation, which can be solved explicitly for the unique positive solution to bring the model to the form given by Eq. (5.4).

For the multicomponent Langmuir isotherm, inversion leads to the solution of a system of quadratic equations, which is much more involved. However, the problem can be simplified by transformation of the equilibrium model to a simpler form by using an adjusted time $\theta = vt - z$ (Helfferich and Klein, 1970). With this adjusted time, the equilibrium model reads

$$\frac{\partial \mathbf{q}(\mathbf{C})}{\partial \theta} + \frac{\varepsilon}{1 - \varepsilon} \frac{\partial \mathbf{C}}{\partial z} = 0. \quad (\text{C.3})$$

In a second step, the well-known inverse of the Langmuir isotherm (Rhee et al., 1989) (p. 252) is applied,

$$C_i(\mathbf{q}) = \frac{\frac{q_i}{H_i}}{1 - \sum_{l=A}^{N_{\text{comp}}} \frac{q_l}{b_l}}, \quad i = A, B, \dots, N_{\text{comp}}, \quad (\text{C.4})$$

and Eq. (C.3), together with Eq. (C.4), is solved for $\mathbf{q}$. For the Langmuir isotherm, this is always possible for any number of components.

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