Looking at Mean-Payoff and Total-Payoff through Windows

K. Chatterjee (IST Austria) L. Doyen (ENS Cachan)
M. Randour (UMONS) J.-F. Raskin (ULB)

Paris - 16.12.2013

LIAFA verification seminar

Aim of this talk

1. Overview of the situation for (multi) MP and TP games
   - No P algorithm known in one dimension
   - In multi dimensions, MP is coNP-complete
   - First contribution: **TP is undecidable in multi dimensions**
   - No timing guarantee

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1. Overview of the situation for (multi) MP and TP games
   - No P algorithm known in one dimension
   - In multi dimensions, MP is coNP-complete
   - First contribution: **TP is undecidable in multi dimensions**
   - No timing guarantee

2. Introduction of **window objectives**
   - Conservative approximation of MP/TP
   - Break the complexity barriers
   - Specifies timing requirements
   - Algorithms, complexity and memory requirements
   - Several flavors of the objective

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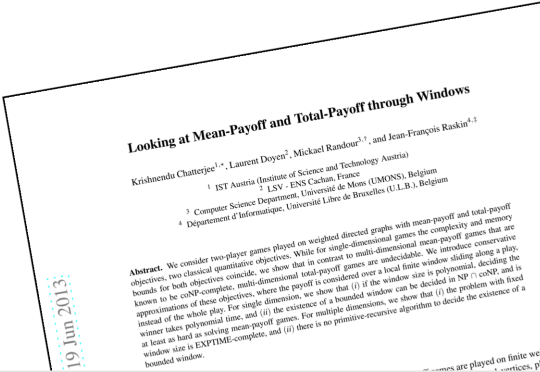
Full paper available on arXiv: abs/1302.4248



19 Jun 2013

Looking at Mean-Payoff and Total-Payoff through Windows

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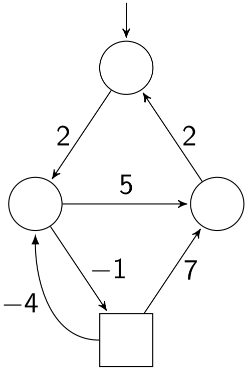
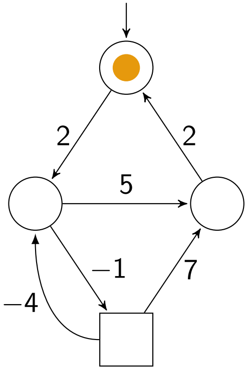
Abstract. We consider two-player games played on weighted directed graphs with mean-payoff and total-payoff objectives, two classical quantitative objectives. While for single-dimensional games the complexity and memory bounds for both objectives coincide, we show that in contrast to multi-dimensional mean-payoff games that are known to be coNP-complete, multi-dimensional total-payoff games are undecidable. We introduce conservative approximations of these objectives, where the payoff is considered over a local finite window sliding along a play, instead of the whole play. For single dimension, we show that (*i*) if the window size is polynomial, deciding the winner takes polynomial time, and (*ii*) the existence of a bounded window can be decided in NP ∩ coNP, and is at least as hard as solving mean-payoff games. For multiple dimensions, we show that (*i*) the problem with fixed window size is EXPTIME-complete, and (*ii*) there is no primitive-recursive algorithm to decide the existence of a bounded window.

MP and TP games



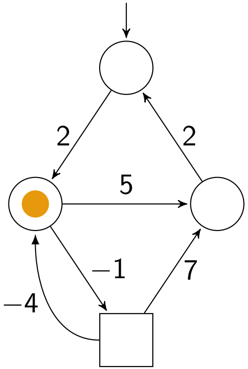
- $G = (S_1, S_2, E, w)$
- $S = S_1 \cup S_2, S_1 \cap S_2 = \emptyset, E \subseteq S \times S$, $w \colon E \to \mathbb{Z}$
- $\mathcal{P}_1$ states = ◯
- $\mathcal{P}_2$ states = □
- Plays, prefixes, **pure** strategies.

MP and TP games



- $\mathrm{TP}(\pi) = \liminf_{n\to\infty} \sum_{i=0}^{i=n-1} w(s_i, s_{i+1})$
- $\mathrm{MP}(\pi) = \liminf_{n\to\infty} \frac{1}{n} \mathrm{TP}(\pi(n))$

MP and TP games



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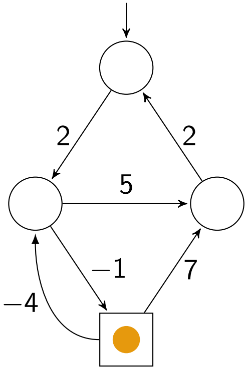

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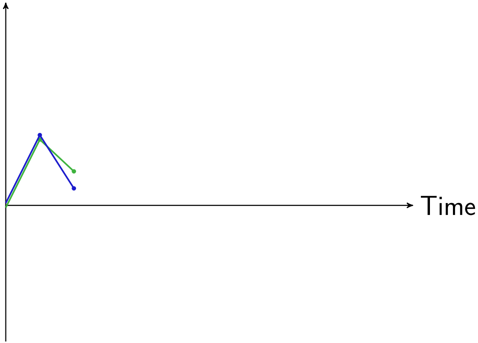

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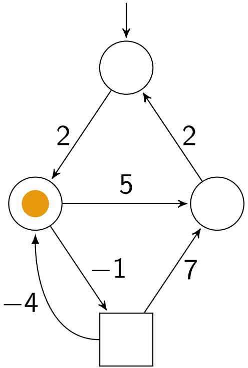

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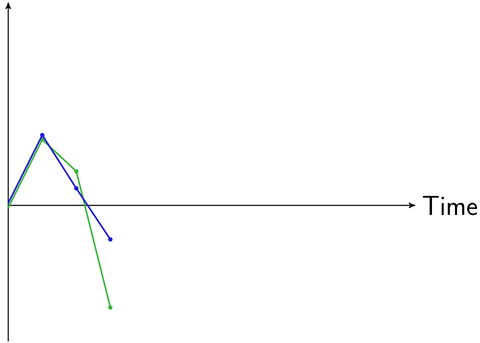
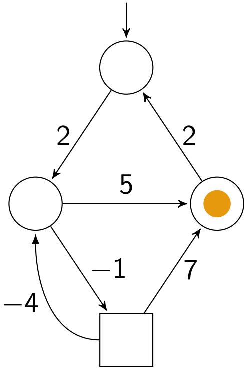


MP and TP games



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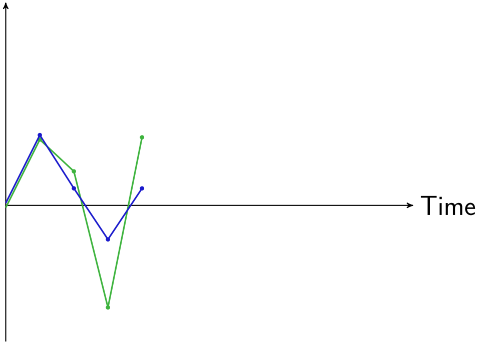

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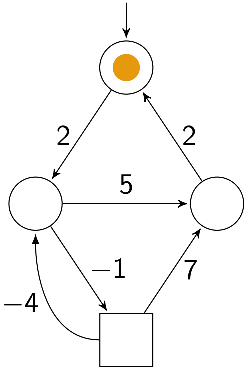

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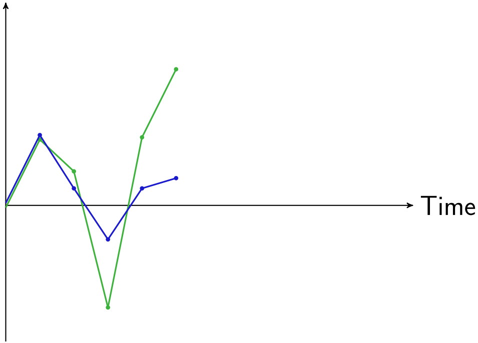

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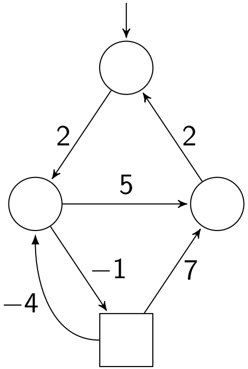

Then, $(2, 5, 2)^\omega$

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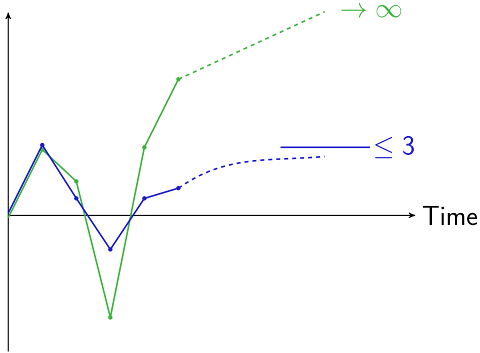
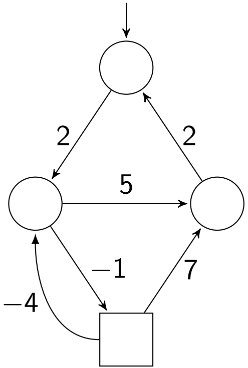


MP and TP games



Then, $(2, 5, 2)^\omega$

- **TP (MP) threshold problem**

  Given $v \in \mathbb{Q}$ and $s_{\mathsf{init}} \in S$,

  $\exists ?\, \lambda_1 \in \Lambda_1$ s.t. $\forall \lambda_2 \in \Lambda_2$,

  $\underline{\mathsf{TP}}(\mathsf{Outcome}_G(s_{\mathsf{init}}, \lambda_1, \lambda_2)) \geq v$

Known results

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\text{MP}}$ / $\overline{\text{MP}}$	NP ∩ coNP	mem-less		coNP-c. / NP ∩ coNP	infinite	mem-less
$\underline{\text{TP}}$ / $\overline{\text{TP}}$	NP ∩ coNP	mem-less		??	??	??

- Long tradition of study. Non-exhaustive selection: [EM79, ZP96, Jur98, GZ04, GS09, CDHR10, VR11, CRR12]
- k-dimension: weights = integer vectors
- *No known polynomial time algorithm for one-dimension*
- *No result on multi-dimension total-payoff*

Multi-dimension TP games are undecidable

Theorem

The threshold problem for infimum and supremum total-payoff objectives is **undecidable** in multi-dimension games, for five dimensions.

Multi-dimension TP games are undecidable

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▷ Reduction from the **halting problem for 2CMs** [Min61]

Two-counter machines

- Finite set of instructions
- Two counters C_1 and C_2 taking values $(v_1, v_2) \in \mathbb{N}^2$
- Instructions:
  - ▷ Increment
    $$C_i ++$$
  - ▷ Decrement
    $$C_i --$$
  - ▷ Zero test and branch accordingly

    If $C_i == 0$ **do** *this* **else do** *that*
- W.l.o.g. if the machine stops, it stops with both counters to zero

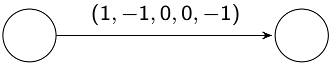
Encoding a 2CM in a 5-dim. TP game

- TP objective (inf or sup) of threshold $(0, 0, 0, 0, 0)$
- $\mathcal{P}_1$ must simulate faithfully
- $\mathcal{P}_2$ retaliates if $\mathcal{P}_1$ cheats
- At the end, $\mathcal{P}_1$ wins the TP game **iff** the 2CM stops

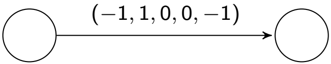
Key idea: after m steps, the TP has value $(v_1, -v_1, v_2, -v_2, -m)$ **iff** the 2CM counters have value (v_1, v_2)

Instructions

- Increment C_1

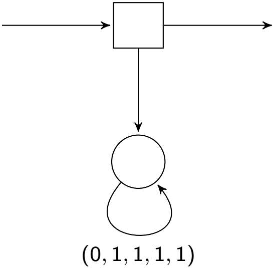


- Decrement C_1



Instructions

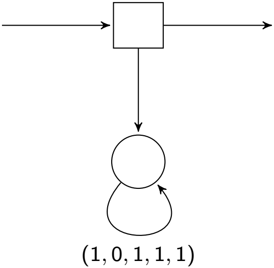
- Checking counter C_1 is non-negative



- ▷ If $\mathcal{P}_1$ cheats, he is doomed!
- ▷ Otherwise, $\mathcal{P}_2$ has no interest in retaliating.

Instructions

- Checking a zero test on C_1



- ▷ If $\mathcal{P}_1$ cheats, he is doomed!
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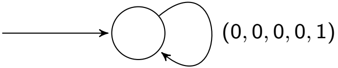
Halting

- If the 2CM halts (with counters to zero w.l.o.g.)



▷ Thanks to the fifth dim., $\mathcal{P}_1$ wins only if the machine halts.

The case is closed

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\text{MP}}$ / $\overline{\text{MP}}$	NP $\cap$ coNP	mem-less		coNP-c. / NP $\cap$ coNP	infinite	mem-less
$\underline{\text{TP}}$ / $\overline{\text{TP}}$	NP $\cap$ coNP	mem-less		**Undec.**	-	-

Motivations

- Classical MP and TP objectives have some drawbacks
  - Complexity issues
    - P membership for the one-dim. case is a long-standing open problem
    - TP undecidable in k-dim.
  - Infimum vs. supremum
  - **no timing guarantee**: the "good behavior" occurs at the limit...

Window objectives: key idea

- **Window** of fixed size **sliding** along a play
  $\rightsquigarrow$ defines a local finite horizon
- Objective: see a **local** $MP \geq 0$ *before hitting the end* of the window
  $\rightsquigarrow$ needs to be verified at *every* step

Window objectives: key idea

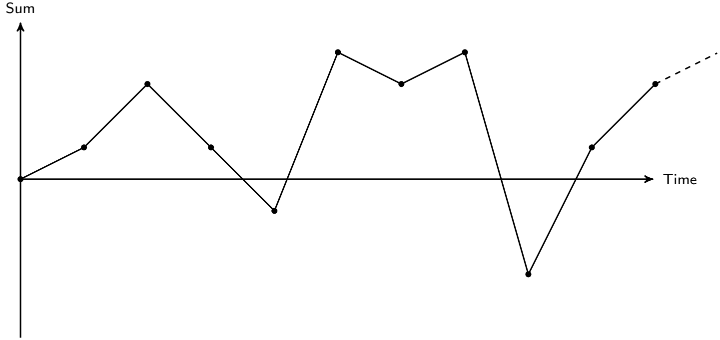
- **Window** of fixed size **sliding** along a play
  $\rightsquigarrow$ defines a local finite horizon
- Objective: see a **local** *MP* ≥ 0 *before hitting the end* of the window
  $\rightsquigarrow$ needs to be verified at *every* step

▷ Conservative approximation of MP/TP

▷ Intuition: local deviations from the threshold must be compensated in a parametrized # of steps

▷ Variety of results and algorithms

Illustration: WMP, threshold zero, maximal window = 4



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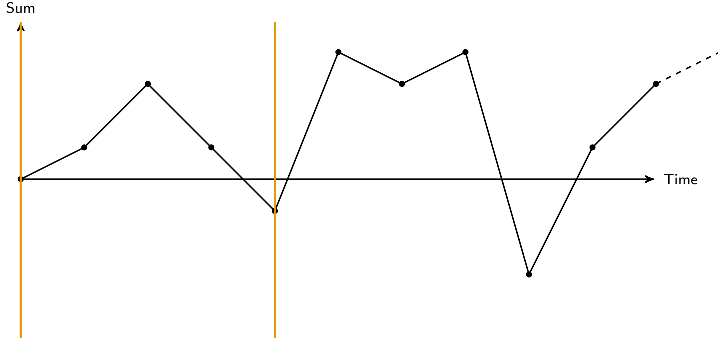

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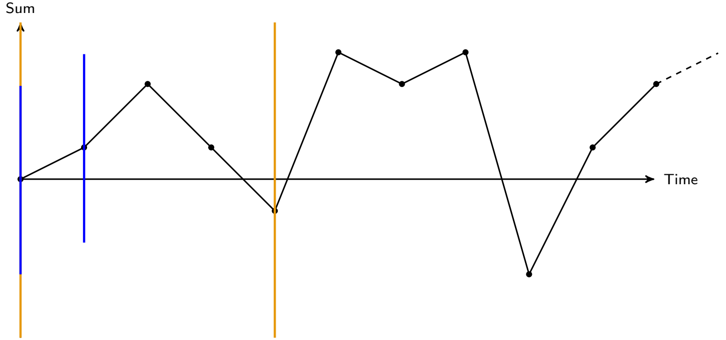

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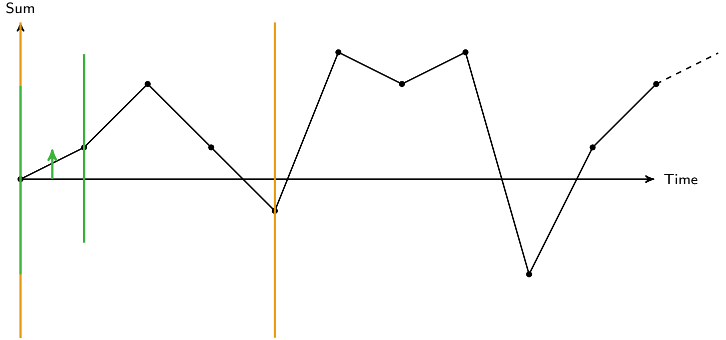

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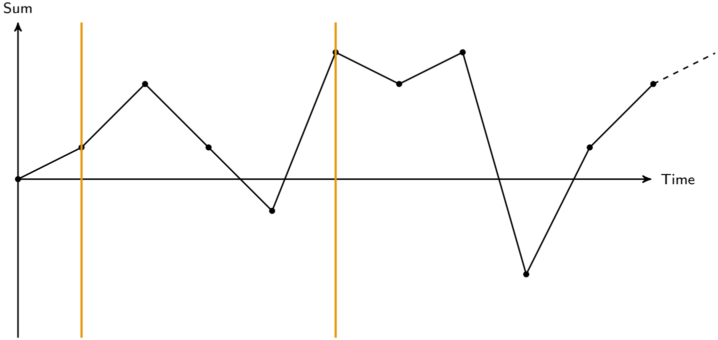

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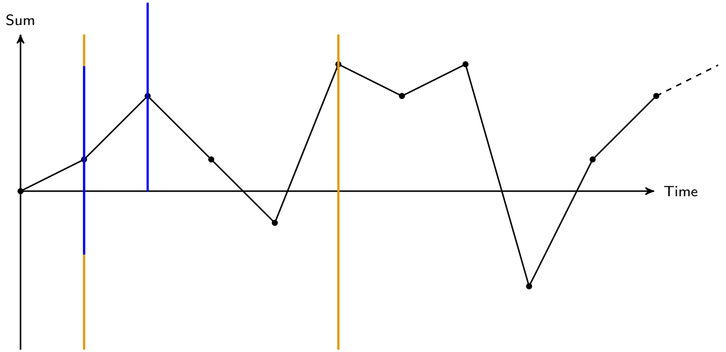

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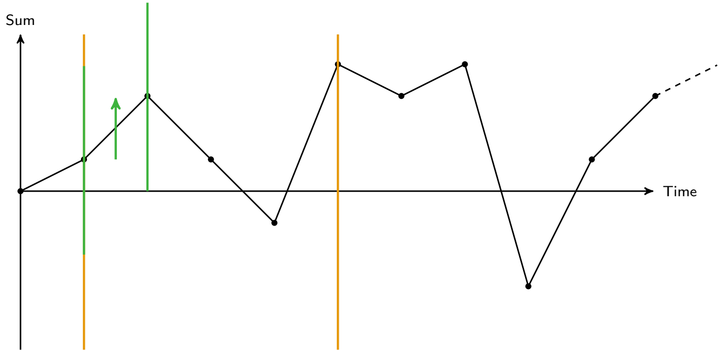

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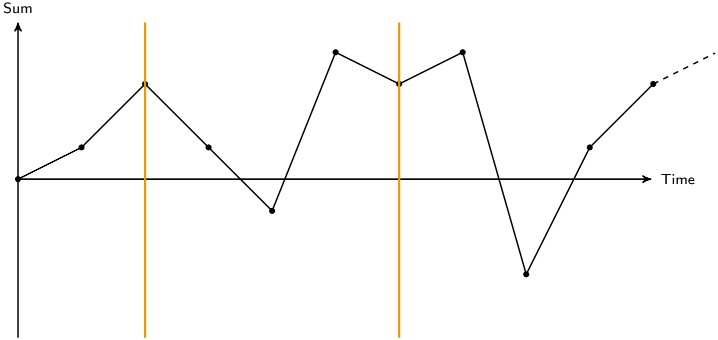

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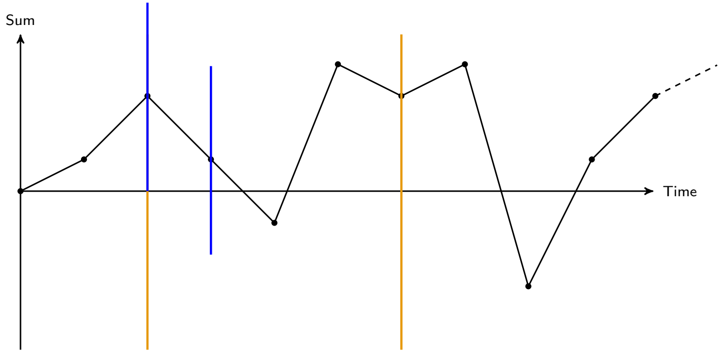

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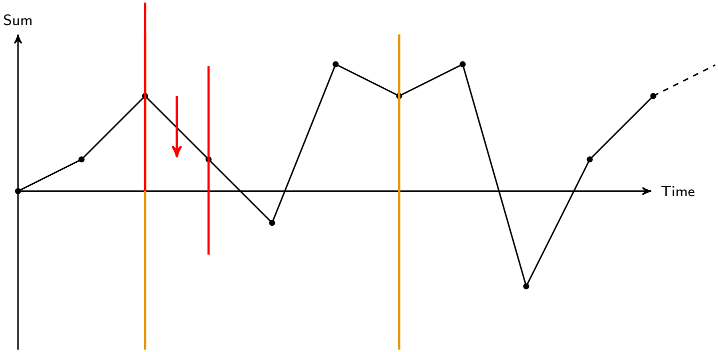

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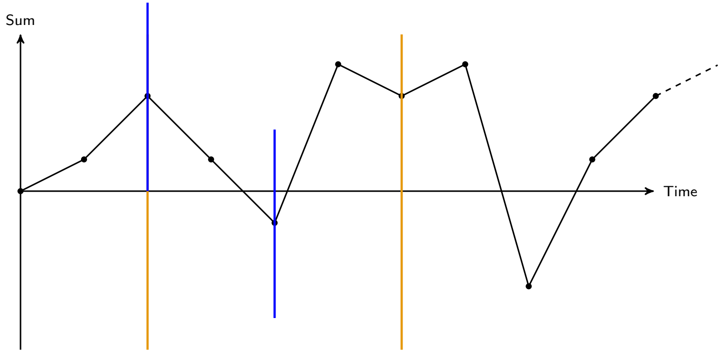

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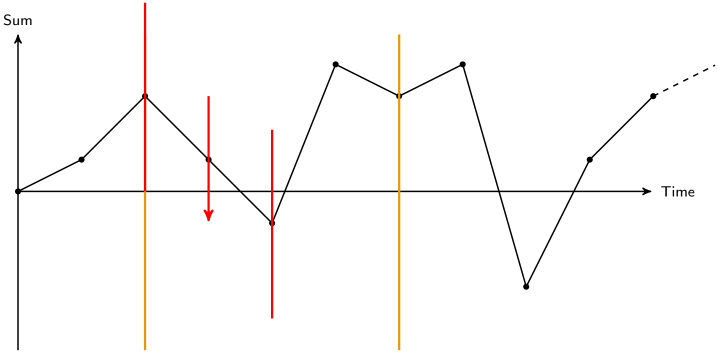

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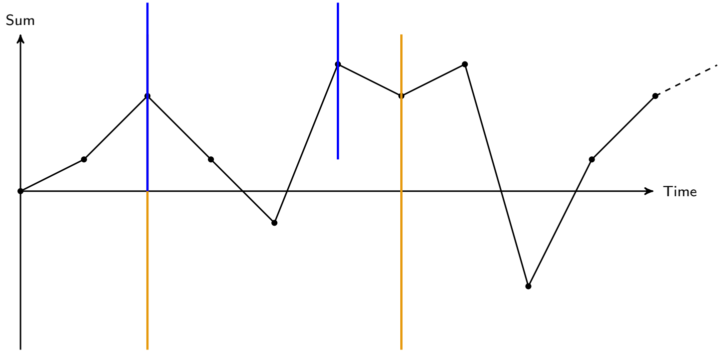

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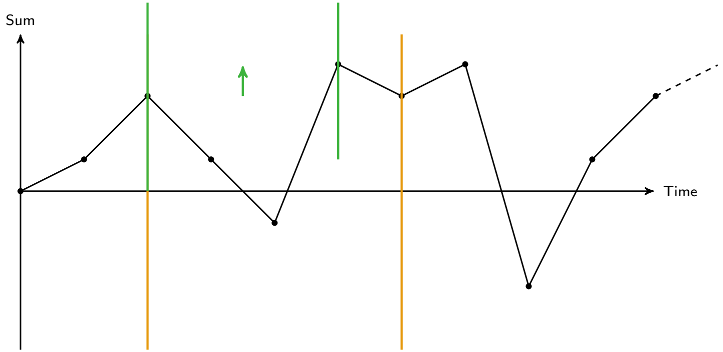

Multiple variants

- Given $l_{\max} \in \mathbb{N}_0$, *good window* $\mathbf{GW}(l_{\max})$ asks for a positive sum in at most $l_{\max}$ steps (one window, from the first state)
- *Direct Fixed Window*: $\mathbf{DFW}(l_{\max}) \equiv \Box\mathbf{GW}(l_{\max})$
- *Fixed Window*: $\mathbf{FW}(l_{\max}) \equiv \Diamond\mathbf{DFW}(l_{\max})$
- *Direct Bounded Window*: $\mathbf{DBW} \equiv \exists\, l_{\max},\ \mathbf{DFW}(l_{\max})$
- *Bounded Window*: $\mathbf{BW} \equiv \Diamond\mathbf{DBW} \equiv \exists\, l_{\max},\ \mathbf{FW}(l_{\max})$

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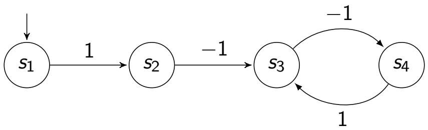
▷ Nice properties: monotonicity in $l_{\max}$, prefix-independence

▷ A window *closes* when the sum becomes positive

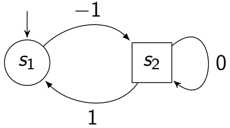
▷ A window is *open* if not yet closed

Example 1



- MP is satisfied
  - ▷ the cycle is non-negative
- **FW**(2) is satisfied
  - ▷ thanks to prefix-independence
- **DBW** is not
  - ▷ the window opened in s_2 never closes

Example 2



- MP is satisfied
  - ▷ all simple cycles are non-negative
- *but* none of the window objectives is
  - ▷ $\mathcal{P}_2$ can force opening windows and delay their closing for as long as he wants (but not forever due to prefix-independence)

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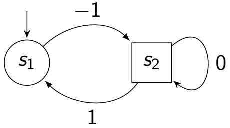

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BW vs. MP

- BW asks for timing guarantees which cannot be enforced here
- Observe that $\mathcal{P}_2$ **needs infinite memory**

Conservative approximation of MP (one-dim.)

The following are true

$$\text{Any window obj.} \Rightarrow \mathbf{BW} \Rightarrow \text{MP} \geq 0$$
$$\mathbf{BW} \Leftarrow \text{MP} > 0$$

Results overview

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\text{MP}}$ / $\overline{\text{MP}}$	NP $\cap$ coNP	mem-less		coNP-c. / NP $\cap$ coNP	infinite	mem-less
$\underline{\text{TP}}$ / $\overline{\text{TP}}$	NP $\cap$ coNP	mem-less		**undec.**	-	-
WMP: fixed polynomial window	**P-c.**	**mem. req.** $\leq$ **linear**$(\lvert S\rvert \cdot l_{\max})$		**PSPACE-h.** **EXP-easy**	**exponential**	
WMP: fixed arbitrary window	**P**$(\lvert S\rvert, V, l_{\max})$			**EXP-c.**		
WMP: bounded window problem	**NP $\cap$ coNP**	**mem-less**	**infinite**	**NPR-h.**	-	-

- $|S|$ the # of states, V the length of the binary encoding of weights, and $l_{\max}$ the window size.

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- $\lvert S\rvert$ the # of states, V the length of the binary encoding of weights, and $l_{\max}$ the window size.
- For one-dim. games with poly. windows, we are in **P**
- For multi-dim. games with fixed windows, we are **decidable**
- Window obj. provide **timing guarantees**

Results overview

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\text{MP}}$ / $\overline{\text{MP}}$	NP $\cap$ coNP	mem-less		coNP-c. / NP $\cap$ coNP	infinite	mem-less
$\underline{\text{TP}}$ / $\overline{\text{TP}}$	NP $\cap$ coNP	mem-less		**undec.**	-	-
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WMP: fixed arbitrary window	**P**($\lvert S\rvert, V, l_{\max}$)			**EXP-c. (1/2)**		
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- $|S|$ the # of states, V the length of the binary encoding of weights, and $l_{\max}$ the window size.

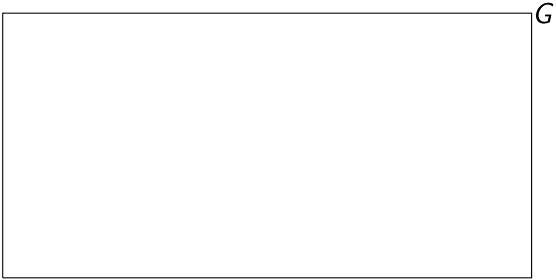
- No time to discuss everything. Focus.

High level sketch: top-down approach

- $\mathbf{FW}(l_{\max}) \equiv \Diamond \mathbf{DFW}(l_{\max})$

▷ Assume we can compute $\mathbf{DFW}(l_{\max})$,

▷ Compute attractor, declare winning and recurse on subgame.



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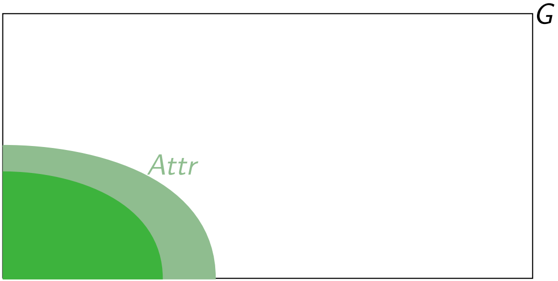

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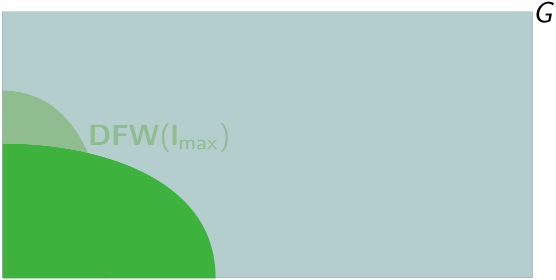

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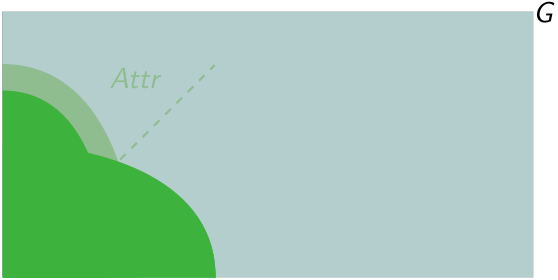

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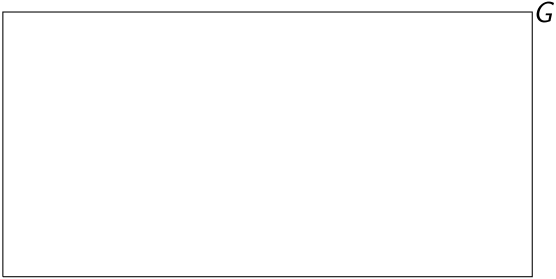

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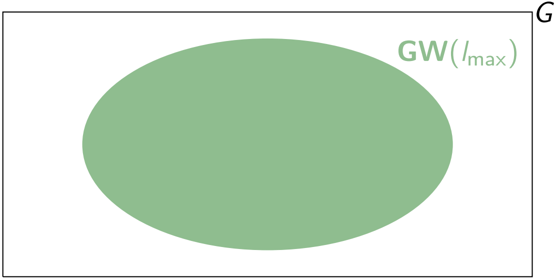

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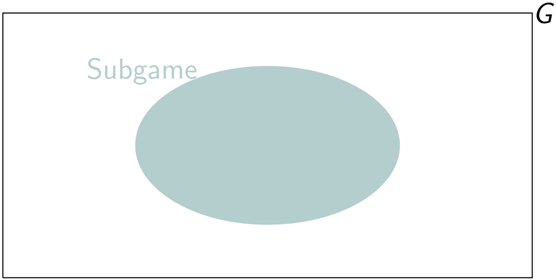

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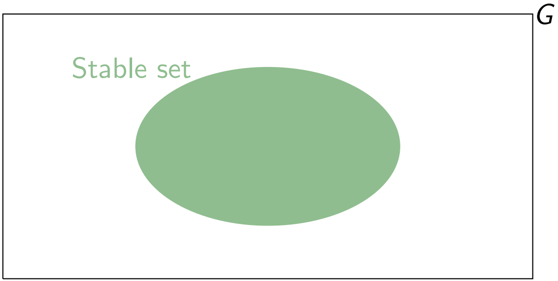

High level sketch: top-down approach

- **GW**($l_{\max}$)

▷ Simply compute the best sum achievable in at most $l_{\max}$ steps and check if positive.

High level sketch: top-down approach

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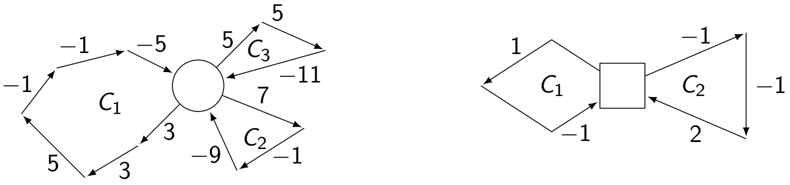
▷ Simply compute the best sum achievable in at most $l_{\max}$ steps and check if positive.

- Finally,

Theorem

In two-player one-dimension games,
(a) the fixed arbitrary window MP problem is decidable in time polynomial in the size of the game and the window size,
(b) **the fixed polynomial window MP problem is P-complete**,
(c) both players require memory, and memory of size linear in the size of the game and the window size is sufficient.

Memory is necessary for both players



$(C_1 C_2 C_3)^\omega, l_{\max} = 4$ $(C_1 C_2)^\omega, l_{\max} = 3$

Choices are based on

- ▷ the # of steps remaining to close the window,
- ▷ the amount to compensate.

EXPTIME algorithm

Winning plays for the FW objective:

- from some point on, on all dimensions, all opening windows are closed within $l_{\max}$ steps
- the closing may be asynchronous

EXPTIME algorithm

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Basically, winning = seeing only a finite number of *bad windows*

- ▷ reduction to an exponentially larger co-Büchi game
- ▷ EXPTIME membership and exponential upper bounds on memory follow

From FW($l_{\max}$) to a co-Büchi game

For each dimension, bookkeeping of

- the amount to compensate to close the window,
- the remaining # of steps to close it.

When a window closes on dim. t, we reset

- ▷ the amount to zero,
- ▷ the # of steps to $l_{\max}$.

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Key elements

- $S \rightsquigarrow S \times (\{-l_{max} \cdot W, \ldots, 0\} \times \{1, \ldots, l_{max}\})^k$
- *bad states* representing windows not closing in time
- co-Büchi objective asks they are visited only finitely often

EXPTIME-hardness for 2 dim. and arbitrary weights

Reduction from **countdown games**.

- Weighted graph $(\mathcal{S}, \mathcal{T})$, with $\mathcal{S}$ the set of states and $\mathcal{T} \subseteq \mathcal{S} \times \mathbb{N}_0 \times \mathcal{S}$ the transition relation.
- Configurations (s, c), $s \in \mathcal{S}$, $c \in \mathbb{N}$.
- Game starts in (s_{init}, c_0).

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- Transitions from a configuration (s, c) performed as follows:
  1. $\mathcal{P}_1$ chooses a duration d, $0 < d \leq c$ such that there exists $t = (s, d, s') \in \mathcal{T}$ for some $s' \in \mathcal{S}$,
  2. $\mathcal{P}_2$ chooses a state $s' \in \mathcal{S}$ such that $t = (s, d, s') \in \mathcal{T}$,
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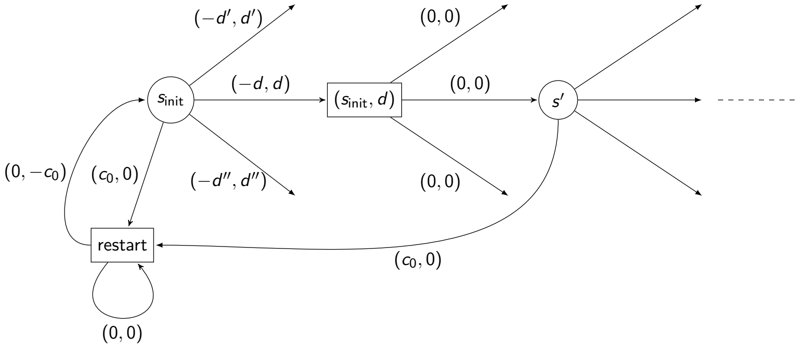
  3. the game advances to $(s', c - d)$.
- Terminal configurations reached whenever no legitimate move is available. $\mathcal{P}_1$ wins iff $(s, 0)$.

Deciding the winner is EXPTIME-complete [JSL08].

From CD games to FW

$(\mathcal{S}, \mathcal{T})$, $(s_{\mathsf{init}}, c_0) \rightsquigarrow G$, $k = 2$, $l_{\mathsf{max}} = 2 \cdot c_0 + 2$



- Two dimensions used to store the counter and its opposite
- $\mathcal{P}_1$ chooses durations and $\mathcal{P}_2$ chooses transitions of the CDG

From CD games to FW

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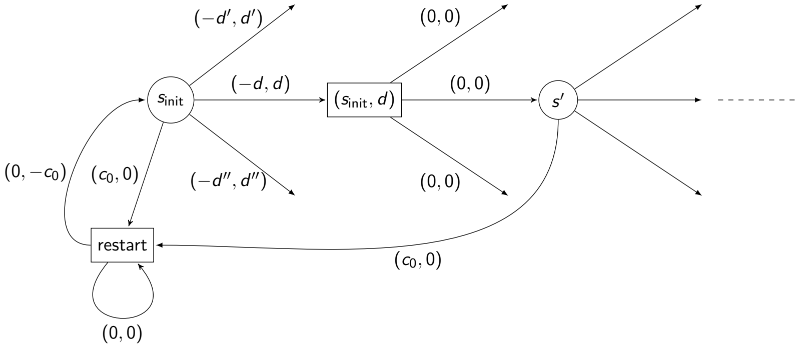

- ▷ $\mathcal{P}_1$ can branch to restart at any time
- ▷ There, $\mathcal{P}_2$ can delay the closing of open windows then restart

From CD games to FW

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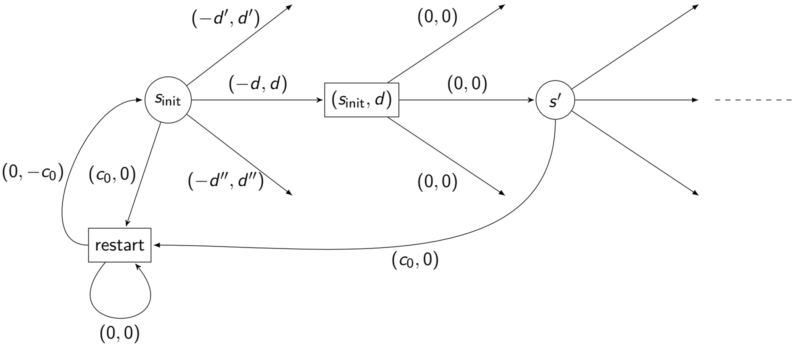
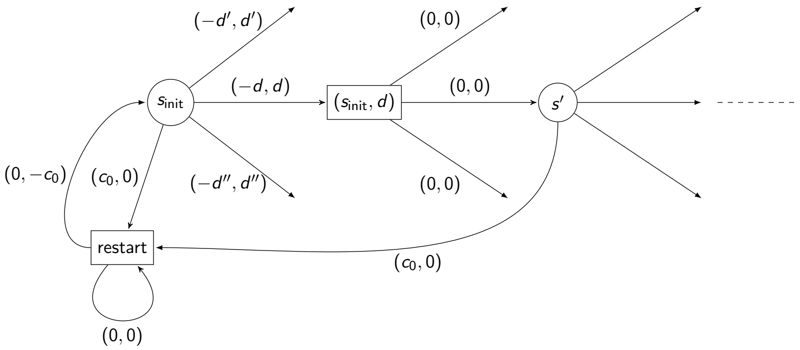


- ▷ To close the window on the 2nd dim., $\mathcal{P}_1$ has to accumulate *at least* c_0 before branching
- ▷ To be safe on the 1st, he must accumulate *at most* c_0

From CD games to FW

$(\mathcal{S}, \mathcal{T})$, $(s_{\text{init}}, c_0) \rightsquigarrow G$, $k = 2$, $l_{\max} = 2 \cdot c_0 + 2$



▷ $\mathcal{P}_1$ wins for FW iff he reaches *exactly* c_0, i.e., iff he can reach a terminal configuration $(s, 0)$ in the CDG

Other results

The multi-dim. FW problem is also

- EXPTIME-hard for weights $\{-1, 0, 1\}$ and arbitrary dimensions
  - ▷ membership problem for APTMs [CKS81]
- PSPACE-hard even for polynomial windows
  - ▷ generalized reachability games [FH10]
  - ▷ also induces that exponential memory is necessary (sufficient thanks to co-Büchi reduction)

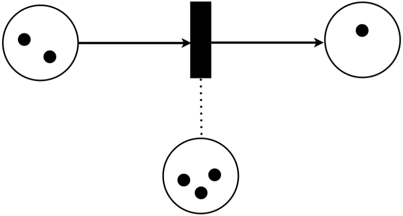
Approach

- We prove **non-primitive recursive**[1] **(NPR) hardness**
- Reduction from the termination problem in **reset nets** (Petri nets with reset arcs) [Sch02]

[1] Cf. Ackermann function

Reset nets

- Classic Petri net (places, tokens, transitions) with added *reset arcs*



▷ Transitions may empty a place from all its tokens

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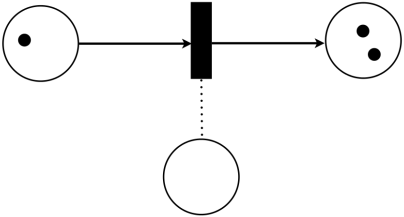

▷ Transitions may empty a place from all its tokens

▷ Given an initial marking, the *termination problem* asks if there exists an infinite sequence of transitions that can be fired

From reset nets to **direct** bounded window games

- Crux of the construction: encoding the markings
  - We use one dimension for each place
  - If a place p contains m tokens, then there will be an open window on dimension p with sum value $-m-1$
  - Hence **during a faithful simulation, all windows remain open** (you cannot consume tokens that do not exist)

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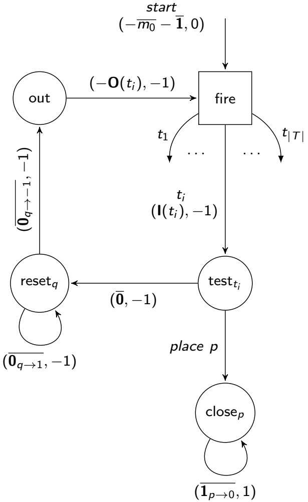
  - Hence **during a faithful simulation, all windows remain open** (you cannot consume tokens that do not exist)

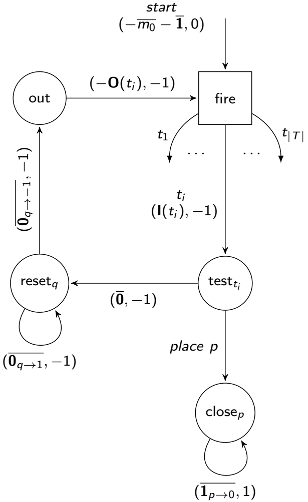
- $\mathcal{P}_2$ simulates the net
- $\mathcal{P}_1$ checks if he is faithful
- $\mathcal{P}_1$ wants to win the direct bounded window MP obj.
  - only able to do so if $\mathcal{P}_2$ cheats, i.e., if all runs terminate

The construction in a nutshell



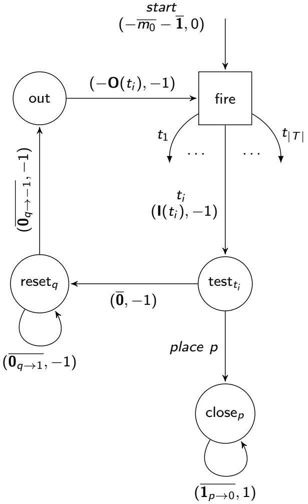
- ▷ The initial marking open corresponding windows in all places
- ▷ $\mathcal{P}_2$ chooses transitions to fire, which consume tokens
- ▷ $\mathcal{P}_1$ can branch or continue (and apply reset, then output)

The construction in a nutshell



- If no infinite execution exists, at some point, $\mathcal{P}_2$ must choose a transition without the needed tokens on some place p
- The window closes on dimension p
- By branching $\mathcal{P}_1$ can close all other windows and ensure winning

The construction in a nutshell



- ▷ If $\mathcal{P}_1$ branches while $\mathcal{P}_2$ is honest, one window stays open forever and he loses
- ▷ The additional dimension ensures that $\mathcal{P}_1$ leaves the reset state

Extension to bounded window objective

- More involved construction

Theorem

In two-player multi-dimension games, the bounded window mean-payoff problem is non-primitive recursive hard.

A new family of objectives

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\text{MP}}$ / $\overline{\text{MP}}$	NP $\cap$ coNP	mem-less		coNP-c. / NP $\cap$ coNP	infinite	mem-less
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- ▷ Conservative approximation of MP/TP
- ▷ For one-dim. games with poly. windows, we are in **P**
- ▷ For multi-dim. games with fixed windows, we are **decidable**
- ▷ Window obj. provide **timing guarantees**
- ▷ *Open question*: is BW decidable in multi-dim. ?

Check the full version on arXiv! abs/1302.4248

Thanks!

Do not hesitate to discuss with us!

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