## PART III

ADDITIONAL COMMUNICATIONS

# NON-MONOTONOUS TEMPERATURE DEPENDENCE OF INTERFACIAL TENSIONS 

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Let us consider two media in coexistence $A$ and $B$ for instance. The interface between these two media is characterized by its interfacial tension $\tau$. As a function of the temperature $T$, it is generally accepted that $\tau$ should be a decreasing function. However, for some substances, $\tau$ may become a more complicated function of $T$. This is the case for water -( n )- alcohol of $4-12$ carbon atoms media. Indeed, as shown in figure $1, \tau$ presents a parabolic profile as a function of $T$ within a certain range of temperature. These data have been obtained using the Wilhelmy plate method [1]


Fig. 1. The interfacial tension $\tau(T)$ : (*) water-1-heptanol; water-1-decanol.

In this note, we would like to introduce a microscopic model which lead to a good description of these data. More details may be found in [2].

Due to the chemical characteristics of the molecules of alcohol, it is expected that these molecules will be perpendicular to the water-alcohol interface as represented in figure 2a. This means that, on a microscopic point of view, the interface may be viewed as a superposition of steps (fig 2.b).


Fig. 2. Sketch of the interface between water and $n$-alcohol: (a) a schematic representation; (b) a step model.

Let $J$ denote the energetic cost associated to this interface per unit of surface. The energy $E_{\Lambda}$ of this interface $A$ may thus be written as

$$
E_{\Lambda}=J . S_{\Lambda}
$$

where $S_{\Lambda}$ represents the surface of $\Lambda$. To evaluate this surface, we shall introduce some variables $h_{i}$ which give the height of the interface associated to the point $i$ with respect to some reference plane (fig. 2.b).

To compute $S_{\Lambda}$ as a function of $h_{0} \cdots h_{N}$, we use some Gaussian approximation

$$
S_{A}=\frac{J}{4} \sum_{\underline{r}, r^{\prime}}\left\{a^{2}+\frac{\left(h_{\underline{\underline{r}}}-h_{\underline{r}^{\prime}}\right)^{2}}{2}\right\}
$$

where $a^{2}$ is the surface of one elementary square. We therefore have the following Hamiltonian for this interface $\Lambda$.

$$
H_{\Lambda}\left(h_{\underline{0}} \cdots h_{\underline{N}}\right)=-\frac{J}{4} \sum_{\underline{r}, \mathbf{r}^{\prime}}\left\{a^{2}+\frac{\left(h_{\underline{r}}-h_{\underline{r}^{\prime}}\right)^{2}}{2}\right\}
$$

The corresponding surface tension is given by

$$
\tau(T)=\lim _{N \nmid \infty}-\frac{1}{N a^{2}} \log \int d h_{\underline{0}} \cdots d h_{\underline{N_{N}}} e^{-\beta H_{\Lambda}\left(h_{\underline{Q}} \cdots h_{\underline{N}}\right) \Pi_{k \in \Omega \Lambda} \delta\left(h_{\underline{k}}\right)}
$$

where $\beta=1 / k_{B} T$ and $\partial \Lambda$ is the boarder of $\Lambda$.
After some calculations [z], we get

$$
\tau(T)=J+\frac{1}{2 a^{2} \beta} \log \frac{\beta J a^{2}}{0.60981}
$$

This explicit result allows us to plot the experimental data in terms of $J$. The corresponding graph is reproduced hereafter (figure 3) and reveals surprisingly a linear behaviour of $J$ as a function of $\beta$.


Fig. 3. The effective cost of energy $J$ as a function of $\beta$ from the experimental data given in fig. 1.

The adjusted constants of the form

$$
J=c \beta+d
$$

are given in the following table with the corresponding "best value" of $a$.

|  | $a(\AA)$ | $c\left(J^{2} m^{-2}\right)$ | $d\left(J m^{-2}\right)$ | $T_{\max }\left({ }^{\circ} K\right) T^{*}=\frac{k T_{\max }}{a \sqrt{-c}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| n-butanol | 4.323 | $-2.81310^{-23}$ | $14.5810^{-3}$ | 293 | 1.764 |
| n-pentanol | 3.574 | $-4.64810^{-23}$ | $23.4810^{-3}$ | 314 | 1.779 |
| n-Hexanol | 3.276 | $-5.79210^{-23}$ | $29.0610^{-3}$ | 322 | 1.783 |
| n-Heptanol | 3.403 | $-5.84410^{-23}$ | $28.7610^{-3}$ | 334 | 1.772 |
| n-octanol | 3.226 | $-7.01910^{-23}$ | $33.2910^{-3}$ | 344 | 1.762 |
| n-nonanol | 2.778 | $-9.75510^{-23}$ | $44.3410^{-3}$ | 347 | 1.746 |
| n-Decanol | 1.972 | $-1.85910^{-22}$ | $82.5310^{-3}$ | 345 | 1.772 |
| n-Dodecanol | 2.679 | $-1.88510^{-22}$ | $79.3910^{-3}$ | 362 | 1.751 |

Let us point out here that the agreement with the experimental data is less than $1 \%$ for more than 150 experimental points.


Fig. 4. The adjusted $J$ as a function of $\beta$ for (*) water-1-heptanol; (■) water-1-decanol.

Let us conclude this note by stressing the surprising linear character of $J$ as a function of $\beta$. The origin of this property remains up to now an open question.

## References

1. Villers, D. and Platten, J.; J. Phys. Chem. 92, 4023 (1988).
2. De Coninck, J; Villers, D. and Platten, J.; J. Phys. Chem. 94, 5057 (1990) and references therein.
