AdS$_5$ rotating non-Abelian black holes

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Abstract

We present arguments for the existence of charged, rotating black holes with equal magnitude angular momenta in $d = 5$ Einstein-Yang-Mills theory with negative cosmological constant. These solutions posses a regular horizon of spherical topology and approach asymptotically the Anti-de Sitter spacetime background. The black hole solutions have also an electric charge and a nonvanishing magnetic flux through the sphere at infinity. Different from the static case, no regular solution with a nonvanishing angular momenta is found for a vanishing event horizon radius.

1 Introduction

The conjectured equivalence of string theory on anti-de Sitter (AdS) spaces and certain superconformal gauge theories living on the boundary of AdS [1, 2] has lead recently to an increasing interest in asymptotically anti-de Sitter (AAdS) black holes. This type of solutions are of special interest since they offer the possibility of studying the nonperturbative structure of some conformal field theories (CFTs).

It is therefore desirable to widen the existing AAdS classes of solutions as much as possible, the case of five dimensional solutions being of particular interest, given the conjectured equivalence between $\mathcal{N} = 4$, $d = 4$ SU(N) YM theory and supergravity solutions in AdS$_5$. Rotating black holes with AdS asymptotics in $d = 5$ have been studied by various authors, starting with Hawking *et.al.* [3] who found the higher dimensional counterparts of the Kerr-AdS$_4$ solution. Charged rotating black holes solutions in $d = 5$ gauged supergravity have been constructed in [4]. Apart from Abelian fields with a Chern-Simons term, these configurations usually contain scalar fields with a nontrivial scalar potential. Rotating black hole solutions in pure Einstein-Maxwell (EM) theory with negative cosmological constant have been constructed numerically in recent work [5].

At the same time, one should remark that gauged supergravity theories playing an important role in AdS/CFT, generically contain non-Abelian matter fields in the bulk, although in the literature mainly Abelian truncations are considered, to date. The lack of attention given to AAdS Einstein-Yang-Mills (EYM) solutions is presumably due to the notorious absence of closed form solutions in this case.

However, one can analyse their properties by using a combination of analytical and numerical methods, which is enough for most purposes. Thus, the examination of AdS_5 gravitating non-Abelian solutions with $\Lambda < 0$ is a pertinent task.

Practically, all that is known in the subject of $d = 5$ AAdS non-Abelian solutions are the EYM-SU(2) spherically symmetric configurations discussed in [6] and the static solutions in [7] of the $N = 4^+$ version of the Romans' gauged supergravity model [8]. These solutions share a number of properties with the better known $d = 4$ EYM AAdS configurations discussed in [9],[10]. In both cases, regular and black hole solutions exist for compact intervals of the parameter that specifies the initial conditions at the origin or at the event horizon. The gauge field approaches asymptotically a configuration which is not a pure gauge, resulting in a nonvanishing magnetic flux through the sphere at infinity.

However, different from the four dimensional case, the mass of an AdS_5 configuration as defined in the usual way presents a logarithmic divergence. In the recent work [11], a counterterm based method has been proposed to regularise the action and mass-energy of the non-Abelian AdS_5 solutions. In this approach, the logarithmic divergence of the action results in a trace anomaly in the dual CFT.

The main purpose of this paper is to present numerical arguments for the existence of rotating $AAdS_5$ non-Abelian black hole solutions. Instead of specializing to a particular supergravity model, we shall consider the simpler case of a EYM-SU(2) theory with negative cosmological constant. Although it seems that this theory is not a consistent truncation of any $d = 5$ supersymmetric model, it enters the gauged supergravities as the basic building block and one can expect the basic features of its solutions to be generic. Also, we shall restrict here to the case of rotating solutions with equal magnitude angular momenta and a spherical topology of the event horizon, which allows us to deal with ordinary differential equations (ODEs).

These solutions share a number of common features with the $d = 4$ counterparts discussed in [12], [13]. In both cases, one finds rotating solutions starting with any static configuration. The rotating black holes have a nonzero electric charge and a nonvanishing flux through the sphere at infinity. However, different from the AdS_4 case [13], here no rotating soliton solutions are found in the limit of zero event horizon radius.

The paper is structured as follows: in Section 2 we present the general framework and analyse the field equations and boundary conditions. The black hole properties are discussed in Section 3. We present the numerical results in Section 4. We conclude with Section 5 where the results are compiled.

2 The model

2.1 The action principle and field equations

We consider the five dimensional SU(2) Einstein-Yang-Mills (EYM) action with negative cosmological constant $\Lambda = -6/\ell^2$

$$I = \int_{\mathcal{M}} d^5x\sqrt{-g}\Big(\frac{1}{16\pi G}(R - 2\Lambda) - \frac{1}{2e^2}\mathrm{Tr}\{F_{\mu\nu}F^{\mu\nu}\}\Big) - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^4x\sqrt{-h}K, \tag{1}$$

Here G is the gravitational constant, R is the Ricci scalar associated with the spacetime metric $g_{\mu\nu}$. $F_{\mu\nu} = \frac{1}{2}\tau^a F_{\mu\nu}^{(a)}$ is the gauge field strength tensor defined as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, with a gauge potential $A_\mu = \frac{1}{2}\tau^a A_\mu^{(a)}$, τ^a being the Pauli matrices and e the gauge coupling constant. K is the trace of the extrinsic curvature for the boundary $\partial\mathcal{M}$ and h is the induced metric of the boundary.

Variation of the action (1) with respect to $g^{\mu\nu}$ and A_μ leads to the field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\ T_{\mu\nu}, \quad \nabla_\mu F^{\mu\nu} - i[A_\mu, F^{\mu\nu}] = 0, \tag{2}$$

where the YM stress-energy tensor is

$$T_{\mu\nu} = \frac{2}{e^2}\mathrm{tr}\{F_{\mu\rho}F_{\nu\lambda}g^{\rho\lambda} - \frac{1}{4}g_{\mu\nu}F_{\rho\lambda}F^{\rho\lambda}\}. \tag{3}$$

2.2 The ansatz

While the general EYM-AdS rotating black holes would possess two independent angular momenta and a more general topology of the event horizon, we restrict here to configurations with equal magnitude angular momenta and a spherical horizon topology. The suitable metric ansatz reads [14]

$$\begin{aligned} ds^2 = \frac{dr^2}{f(r)} &+ g(r)d\theta^2 + h^2(r)(\sin^2\theta(d\varphi - w(r)dt)^2 + \cos^2\theta(d\psi - w(r)dt)^2) \\ &-(h^2(r) - g(r))\sin^2\theta\cos^2\theta(d\varphi - d\psi)^2 - f(r)\sigma^2(r)dt^2\ , \end{aligned} \tag{4}$$

where $\theta \in [0, \pi/2]$, $(\varphi, \psi) \in [0, 2\pi]$, r and t being the radial and time coordinates. This line element presents five Killing vectors

$$\begin{aligned} K_1 &= \frac{1}{2}\sin(\psi - \varphi)\partial_\theta - \frac{1}{2}\cos(\psi - \varphi)\cot\theta\partial_\varphi - \frac{1}{2}\cos(\psi - \varphi)\tan\theta\partial_\psi, \\ K_2 &= \frac{1}{2}\cos(\psi - \varphi)\partial_\theta + \frac{1}{2}\sin(\psi - \varphi)\cot\theta\partial_\varphi + \frac{1}{2}\sin(\psi - \varphi)\tan\theta\partial_\psi, \\ K_3 &= -\frac{1}{2}\partial_\varphi + \frac{1}{2}\partial_\psi, \quad K_4 = \frac{1}{2}\partial_\varphi + \frac{1}{2}\partial_\psi, \quad K_5 = \partial_t. \end{aligned} \tag{5}$$

The computation of the appropriate SU(2) connection compatible with the symmetries of the metric ansatz (4) can be done by applying the standard rules for calculating the gauge potentials for any spacetime group [15, 16]. According to Forgacs and Manton, a gauge field admit a spacetime symmetry if the spacetime transformation of the potential can be compensated by a gauge transformation [15], $\mathcal{L}_{K_i}A_\mu = D_\mu U_i$, where $\mathcal{L}$ stands for the Lie derivative. The expression we find in this way for the gauge field ansatz is

$$\begin{aligned} A_r = 0,\ A_\theta = (2W(r), 0, 0)\ ,\ A_\varphi = (0, -W(r)\sin 2\theta, H(r) - \cos 2\theta(H(r) + 1))\ , \\ A_\psi = (0, W(r)\sin 2\theta, H(r) + \cos 2\theta(H(r) + 1))\ , \quad A_t = (0, 0, V(r))\ , \end{aligned} \tag{6}$$

the only nonvanishing components of the compensating potentials U_i being

$$U_1 = \frac{1}{2}\frac{\cos(\psi - \varphi)}{\sin 2\theta}\tau_3, \quad U_2 = \frac{1}{2}\frac{\sin(\psi - \varphi)}{\sin 2\theta}\tau_3. \tag{7}$$

The general ansatz (4), (6) can be proven to be consistent, and, as a result, the EYM equations reduce to a set of seven ODEs (in the numerics, we fix the metric gauge by taking $h(r) = r$). The solutions have a spherically symmetric limit with

$$g(r) = r^2,\ h(r) = r,\ w(r) = 0, \quad W(r) = \frac{1}{2}(\tilde{w}(r) + 1), \qquad H(r) = \frac{1}{2}(\tilde{w}(r) - 1), \quad V(r) = 0, \tag{8}$$

whose basic properties were discussed in [6]. The vacuum rotating black holes in [3] with two equal angular momenta are recovered for a vanishing gauge field, $H = -1,\ W = 0$ (or, equivalently, $H = 0,\ W = 1$), $V = 0$ and

$$f(r) = 1 + \frac{r^2}{\ell^2} - \frac{2\hat{M}}{r^2}(1 - \frac{a^2}{\ell^2}) + \frac{2\hat{M}\hat{a}^2}{r^4}, \quad h^2(r) = r^2\left(1 + \frac{2\hat{M}\hat{a}^2}{r^4}\right), \qquad (9)$$
$$w(r) = \frac{2\hat{M}\hat{a}}{rh^2(r)}, \quad g(r) = r^2, \quad b(r) = \frac{r^2 f(r)}{h^2(r)},$$

where $\hat{M}$ and $\hat{a}$ are two constants related to the solution's mass and angular momenta. The Einstein-Maxwell solutions in [5] are recovered for the metric ansatz (4) written in an isotropic coordinate system and an U(1) subgroup of (6), obtained for $W(r) \equiv 0$.

3 Black Hole Properties

3.1 Asymptotic expansion and boundary conditions

Similar to the vacuum case (9), the horizon of these rotating black holes is a squashed S^3 sphere and resides at a constant value of the radial coordinate $r = r_h$, being characterized by $f(r_h) = 0$. At the horizon, the solutions satisfy the boundary conditions

$$f|_{r=r_h} = 0, \quad g|_{r=r_h} = g_h, \quad \sigma|_{r=r_h} = \sigma_h, \quad w|_{r=r_h} = \Omega_H, \qquad (10)$$
$$H|_{r=r_h} = H_h, \quad W|_{r=r_h} = W_h, \quad V|_{r=r_h} = -2\Omega_H H_h,$$

where $g_h,\ \sigma_h, \Omega_H,\ H_h$ and W_h are free parameters (with $(g_h,\ \sigma_h) > 0$).

We find also the following asymptotic expansion as $r \to \infty$

$$f(r) = 1 + \frac{r^2}{\ell^2} + \frac{f_2}{r^2} - \frac{128\pi G}{e^2}(W_0 - 1)^2 W_0^2 \frac{\log(r/\ell)}{r^2} + \dots, \quad \sigma(r) = 1 + \frac{s_4}{r^4} + \dots,$$
$$g(r) = r^2 - \frac{s_4}{r^2} + \dots, \quad w(r) = \frac{\hat{J}}{r^4} + \dots, \qquad (11)$$
$$H(r) = W_0 - 1 + \frac{H_2}{r^2} - 2\ell^2 W_0(W_0 - 1)(2W_0 - 1)\frac{\log(r/\ell)}{r^2} + \dots,$$
$$W(r) = W_0 + \frac{W_2}{r^2} - 2\ell^2 W_0(W_0 - 1)(2W_0 - 1)\frac{\log(r/\ell)}{r^2} + \dots, \quad V(r) = \frac{q}{r^2} + \dots,$$

where $f_2, s_4, \hat{J}, W_0,\ W_2,\ H_2$ and q are real constants. Note that these asymptotics preserve the full AdS symmetry group.

One can see that, similar to the static case, the g_{tt} component of the metric has a term proportional with $(1 - W_0)^2 W_0^2 (\log r)/r^2$, which leads to a divergent value of the mass-energy as defined in the usual way, unless $W_0 = 0$ or $W_0 = 1$. However, we could not find rotating non-Abelian solutions with these values of W_0. This agrees with the physical intuition based on a heuristic Derick-type scaling argument, although a rigorous proof exists for the spherically symmetric limit only [6].

3.2 Global charges

3.2.1 The mass and angular momenta

The mass-energy and angular momenta or these solutions is computed by using the procedure proposed by Balasubramanian and Kraus [17], which furnishes a means for calculating the gravitational action and conserved quantities without reliance on any reference spacetime. This technique was inspired by AdS/CFT correspondence and consists of adding suitable counterterms I_{ct} to the action of the theory in order to ensure the finiteness of the boundary stress tensor [18]. As found in [17], the following counterterms are sufficient to cancel divergences in five dimensions, for AdS_5 vacuum black hole solutions[1] (here R is the Ricci scalar for the boundary metric h)

$$I_{\rm ct} = -\frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^4x\sqrt{-h}\left[\frac{3}{\ell}+\frac{\ell}{4}\mathrm{R}\right]\,. \tag{12}$$

Using these counterterms one can construct a divergence-free stress tensor from the total action $I_{tot}=I+I_{\rm ct}$ by defining

$$\mathrm{T}_{AB} \;=\; \frac{2}{\sqrt{-h}}\frac{\delta I_{tot}}{\delta h^{AB}} = \frac{1}{8\pi G}(K_{AB}-Kh_{AB}-\frac{3}{\ell}h_{AB}+\frac{\ell}{2}E_{AB}), \tag{13}$$

where E_{AB} is the Einstein tensor of the intrinsic metric h_{AB}.

The presence of the additional matter fields in the bulk action brings the potential danger of having divergent contributions coming from both the gravitational and matter action [19]. Various examples of AAdS solutions whose action and mass *cannot* be regularized by employing only the counterterm (12) have been presented in the literature. This is also the case of the AdS_5 non-Abelian solutions, where the backreaction of the gauge fields causes certain metric components to fall off slower than usual. As a result, the action and the mass-energy present generically a logarithmic divergence, unless one considers corrections to the YM Lagrangean consisting of higher order terms of the Yang–Mills hierarchy [20].

However, in such cases, it is still possible to obtain a finite mass and action by allowing I_{ct} to depend not only on the boundary metric h_{AB}, but also on the matter fields. The matter counterterm expression which is added to I_{tot} for AdS_5 non-Abelian solutions is [11] (with A, B boundary indices)

$$I_{ct}^{(m)} = -\log\left(\frac{r}{\ell}\right)\int_{\partial M} d^4x\sqrt{-h}\frac{\ell}{2e^2}\ \mathrm{tr}\{F_{AB}F^{AB}\}\ , \tag{14}$$

which yields a supplementary contribution to (13)

$$\mathrm{T}_{AB}^{(m)} = -\log\left(\frac{r}{\ell}\right)\frac{2\ell}{e^2}\ \mathrm{tr}\{F_{AC}F_{BC}h^{CD}-\frac{1}{4}h_{AB}\ F_{CD}F^{CD}\}. \tag{15}$$

Provided the boundary geometry has an isometry generated by a Killing vector ξ, a conserved charge

$$\mathfrak{Q}_\xi = \oint_\Sigma d^3S^i\ \xi^j\mathrm{T}_{ij} \tag{16}$$

can be associated with a closed surface Σ [17]. If $\xi = \partial/\partial t$ then $\mathfrak{Q}$ is the conserved mass/energy E; there are also two angular momenta associated with the Killing vectors $\partial/\partial\varphi$ and $\partial/\partial\psi$.

[1] These counterterms regularize also the mass-energy and action of rotating Einstein-Maxwell-AdS solutions in [5].

As a result, we find the following expressions for mass-energy and angular momentum of the solutions in this paper[2]:

$$E = -\frac{V_3}{8\pi G}\left(\frac{3f_2}{2} + \frac{4s_4}{\ell^2}\right) + E_c, \quad J_\varphi = J_\psi = J = -\frac{\hat{J}V_3}{16\pi G}\ , \tag{17}$$

where $E_c = 3\pi\ell^2/32G$ is a constant terms interpreted as the mass-energy of the AdS_5 background [17] and $V_3 = 2\pi^2$ is the area of the three sphere. One can prove that the term (14) regularizes also the tree level action of the solutions[3].

3.2.2 Other relations

These solutions have also an electric charge

$$Q_e = -\frac{1}{V_3}\lim_{r\to\infty}\int dS_k \text{tr}\{F^{kt}\frac{\tau_3}{2}\} = q. \tag{18}$$

By using the fact that the integral of the angular momentum density can be written as a difference of two boundary integrals [22], one writes

$$\int d^4x\ T_\varphi^t\sqrt{-g} = \oint_\infty 2\text{tr}\{A_\varphi F^{\mu t}\}dS_\mu - \oint_{r=r_h} 2\text{tr}\{A_\varphi F^{\mu t}\}dS_\mu. \tag{19}$$

(a similar relation holds for T_ψ^t). Making use of the Einstein equations, one finds the following relation

$$J - 2Q_e(W_0 - 1) = \frac{g_h r_h H_h}{\sigma_h}\Big(V'(r_h) + 2\Omega_H H'(r_h)\Big) + \frac{g_h r_h^3\Omega'(r_h)}{8\sigma_h}\ , \tag{20}$$

relating global charges to event horizon quantities. It is also of interest to evaluate the integral of $\text{tr}\{F_{\mu t}F^{\mu t}\}$. This measures the contribution of the non-Abelian electric field to the mass/energy of the system. Similar to the four dimensional case, by using the YM equations this integral can be expressed as

$$-E_e = \int \text{tr}\{F_{\mu t}F^{\mu t}\}\sqrt{-g}d^4x = \oint_\infty \text{tr}\{A_t F^{\mu t}\}dS_\mu - \oint_{eh} \text{tr}\{A_t F^{\mu t}\}dS_\mu. \tag{21}$$

Thus, for globally regular configurations, a vanishing magnitude of the electric potentials at infinity implies a purely magnetic solution. In contrast, one finds rotating black hole solutions with $A_t(\infty) = 0$ which are supported by the event horizon contribution. Since the asymptotic expansion (11) holds for both globally regular and black hole solutions, we conclude that there are no rotating EYM-SU(2) solitons in AdS_5 (the condition $V(\infty) = 0$ follows from the physical requirement that the spacetime approach the AdS background at infinity). However, rotating soliton solutions are likely to exist for a larger gauge group.

[2]Note that these quantities are evaluated in a frame which is nonrotating at infinity.

[3]In the absence of closed form solutions, there is no obvious way to perform a meaningful Wick rotation and obtain a real Euclidean solution for a rotating non-Abelian black hole. However, one can use a quasi-Euclidean approach as described in [21].

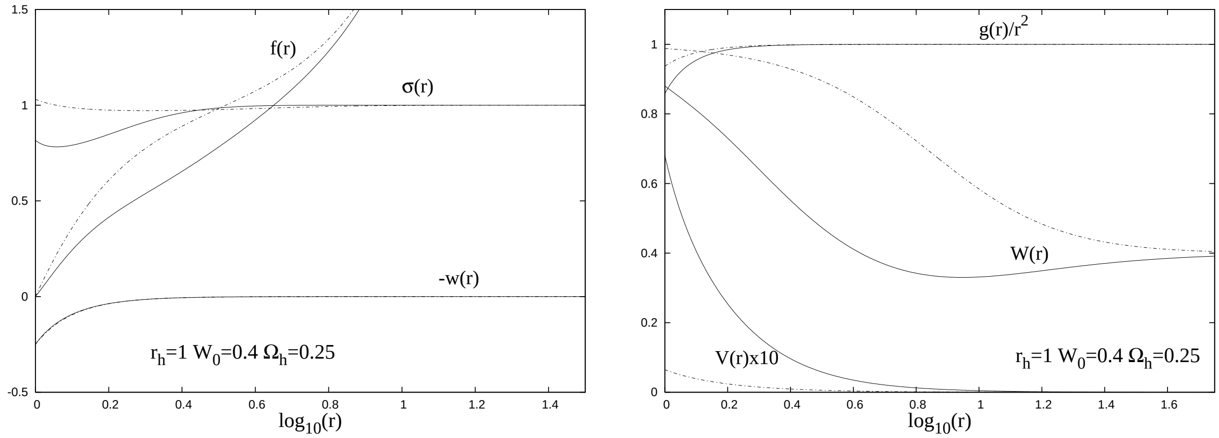

Figure 1: The profiles of the metric functions $f(r)$, $g(r)/r^2$, $w(r)$, $\sigma(r)$ and the non-Abelian gauge potentials $W(r)$, $V(r)$ are shown for two typical charged rotating black hole solutions with the same values of event horizon radius r_h, event horizon angular velocity Ω_H and magnitude of the magnetic potential at infinity W_0.

The Killing vector $\chi = \partial/\partial_t + \Omega_\varphi \partial/\partial\varphi + \Omega_\psi \partial/\partial\psi$ is orthogonal to and null on the horizon. For the solutions within the ansatz (4), the event horizon's angular velocities are all equal, $\Omega_\varphi = \Omega_\psi = w(r)|_{r=r_h}$. The Hawking temperature as found by computing the surface gravity is

$$T_H = \frac{\sqrt{b'(r_h)f'(r_h)}}{4\pi}. \tag{22}$$

Another quantity of interest is the area A_H of the rotating black hole horizon

$$A_H = r_h g_h V_3. \tag{23}$$

As usual, one identifies the entropy of black hole solutions with one quarter of the even horizon area, $S = A_H/4G$.

To have a measure of the deformation of the horizon, we introduce a deformation parameter defined as the ratio of the equatorial circumference L_e and the polar one L_p, which for these solutions we are considering takes the form

$$\frac{L_e}{L_p} = \frac{r_h}{\sqrt{g(r_h)}} \ . \tag{24}$$

These rotating solutions present also an ergoregion inside of which the observers cannot remain stationary, and will move in the direction of rotation. The ergoregion is the region bounded by the event horizon, located at $r = r_h$ and the stationary limit surface, or the ergosurface, $r = r_e$. The Killing vector $\partial/\partial t$ becomes null on the ergosurface , *i.e.* $g_{tt}(r_e) = -b(r_e) + r^2 w^2(r_e) = 0$. The ergosurface does not intersect the horizon.

4 The properties of solutions

Although we have considered other values as well, the numerical results reported in this section corresponds to $\ell = 10$, which is also the value taken in the study [6] of the static solutions. Dimensionless

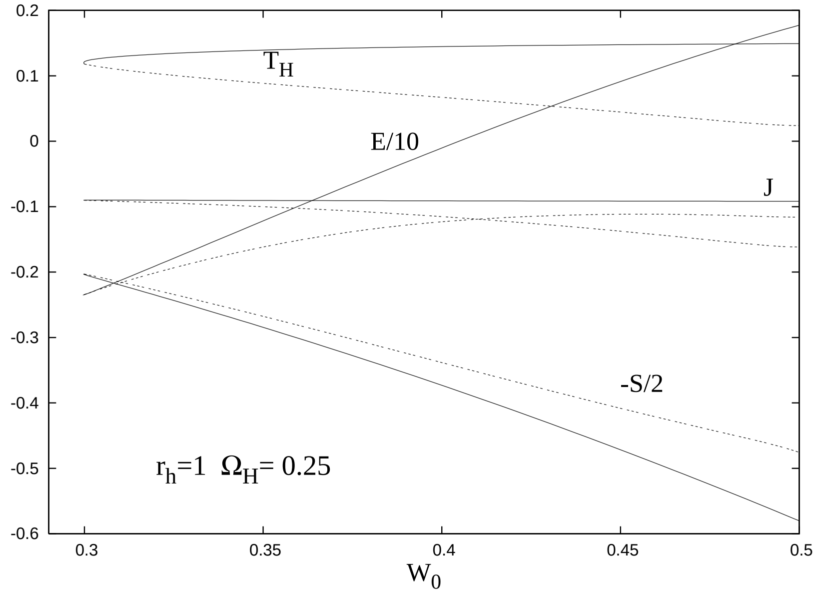

Figure 2: Some relevant parameters are plotted as a function of W_0 (with W_0 near the critical value $W_0^{(cr)}$) for rotating black hole solutions with $r_h = 1$, $\Omega_H = 0.25$.

quantities are obtained by using the rescaling $r \to r\sqrt{4\pi G}/e$, and $\Lambda \to \Lambda e^2/(4\pi G)$. To integrate the equations, we used the differential equation solver COLSYS which involves a Newton-Raphson method [23].

We start the description here by recalling the situation in the static limit. Spherically symmetric non-Abelian black holes exist for any value of the event horizon radius, a globally regular configuration being approached as $r_h \to 0$. The parameter W_0 in the boundary conditions at infinity is not fixed; however, one finds the existence of a minimal value of W_0, which depends on r_h. The mass of spherically symmetric black holes as defined in (17) may take negative values as well, for a range of $(W_0,\ r_h)$. One notice also the possible existence of several configurations for the same set (W_0, r_h).

The rotating solutions we have found preserve this general picture. As expected, we could find rotating solutions starting with any static black hole. Rotating solutions are obtained by increasing the value of Ω_H or $\hat{J}$ [4]. For all the solutions we studied, the metric functions $f(r)$, $g(r)$, $\sigma(r)$ and $w(r)$ interpolate monotonically between the corresponding values at $r = r_h$ and the asymptotic values at infinity, without developing any pronounced local extrema. (The magnetic gauge potentials present, however, a more complicated behaviour.) As a typical example, we present in Figure 1 the profile of two solutions with the same values of r_h, $W_0 = 0.3$ and Ω_H. These configurations are clearly distict and have different global charges.

The basic geometrical features of these rotating solutions are rather similar to the vacuum or U(1) case (*e.g.* the presence of an ergosphere and the fact that the horizon is deformed[5]). In the numerics we have paid special attention to the solutions' dependence on the magnitude of the magnetic potentials

[4]In the numerical procedure, we have fixed the values of W_0, Ω_H (or $\hat{J}$) together with $V(r_h) = -2\Omega_H H_h$, $f(r_h) = 0$, and a set of three more complicated conditions at the horizon involving both the functions and their derivatives.

[5]However, different from the U(1) case [5], the rotating non-Abelian solutions we have studied have $L_e/L_p > 1$ only.

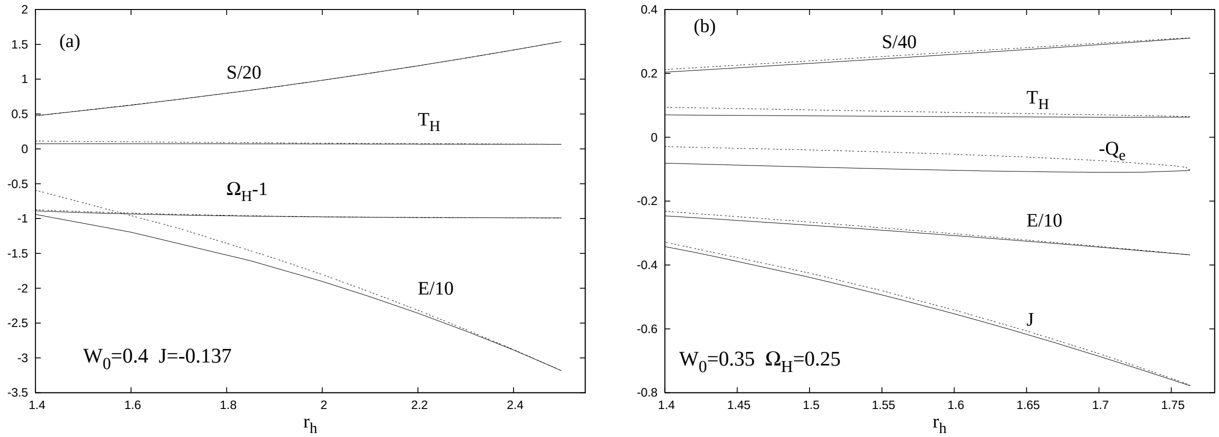

Figure 3: Some relevant parameters are plotten as a function of event horizon radius for rotating black holes with a fixed value of magnetic potentials at infinity and the same angular momenta (a) or event horizon velocity (b).

at infinity W_0, which is a purely non-Abelian feature. Even in this case, the configurations present a very rich structure, which makes their complete classification in the space of physical parameters a considerable task which is not aimed in this paper. Instead, we analyzed in details a few particular classes of solutions which, hopefully, reflect all the properties of the general pattern.

A feature of the rotating solutions we have studied so far is the existence of two different solutions for the same values of (r_h, Ω_H, W_0). These solutions have different global charges and distinct temperatures. No upper limit on W_0 seems to exist (although the numerics become very difficult for large W_0). When fixing the event horizon radius and the rotation parameter Ω_H (or $\hat{J}$), we have noticed, similar to the static case, the existence of a minimal value of W_0, say $W_0^{(cr)}(r_h, \Omega_H)$. At that point, a secondary branch of solutions emerges, which extends to larger values of W_0. This behaviour is illustrated in Figure 2. The occurrence of a minimal value of $W_0 > 0$ makes it unlikely that the non-Abelian black holes constructed here are bifurcations of the Abelian solutions which correspond to setting $W = 0$ in the equations.

We have also studied the dependence of solutions properties on the value of the event horizon radius for fixed w_0 and J. The numerical results strongly support the existence of two branches of rotating black hole solutions which join at a maximal value of r_h (see Figure 3a). This result further suggests that no solution exist for higher values of the event horizon r_h. As expected, the same pattern is found when taking instead a constant value of the event horizon velocity instead of J, see Figure 3b.

When a rotating black hole solution is considered for $r_h \to 0$ with the other parameters fixed, we observe that $V(r_h)$ converge to zero (in fact the electric potential V tends to zero uniformly in this limit) as well as J. In the same time the value $|w'(r_h)|$ goes to infinity, so that the rotation function $w(r)$ becomes more and more peaked at $r = r_h$. As a result, no rotating soliton solution is found.

Finally, we examine the properties of the solutions on the value Ω_H of the event horizon velocity. The numerical results obtained suggest that $W_0^{(cr)}$ depends weakly on Ω_H. Similar to the vacuum of EM cases, we noticed here again the existence of two branches of solutions. However, no definite conclusion is unfortunately available due to severe numerical difficulties that we met when increasing the value of Ω_H. The evolution of different parameter characterizing this family of solutions is reported on Figure 4 for $r_h = 1, W(r_h) = 0.4$; as far as we could see there is no signal that the two branches will meet at a

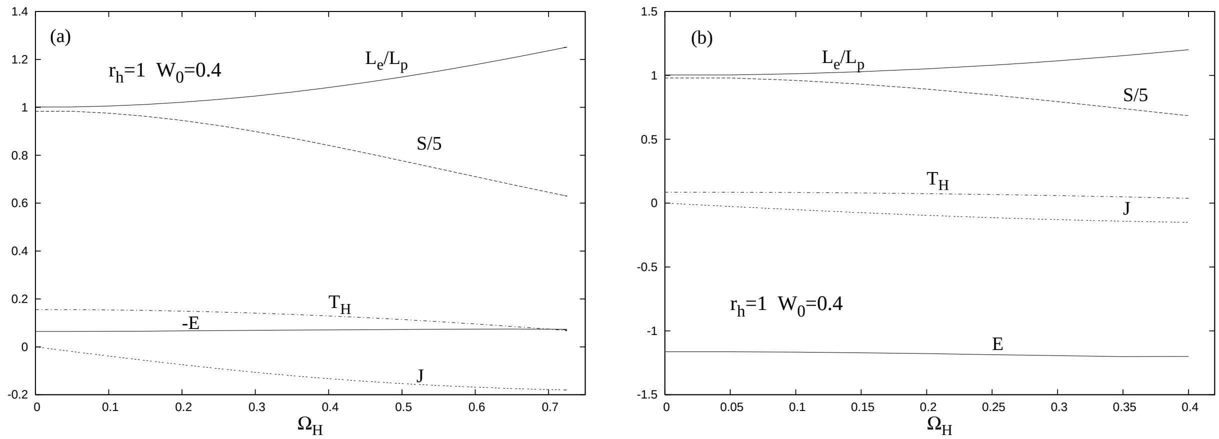

Figure 4: The evolution of some relevant parameters is plotten as a function of event horizon velocity for two branches rotating black holes with the same event horizon radius r_h and the same values of the magnetic potential at infinity.

maximal value of Ω_H, as happens for vacuum or EM rotating solutions (for $r_h = 1,\ W_0 = 0.4$ we could integrate the first branch up to $\Omega_H \simeq 0.75$ and the second branch up to $\Omega_H \simeq 0.45$, the numerical results becoming unreliable for larger values of Ω_H). This suggests that the two branches remain may open and exist for large values of the angular momentum. Integration of the equations with a different technique and/or a different metric parametrization may clarify this issue.

5 Further remarks

The main purpose of this paper was to present arguments for the existence of a general class of five dimensional AdS charged rotating solutions in EYM theory, in which the two angular momenta are equal. These solutions depend on four nontrivial parameters, namely the the mass, the angular momenta, the electric charge and the essential value of a magnetic potential at infinity.

This class of solutions may provide a fertile ground for further study of charged rotating configurations in gauged supergravity models and one expects some of their properties to be generic. Our preliminary results indicate the presence of similar solutions in the $\mathcal{N} = 4^+$ version of the Romans' gauged supergravity model, with a dilaton potential presenting a stationary point [8]. Rotating EYM topological black holes with an horizon of negative curvature are also likely to exist for $\Lambda < 0$. In addition, it would be interesting to generalize these solutions to higher dimensions, thus extending the study of charged U(1) black holes in [5] to EYM-Λ theory.

The study of the solutions discussed in this paper in an AdS/CFT context is an interesting open question. A generic property of the non-Abelian fields in AAdS backgrounds is that they do not approach asymptotically a pure gauge configuration. The boundary form of the non-Abelian potential (6) is

$$A_{(0)} = W_0 \tau_1 d\theta + \left(-W_0 \sin 2\theta \frac{1}{2}\tau_2 + (-W_0 \cos 2\theta + W_0 - 1)\frac{1}{2}\tau_3 \right) d\varphi + \left(W_0 \sin 2\theta \frac{1}{2}\tau_2 + (W_0 \cos 2\theta + W_0 - 1)\frac{1}{2}\tau_3 \right) d\psi \ , \tag{25}$$

with a nonzero boundary field strength tensor $F_{(0)}^{\mu\nu}$ (note that $A^{(0)}$ can be gauged away in the Abelian limit $W_0 = 0$). On the CFT side, these fields corresponds to external source currents coupled to various operators.

The metric on which the boundary CFT is defined is found by as $\gamma_{ab} = \lim_{r\to\infty} \frac{\ell^2}{r^2} h_{ab}$, and corresponds to a static Einstein universe in four dimensions,

$$\gamma_{ab} dx^a dx^b = -dt^2 + \ell^2 d\Omega_3^2. \tag{26}$$

One can use the AdS/CFT "dictionary" to predict qualitative features of a quantum field theory in this background. For example, the expectation value of the dual CFT stress-tensor can be calculated using the relation [24] $\sqrt{-\gamma}\gamma^{ab} < \tau_{bc} >= \lim_{r\to\infty} \sqrt{-h} h^{ab} \mathrm{T}_{bc}$. For these solutions we find the following non-vanishing components of the dual CFT stress-energy tensor (with $x^1 = \theta,\ x^2 = \varphi,\ x^3 = \psi,\ x^4 = t$)

$$< \tau_b^a >= \frac{N^2}{4\pi^2\ell^4}\Bigg[\frac{1}{2}\Big(\frac{1}{4} - \frac{f_2}{\ell^2} - \frac{4s_4}{\ell^4}\Big)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} + \frac{2s_4}{\ell^4}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \sin^2\theta & \sin^2\theta & 0 \\ 0 & \cos^2\theta & \cos^2\theta & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{27}$$
$$+2\hat{J}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sin^2\theta \\ 0 & 0 & 0 & \cos^2\theta \\ 0 & -\frac{1}{\ell^2} & -\frac{1}{\ell^2} & 0 \end{pmatrix}\Bigg] - \frac{8}{e^2}\frac{(W_0-1)^2W_0^2}{\ell^3}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

where we have replaced $8\pi G = 4\pi^2\ell^3/N^2$ [2], with N the rank of the gauge group of the dual $\mathcal{N} = 4,\ d = 4$ theory. The first three terms in this relation appear also for other known rotating solutions with equal magnitude angular momenta. The last term, however, is due to the existence of a non-Abelian matter content in the bulk and implies a nonvanishing trace of the $d = 4$ CFT stress tensor, $< \tau_a^a >= -24(W_0-1)^2W_0^2/(\ell^3e^2)$. From (25), this can be written as $< \tau_a^a >= -\frac{\ell}{4e^2}F_{(0)}^2$, in agreement with the general results [11].

Further progress in this direction may require to embed these solutions in a supergravity model.

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References

[1] E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253 [arXiv:hep-th/9802150].

[2] J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231 [Int. J. Theor. Phys. **38** (1999) 1113] [arXiv:hep-th/9711200].

[3] S. W. Hawking, C. J. Hunter and M. M. Taylor-Robinson, Phys. Rev. D **59** (1999) 064005 [arXiv:hep-th/9811056].

[4] Z. W. Chong, M. Cvetic, H. Lu and C. N. Pope, Phys. Rev. Lett. **95** (2005) 161301 [arXiv:hep-th/0506029]; Z. W. Chong, M. Cvetic, H. Lu and C. N. Pope, Phys. Lett. B **644** (2007) 192 [arXiv:hep-th/0606213]; Z. W. Chong, M. Cvetic, H. Lu and C. N. Pope, Phys. Rev. D **72** (2005) 041901 [arXiv:hep-th/0505112]; M. Cvetic, H. Lu and C. N. Pope, Phys. Rev. D **70** (2004) 081502 [arXiv:hep-th/0407058]; M. Cvetic, H. Lu and C. N. Pope, Phys. Lett. B **598** (2004) 273 [arXiv:hep-th/0406196].

[5] J. Kunz, F. Navarro-Lerida and E. Radu, arXiv:gr-qc/0702086.

[6] N. Okuyama and K. i. Maeda, Phys. Rev. D **67** (2003) 104012 [arXiv:gr-qc/0212022].

[7] E. Radu, Class. Quant. Grav. **23** (2006) 4369 [arXiv:hep-th/0601135].

[8] L. J. Romans, Nucl. Phys. B **267** (1986) 433.

[9] E. Winstanley, Class. Quant. Grav. **16** (1999) 1963 [arXiv:gr-qc/9812064].

[10] J. Bjoraker and Y. Hosotani, Phys. Rev. D **62** (2000) 043513 [arXiv:hep-th/0002098].

[11] Y. Brihaye and E. Radu, "Magnetic solutions in AdS_5 and trace anomalies", arXiv:0706.4378 [hep-th].

[12] R. B. Mann, E. Radu and D. H. Tchrakian, Phys. Rev. D **74** (2006) 064015 [arXiv:hep-th/0606004].

[13] E. Radu, Phys. Lett. B **548** (2002) 224 [arXiv:gr-qc/0210074].

[14] J. Kunz, F. Navarro-Lerida and A. K. Petersen, Phys. Lett. B **614** (2005) 104 [arXiv:gr-qc/0503010];

[15] P. Forgacs and N. S. Manton, Commun. Math. Phys. **72** (1980) 15.

[16] P. G. Bergmann and E. J. Flaherty, J. Math. Phys. **19** (1978) 212.

[17] V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208** (1999) 413 [arXiv:hep-th/9902121].

[18] J. D. Brown and J. W. York, Phys. Rev. D **47** 1407 (1993).

[19] M. M. Taylor-Robinson, arXiv:hep-th/0002125.

[20] E. Radu and D. H. Tchrakian, Phys. Rev. D **73** (2006) 024006 [arXiv:gr-qc/0508033].

[21] J. D. Brown, E. A. Martinez and J. W. York, Phys. Rev. Lett. **66** 2281 (1991); J. D. Brown and J.W. York, Phys. Rev. **D47** 1420 (1993), [arXiv:gr-qc/9405024]; I. S. Booth and R. B. Mann, Phys. Rev. Lett. **81**, 5052 (1998) [arXiv:gr-qc/9806015]; I. S. Booth and R. B. Mann, Nucl. Phys. B **539**, 267 (1999) [arXiv:gr-qc/9806056].

[22] J. J. Van der Bij and E. Radu, Int. J. Mod. Phys. A **17** (2002) 1477 [arXiv:gr-qc/0111046].

[23] U. Ascher, J. Christiansen, R. D. Russell, Math. of Comp. **33** (1979) 659; U. Ascher, J. Christiansen, R. D. Russell, ACM Trans. **7** (1981) 209.

[24] R. C. Myers, Phys. Rev. D **60** (1999) 046002.