Building functionality assignment in dense and compact blocks using graph theory and game theory

UCS 2021

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The CoMod Project

- Coordinated project supervised by the Faculty of Sciences and the Faculty of Architecture and Urban Planning of the University of Mons.
- In prospect of sustainable densification, we were interested in the notion of *compactness*.
- Compactness is characterised by high population density with respect to a set of comfort, legacy and privacy constraints.
- Previous study [DS18] identified many parameters and criterion on compactness and built a catalogue of more than 1600 compact typo-morphologies on the urban block scale.
- However, targeting urban compactness faces a difficult conciliation between various quantitative and qualitative parameters and completing the catalogue would need a very annoying work of encoding and diagnosis.

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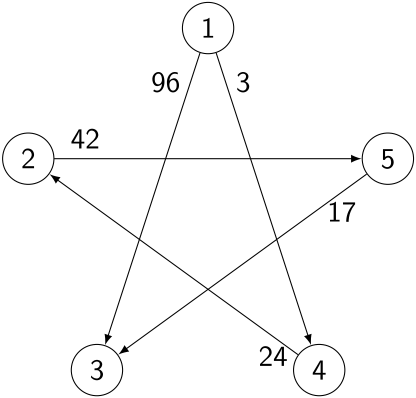
$\Longrightarrow$ We need mathematics and computer science to simplify our works

Question of search

"What is the optimal location for a certain type of building?"

Graph theory



A weighted directed graph [CLRS09] is defined by:

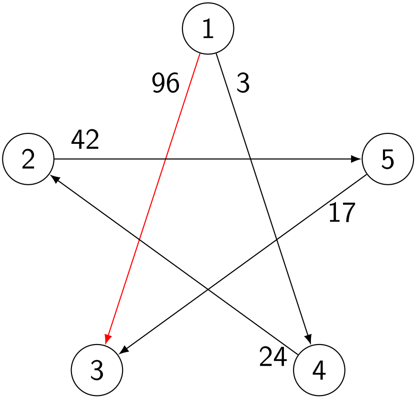
- a finite set of vertexes (also called nodes) V;
- a finite set of ordered pair called edges E. We note (u, v) the edge between the nodes u and v;
- a weight function $w : E \longrightarrow \mathbb{R}$.

Path in a graph

Let $G = (V, E)$ be a graph. A *path* of G is a sequence of nodes $P = (n_1, \ldots, n_k)$ with $k \geq 1$, such as $(n_i, n_{i+1}) \in E$ for all $0 \leq i \leq k-1$. We note $W(P)$ the weight or the length of P as:

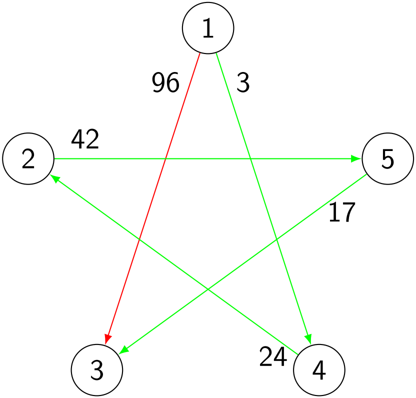
$$W(P) = \begin{cases} k-1 & \text{if } G \text{ is not weighted} \\ \sum_{i=0}^{k-1} w(s_i, s_{i+1}) & \text{if } G = (V, E, w) \text{ is a weighted graph.} \end{cases}$$

Path in a graph



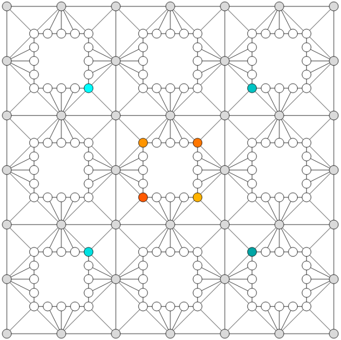
- $(1, 3)$ is a path from the node 1 to node 3 of cost 96.

Path in a graph



- $(1, 3)$ is a path from the node 1 to node 3 of cost 96.
- $(1, 4, 2, 5, 3)$ from the node 1 to node 3 of cost 86. This path is a *shortest path* from 1 to 3.

A urban block layout modeled by a graph



- The gray nodes are road nodes (crossroads, pedestrian crossing, *etc.*).
- The orange nodes are convenience stores.
- The blue nodes are schools.
- The white nodes are initially potential location for each type of building.
- After deciding, what is the best location for each building, white nodes are considered as housing.

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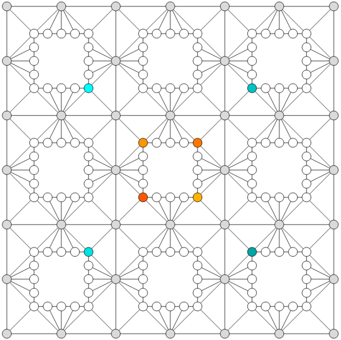

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To decide what node must have what color, i.e. what is the best location for each type of building, we are using some concepts from *game theory*

Notions of game theory

A strategic game [LCS16] is given by $\mathcal{G} = (N, (S_i)_{i \in N}, (\varphi_i)_{i \in N})$ where :

- $N = \{1, 2, \ldots, n\}$ is the set of players;
- For all $i \in N$, S_i is the set of strategies of player i.
  We note $\mathcal{S} = \prod_{i=1}^{n} S_i$ and we call an item of $\mathcal{S}$ a *strategies profile*.
- For all $i \in N, \varphi_i : \mathcal{S} \longrightarrow \mathbb{R}$ is the payoff function of the player i.

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- We assume that each players are *rational* and *selfish*.
- The objective of each player is to optimize his payoff.
- The players choose their strategy without worrying about the other players.

Nash equilibrium

The strategy profile $s^* \in \mathcal{S}$ is a *Nash Equilibrium* (NE) if and only if (assuming that the players want to maximize their payoff function)

$$\varphi_i(s_i^*, s_{-i}^*) \geq \varphi_i(s_i', s_{-i}^*) \ \forall s_i' \in S_i, \forall i \in N$$

- A NE is a strategies profile where no player is interested in changing his strategy.
- Following some hypothesis, there exist some techniques to find a NE in a game [LCS16].

Our modeling

Convenience store

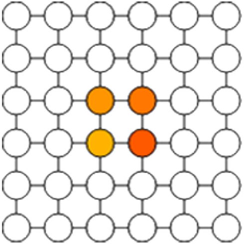
Given a graph G, we note $White(G)$ the set of white nodes of G. Let $N = \{1, \ldots, n\}$. We model our problem with the convenience store as the game $\mathcal{G} = (N, (S_i)_{i \in N}, (\varphi_i)_{i \in N})$, where for all $i \in N$, $S_i = White(G)$, we note $v_i \in White(G)$ the node in G selected by the player i and where

$$\varphi_i(v_i, v_{-i}) = \sum_{u \in White(G)} d_G(u, v_i).$$

The players want to *minimize* their payoff function.

Our modeling

Convenience store



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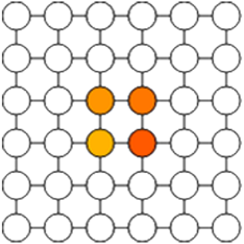

- Each player has an orange token that he can move on a white node of the graph.

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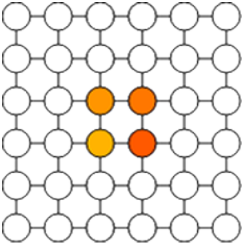

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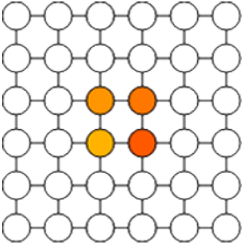

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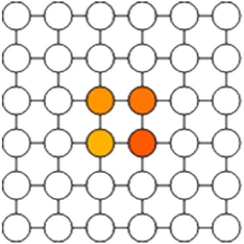

- Each player has an orange token that he can move on a white node of the graph.
- In a first time the tokens are randomly placed on the graph.
- Turn by turn, each player can move his token to an *adjacent* node to improve his payoff.
- When no more player has interest to move his token, a NE is reached.

Our modeling

School

As for the convenience stores, we use a multi-players game to find the best location for the schools. The difference with the orange players is their payoff functions. If for a given graph G we note $Blue(G)$ the set of blue nodes, the payoff function of the player i is given by:

$$\varphi_i(v_i, v_{-i}) = \sum_{u \in White(G)} \min_{b \in Blue(G)} d_G(u, b)$$

where v_i is the current vertex chosen by the player i.

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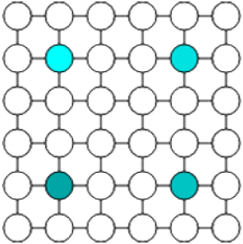
where v_i is the current vertex chosen by the player i.

- The payoff functions are the same for all the blue players. So they will cooperate.
- That symbolize the fact that schools are not directly in competition. Their objective is that all the citizens have access to a school close to their home.

Our modeling

School



Futur works

- We are now modeling this problem as an instance of the Facility Location Problem (FLP) [Gho03].
- In order to ensure that all residents have quick access to schools, it may be interesting not to have a fixed number of schools.
- The FLP allows us to find the minimal number of school to open and their optimal position.

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