A curious difference between $H(\mathbb{D})$ and $H(\mathbb{C})$

by

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It is a classical result of G. R. MacLane that the differentiation operator $Df = f'$ has a dense orbit when viewed as an operator on the space $H(\mathbb{C})$ of entire functions; in other words, it is hypercyclic. The same is then true if one regards D as an operator on the space $H(\mathbb{D})$ of holomorphic functions on the unit disk. More recent research has shown that the operator is even chaotic and frequently hypercyclic. If one widens the perspective then a curious difference appears between the spaces $H(\mathbb{D})$ and $H(\mathbb{C})$. Indeed, the differentiation operator is a special weighted shift operator. Now, it turns out that, on one of the two spaces, every frequently hypercyclic weighted shift is chaotic, while on the other one there are weighted shifts that are frequently hypercyclic but not chaotic. Which is the pathological space, and why?

Joint work with Stéphane Charpentier and Quentin Menet