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OPTIMAL DESIGN OF MULTIBODY SYSTEMS BY USING GENETIC ALGORITHMS

Sélim Datoussaïd
Faculté Polytechnique de Mons
Mécanique Rationnelle, Dynamique et Vibrations
Boulevard Dolez, 31
Mons, Hainaut 7000
Belgium
Email: selim@mecara.fpms.ac.be

Olivier Verlinden
Calogéro Conti
Mécanique Rationnelle, Dynamique et Vibrations
Faculté Polytechnique de Mons

ABSTRACT

The design step of multibody systems requires in some specific cases an optimization process, in order to determine the set of parameters which lead to optimal kinematic or dynamic performances. The aim of this paper is to propose an optimal design method adapted to general multibody systems and submitted to kinematic and/or dynamic time-dependent criteria. The optimization process is based on stochastic techniques referred to *genetic algorithms*, which are inspired from natural evolution and can often overtop classical optimization methods when applied to practical problems. Illustrative examples are given in the context of the optimization of the kinematics of a motorcar suspension and the lateral dynamics of an urban railway vehicle.
Keywords : multibody, optimization, design, genetic algorithms

NOMENCLATURE

$\underline{\mathbf{b}}$ vector of the n_b design variables
$\underline{\mathbf{q}}$ vector of the N configuration parameters
ψ_0 objective function
$\underline{\psi}$ vector of the m inequality constraints
$\phi_i(t)$ time dependent function
$< \phi_i >$ equivalent form of the time function
$\underline{\mathbf{F}}$ algebro-differential equations of motion
$\mathsf{F}(A)$ fitness associated with the alternative A

Introduction

When designing a multibody system, the dynamic analysis step predicts the behaviour of the mechanism for a special set of *design variables*. These ones have to be optimized in order to improve the performances of the system while respecting conceptual and technological constraints. Nevertheless when dynamic behaviour is involved, the optimization process becomes harder because both performances and constraints are explicitly time dependent.

The purpose of this paper is to describe a general method for the optimization of the dynamic behaviour of multibody systems. A stochastic method based on genetic algorithms has been applied as optimization engine. The kinematics of the multibody system is described in terms of relative coordinates and the equations of motion are written from a residual formulation (Verlinden, 1994).

Optimization of multibody systems

The optimization of the dynamic behaviour of multibody systems becomes a complicated task because of the explicit time dependency. On one hand, the equations of motion, so-called *state equations*, are time dependent, as well as the objective and constraint functions. On the other hand, the configuration parameters $\underline{\mathbf{q}}$, so called *state variables*, describe the motion of the multibody system and cannot be considered as design variables because their values are obtained from the integration of the state equations. The system of differential equations of motion (1) is expressed in its residual form by (Hiller, 1990; Verlinden, 1994)

$$\underline{\mathbf{F}}(\underline{\mathbf{b}},\underline{\mathbf{q}}(t),\underline{\dot{\mathbf{q}}}(t),\underline{\ddot{\mathbf{q}}}(t),t)=\underline{\mathbf{0}} \tag{1}$$

and eventually involves constraint equations if the mechanical system contains kinematic loops. Usually the optimization process deals with extreme dynamic response, subject to performance constraints that must hold over the entire time interval $[0\,\tau]$. The objective function ψ_0 which has to be maximized is written as

$$\psi_0=\max_{t\in[0\,\tau]}\left(M-f_0(\underline{\mathbf{b}},\underline{\mathbf{q}}(t),\underline{\dot{\mathbf{q}}}(t),\underline{\ddot{\mathbf{q}}}(t),t)\right) \quad M>>0 \tag{2}$$

The criterion f_0 can be for example the maximum acceleration of one of the bodies of the multibody system under operating conditions. The scalar M is chosen in such a way that ψ_0 remains positive.

The constraints are usually time dependent and express, for example, the limited range of the relative distance between two bodies

$$\underline{\phi}(\underline{\mathbf{b}},\underline{\mathbf{q}}(t),t)\leq 0 \quad t\in[0\,\tau] \tag{3}$$

Moreover the design variables $\underline{\mathbf{b}}$, which concern geometric characteristics, inertia, stiffness or damping properties, have to stay between technological lower and upper boundaries $\underline{\mathbf{b}}^L$ and $\underline{\mathbf{b}}^U$ that lead to further constraints

$$\underline{\psi}(\underline{\mathbf{b}})\equiv\begin{cases}\underline{\mathbf{b}}^L-\underline{\mathbf{b}}\leq\underline{\mathbf{0}}\\ \underline{\mathbf{b}}-\underline{\mathbf{b}}^U\leq\underline{\mathbf{0}}\end{cases} \tag{4}$$

To avoid dealing with time dependent functions, the initial problem is reformulated in the form of an equivalent functional formulation so that the constraints are transformed into time non-dependent functions. These are equal to zero when the constraints are satisfied and become positive otherwise. if the *equivalent* operator $<>$ is defined by (Haug, 1979)

$$<\phi_i(t)>=\begin{cases}\phi_i(t)\ if & \phi_i(t)\geq 0\\ 0\ if & \phi_i(t)<0\end{cases} \tag{5}$$

the equivalent constraint during $[0\,\tau]$ is replaced by

$$\underline{\psi}=\int_0^\tau<\underline{\phi}(t)>dt \tag{6}$$

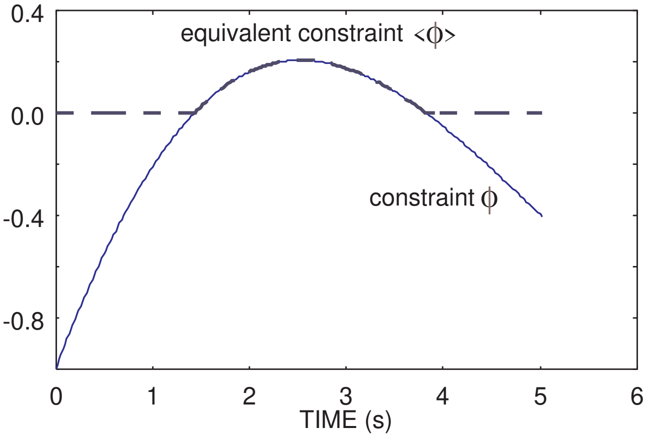

Figure 1. EQUIVALENT CONSTRAINT

Figure 1 shows the effect of the equivalent operator (5). We can see that $<\underline{\phi}(t)>$ is discontinuous when $\underline{\phi}(t)$ becomes positive. Then the derivatives don't exist in several points.
The non linear programming problem associated with the optimization of a multibody system has thus for objective to

> search the design variables set $\underline{\mathbf{b}}_{opt}$ that minimizes the objective function (2) and verifies the equations of motion (1) and the technological constraints (4) and (6).

Genetic algorithm

A genetic algorithm can be considered as an iterative scheme, where each iteration cycle forms a *generation* of an *evolutionary process* (Goldberg, 1989). Genetic algorithms don't directly use the design variables $\underline{\mathbf{b}}$ but a string code usually defined by a binary coding.

$$\text{design variables} \quad \underline{\mathbf{b}} \quad \underset{coding}{\overset{decoding}{\Longleftrightarrow}} \quad A=\{1,0,0,\ldots,1,1\} \quad \text{string}$$

$$\underline{\mathbf{b}}=\underline{\mathbf{g}}(A) \tag{7}$$

For an admissible individual A, the *fitness* can be defined as the value of the objective function. In order to eliminate the illicit individuals, a penalty transformation is used to significantly decrease their fitness

$$\mathsf{F}(A)=\psi_0-\alpha\sum_{i=1}^{m}\psi_i \qquad \alpha>0 \tag{8}$$

with :
α : penalty factor;
ψ_0 : objective function;
ψ_i : equivalent constraints $i = 1, \ldots, m$
Starting from a random initial *population*, the basic genetic algorithm consists of the following cycle :

1. selection
   It is a process in which individual strings are copied according to their fitness values. Strings with a higher value have a higher probability of contributing one or more offsprings in the next generation. Perhaps the easiest reproduction operator is to created a biased roulette wheel where each current string in the population has a roulette wheel slot sized in proportion to its fitness. One string is selected by simply spinning the roulette wheel.
2. crossover
   After reproduction, simple crossover may proceed into two steps (figure 2)
   (a) Two individuals from the newly population are selected at random.
   (b) For each pair of strings, a position k along them is selected uniformly at random. Two new strings are created by swapping the substrings of those individuals.

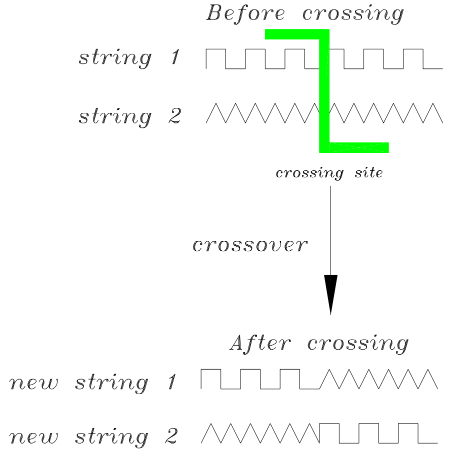

Figure 2. SCHEMATIC OF SIMPLE CROSSOVER

3. mutation
   It is the occasional random alteration of the value of a string position. This simply means changing a 1 to a 0 and vice versa.

In spite of the simplicity of these operators, it has been shown that genetic algorithms are robust optimization methods well adapted to practical problems (Goldberg, 1989). Moreover we can see that they don't use the derivatives and they don't make assumptions such as convexity to converge.

Using genetic algorithms for the multibody optimization

When a genetic algorithm is used for multibody optimization, the fitness associated to a given set of design variables, is described by the behaviour of the mechanical system. The objective function and the constraints involve dynamics (transient and steady state response) as well as kinematics.
The objective function and the time dependent constraints are transformed into their equivalent forms. The constraints that bound the design variables are chosen as the boundaries of the feasible domain. Then they are always verified. The use of a genetic algorithm needs the size of the population, the length of the strings, the coding function and the probability of crossing and mutation. Those parameters depend on the problem and the feasible domain. For multibody systems, populations of 40 to 80 individuals seem to be enough. In fact, the population size deeply depends on the feasible domain and the constraints. We use a linear binary coding function, as described in (9)

$$b_i = \frac{b_{i,max} - b_{i,min}}{2^n} Dec(A_i) \qquad i = 1, \ldots, n_b \tag{9}$$

with :
$Dec(A_i)$: decimal value of the n bits associated to the design variable b_i
$b_{i,min}$, $b_{i,max}$: minimal and maximal boundary values of the i$^{\text{th}}$ design variable
The string length depends on the number of design variables and their desired precision. More they should be accurate, longer will be the string. The crossover probability is usually between 0.6 and 0.8.
The mutation process destroys the strings. Then its probability of occurrence is small but it should be enough to guaranty that the research is performed through the whole domain, in order to get the global optimum. We propose to choice the mutation probability (p_m) by the following rule

$$p_m \approx \sigma \frac{1}{l} \qquad \text{with } \frac{1}{5} \leq \sigma \leq \frac{1}{3} \tag{10}$$

σ is the part of the population devoting to the exploration of the feasible domain. l is the length of the string. Then the ratio $\frac{1}{l}$ yields that only one site of the string mutates on average.

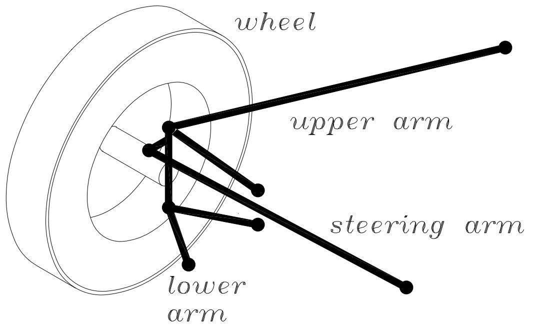

Figure 3. KINEMATIC OPTIMIZATION

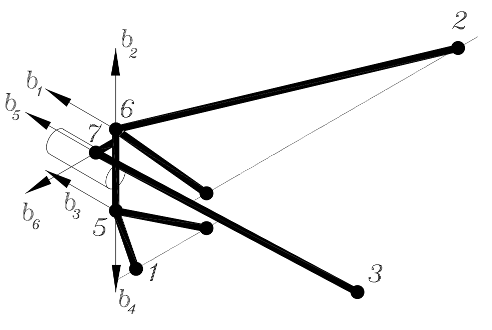

Figure 4. DESIGN VARIABLES

lateral amplitude of node 6	$0.1207 \leq b_1 \leq 0.2007\,m$
vertical amplitude of node 6	$0.1830 \leq b_2 \leq 0.3030\,m$
lateral amplitude of node 5	$0.1167 \leq b_3 \leq 0.1967\,m$
vertical amplitude of node 5	$0.1167 \leq b_4 \leq 0.1967\,m$
lateral amplitude of node 7	$0.0900 \leq b_5 \leq 0.2100\,m$
longitudinal amplitude of node 7	$0.2965 \leq b_6 \leq 0.4165\,m$

Table 1. RANGE VARIATION OF THE DESIGN VARIABLES

population size	80
string length	48
number of iterations	50
crossing probability	0.65
mutation probability	0.005
Penalty factor	1000

Table 2. OPTIMIZATION PARAMETERS FOR THE SUSPENSION

$b_1 = -0.1796\,m$	$b_2 = -0.2968\,m$	$b_3 = 0.1967\,m$
$b_4 = -0.1573\,m$	$b_5 = -0.2072\,m$	$b_6 = -0.4014\,m$
$\psi_{0_{opt}} = 0.064788\,rad$		

Table 3. OPTIMAL VALUES FOR THE SUSPENSION

Example of kinematic optimization

The short/long arm front suspension of figure 3 is composed of the upper and lower control arms and the steering arm connected to the wheel carrier and the car frame. The design of the links should give the smallest steer angle of the wheel when the suspension is subjected to a vertical motion. The kinematics of the mechanism is described in terms of relative coordinates. The optimization is performed on some coordinates of nodes 5, 6 and 7 (figure 4) with a total of six design variables. Table 1 shows their range variation. The coding function (9) associates an admissible design variables set with a string. In that case, 8 bits are used per design variables ($n = 8$), that yields to a 48 bits length string ($l = 48$). Table 2 shows the defining parameters of the genetic algorithm. Table 3 gives the optimal values.

The population concerns 80 individuals. The evolution is performed on 50 generations. Figure 5 shows the maximum steer angle of the wheel for the best string during the evolution. At the end of the optimization process the best individual minimizes the steer angle (figures 6).

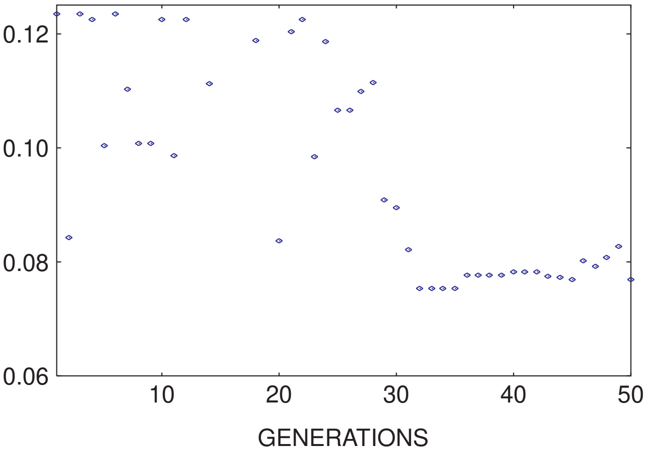

Figure 5. BEST SOLUTION DURING THE EVOLUTION (*rad*)

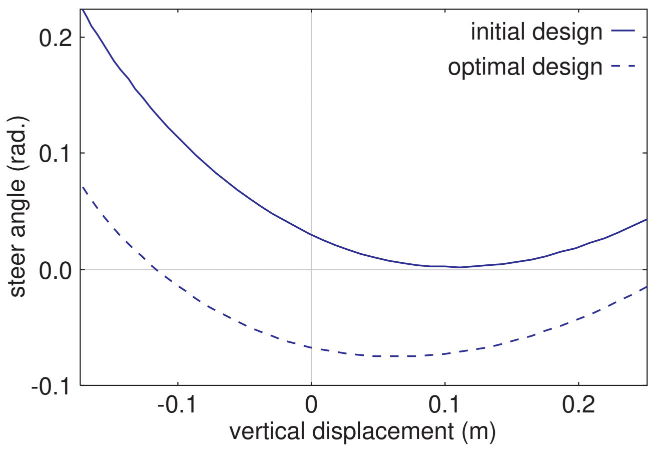

Figure 6. INITIAL AND OPTIMAL EVOLUTIONS

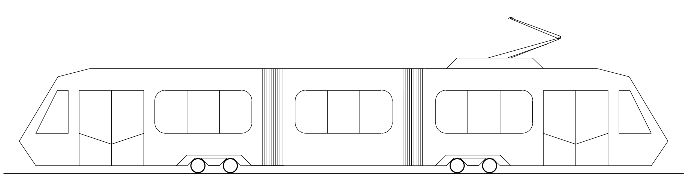

Figure 7. RAILWAY VEHICLE WITH THE BRIDGE CAR

yaw stiffness, front bogie	$5 \leq b_1 \leq 500\,KNm/rad$
yaw stiffness, rear bogie	$5 \leq b_2 \leq 500\,KNm/rad$
yaw damper, front bogie	$5 \leq b_3 \leq 50\,KNms/rad$
yaw damper, rear bogie	$5 \leq b_4 \leq 50\,KNms/rad$
yaw damper, front car articulation	$5 \leq b_5 \leq 50\,KNms/rad$
yaw damper, rear car articulation	$5 \leq b_6 \leq 50\,KNms/rad$
lateral stiffness, front bogie	$1 \leq b_7 \leq 100\,KN/m$
lateral stiffness, rear bogie	$1 \leq b_8 \leq 100\,KN/m$
lateral damper, front bogie	$0.5 \leq b_9 \leq 30\,KNs/m$
lateral damper, rear bogie	$0.5 \leq b_{10} \leq 30\,KNs/m$

Table 4. RANGE VARIATION OF THE DESIGN VARIABLES

population size	40
string length	100
number of iterations	80
crossing probability	0.65
mutation probability	0.005
Penalty factor	1000

Table 5. OPTIMIZATION PARAMETERS FOR THE STREETCAR

Example of dynamic optimization

When designing the suspension of a railway urban vehicle, one of the main criteria concerns the lateral behaviour of the tramway when it is running on narrow curves with small radius. In order to maximize the comfort when crossing such curves, the lateral acceleration has to be minimized by choosing efficient suspension properties. Figure 7 illustrates the case of a railway vehicle composed of three cars and two motor bogies (Datoussaïd, 1993), (Petz, 1991). It has to be adapted to specific traffic constraints corresponding to small radius curves (14 m) at low constant speed (4 m/s). In practice the problem consists to choose the secondary suspension characteristics between the cars and the bogies in such a manner that the maximum acceleration of the cars is minimum. The 10 design variables concern the lateral and yaw stiffness and damping values of the front and the rear bogies and the yaw damping value of the articulations. Table 4 shows their range variation. The constraints limit the relative yaw motion between cars and bogies. A 100 bits length string is used to code the 10 design variables ($n = 10$ bits per design variable, $l = 100$). Table 5 gives the parameters used.

Figure 8 shows the benefit for the bridge car between the initial design and the optimized suspension. The optimal design verifies the kinematic constraints. Figure 9 illustrates the yaw angle between the front car and the bogie. The optimal design decreases the angle and the acceleration in the same time.
We have repeated the optimization with 10 different initial populations chosen at random. Table 6 compares the optima. The values of the objective function are quite the same. But the design variables aren't. The lateral stiffness b_7 hasn't any influence on the solution. The yaw damper b_3 always reaches its upper limit.
In that way, we can depict the shape of the optimal zone but it has needed several optimizations. An other method to describe the optimal zone shape is based on the cluster theory (Hardy A., 1996).

Conclusion

The general method proposed for the optimization of the dynamic and kinematic behaviour of multibody systems has been successfully implemented. It combines computer aided dynamic analysis of mechanisms and non determinist search methods so called genetic algorithms. Such methods are interesting because

trial	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	ψ_0
	(KNm/rad)		$(KNms/rad)$				(KN/m)		(KNs/m)		(m/s^2)
1	13.90	17.24	47.10	32.54	15.95	47.36	79.29	7.19	16.01	27.95	2.767
2	25.66	5.39	48.426	41.25	30.25	47.32	59.45	20.16	23.91	28.01	2.844
3	26.00	36.84	48.55	26.64	16.39	10.41	75.71	4.10	14.80	10.62	2.793
4	7.42	20.58	47.32	24.00	15.42	29.28	30.32	7.29	26.02	22.56	2.732
5	13.61	16.85	48.06	10.67	21.85	49.82	34.19	25.77	15.98	29.91	2.806
6	27.60	6.16	46.74	48.94	13.84	41.82	31.19	56.55	5.63	26.25	2.844
7	16.23	16.32	47.89	32.32	18.81	41.73	20.16	11.93	9.78	15.18	2.733
8	17.05	19.08	49.91	21.32	10.06	27.39	60.81	2.93	9.47	11.40	2.706
9	36.84	44.14	49.08	24.35	8.17	47.45	89.84	7.39	18.12	29.83	2.820
10	30.40	27.79	49.65	23.52	30.91	47.32	83.64	13.88	15.87	29.88	2.814
average	21.47	21.04	48.27	28.56	18.17	38.99	53.73	15.72	15.56	23.16	2.786
deviation	2.75	3.68	0.318	3.23	2.28	3.82	8.95	4.81	1.89	2.35	0.01

Table 6. OPTIMA FROM 10 RANDOM INITIAL POPULATIONS

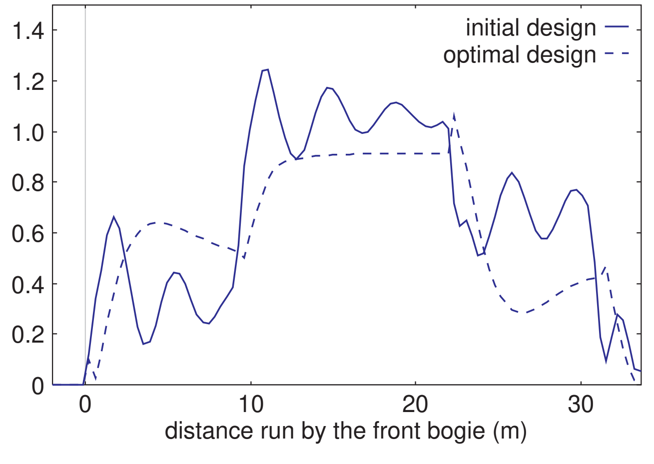

Figure 8. ACCELERATION OF THE BRIDGE CAR (m/s^2)

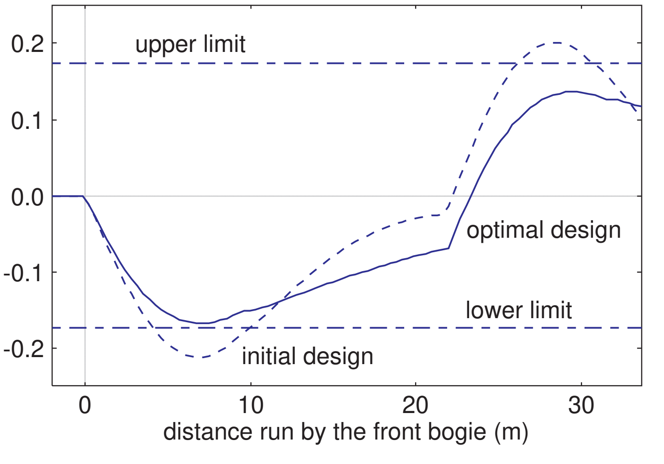

Figure 9. RELATIVE YAW ANGLE BETWEEN FRONT CAR AND BOGIE (rad)

they only use the value of the functions. Non-linear time dependent constraints are easily taken into account through a penalty transformation. So are the design variables through a binary string code.

The main advantages of such stochastic methods are that they don't need the derivatives to seek for the global optimum and they don't make assumptions on continuity or convexity, which are difficult to assure in multibody optimization because of the time dependent functions.

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