Backscattering of light from a water droplet: The glory effect

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We discuss the theoretical attempts at explaining the glory effect and describe a simple setup that permits visualization of the glory ray.

I. INTRODUCTION

When traveling by air on a sunny day, it may happen that you can see, out the window, the shadow of the airplane on the clouds below. This shadow may be surrounded by a halo of spectral colors: a phenomenon known as the "glory effect."

Many descriptions of this optical phenomenon can be found. As far as we know, one of the earliest was written in the 16th century by Benvenuto Cellini in his autobiography[1]: from the top of a hill with the sun behind him, he noticed that the shadow of his head on the wet grass was crowned with a bright halo.

Two centuries later, Thomas De Quincey[2] spoke about a German hill, the "Brocken" from the top of which, when the sun is behind him and near the horizon, the observer can see his magnified silhouette on the clouds, provided that there is some water vapor in the air. In the 19th century, Gaston Tissandier, a well-known balloonist, noticed several times that the shadow of his balloon on the clouds below was surrounded by a shiny halo.[3]

The conditions under which these colored halos are generally seen has led to giving them the name of "pilot's bow" or "Brocken bow" and also the name of "glory effect" because of the extraordinary brightness surrounding the shadow.

After a brief discussion of the phenomenon (Sec. II) we present the earliest attempts at scientifically explaining the glory (Sec. III); these descriptions used geometrical optics, as it was thought that water drops could reflect the light intensively. However, these explanations were deficient in many respects and physicists were led to use wave theory instead. This approach will be covered in Sec. VI. The results of a simulation of the glory by a method first proposed by H. C. Bryant and N. Jarmie[4] will then be reviewed (Sec. V). In Sec. IV, we present the simple setup we used to take a photograph of the light backscattered by a water droplet and we show a picture on which the glory ray may be identified.

II. SHORT DESCRIPTION OF THE GLORY

At the end of last century, C. T. R. Wilson built the first cloud chamber hoping that he would be able to create in the laboratory the necessary natural conditions for the observation of the glory. He gave up in order to experiment with tracks of charged particles in a water-saturated atmosphere and eventually developed the famous "cloud chamber," which played such an important role as a detector of charged particles in nuclear and particle physics. The presence of water drops whenever the glory effect was observed and the occurrence of the colors of the spectrum around the shadow of objects led to believe that this phenomenon was similar to the rainbow. But a critical detailed description of the classical rainbow and of the glory allows a clear distinction between these two phenomena. The ring of the first rainbow makes invariably an angle of 42° with the direction of the shadow received by the observer.[5] In the case of the glory the angular diameter is inversely proportional to the diameter of the water drops responsible for the phenomenon; besides the observer must make an angle of precisely 180° with the direction of the rays of light impinging on the drops if he is to observe the effect.

Figure 1 is a good illustration of the first rainbow. A water droplet of about 2 mm in diameter is hung on a hypodermic needle (of about 0.5 mm in diameter). A narrow beam from a He–Ne laser reaches the drop at half height. The plane of the picture is perpendicular to the needle; one can see the incident beam (which is thicker), the rays coming out of the droplet after the first rainbow making an angle of 138° (i.e., 180°–42°) with respect to the direction of the incident beam. This corresponds to the angle of 42° as seen by the observer.

With this method it is not possible to observe the rays of the second rainbow, nor the glory rays because the photons that impress the film are scattered by the particles of the surrounding air. Compared to the other beams the backscattered light has too weak an intensity so that we cannot record it on the film.

III. GEOMETRICAL OPTICS

Let us consider the path of an incident light ray inside a spherical water droplet with a refractive index n, coming in with an incident angle θ_i varying from 0 to 90°. Passing from air into the sphere the ray will be refracted and will form an angle θ_t with the normal, which is obtained from Snell's law (see Fig. 2):

$$\sin\theta_i = n \sin\theta_t,$$

where we write for simplicity that $n_{\mathrm{air}} = 1$. Each ray inside the sphere forms an isosceles triangle with two radii of the sphere; consequently it will get out of the sphere, backscattered, at any point in the succession of the reflections, with an angle θ_i equal to the incidence angle.

We can easily formulate the relation between θ_i and θ_t after a certain number of internal reflections. If p is the number of internal reflections + 1, and t the number of complete circles made by the ray, we get the following formula:

$$\theta_i = \theta_t\, p + (2t + 2 - p)\pi/2. \tag{1}$$

With this formula and Snell's law we conclude that water droplets can produce a backscattered ray only after four internal reflections (the refractive index of water is 1.33). As one of the first characteristics of the Brocken spectrum is that it is particularly bright, we see that geometrical optics cannot give a satisfactory explanation to the glory phenomenon.

A deeper understanding of the physics involved is necessary.

IV. WAVE THEORY

H. C. van de Hulst[6] was the first to suggest a new hypothesis: the glory is the result of surface waves provoked by grazing incident rays that enter the drop and "get out" after one internal reflection by hanging on themselves at the surface. The path along the surface will be rather short (an arc of 15°, see Fig. 2). But the assumption of van de Hulst was not justified in a quantitative manner.

The scattering theory of a plane wave by a transparent sphere was given by Nussenzveig.[7] He based his theory on

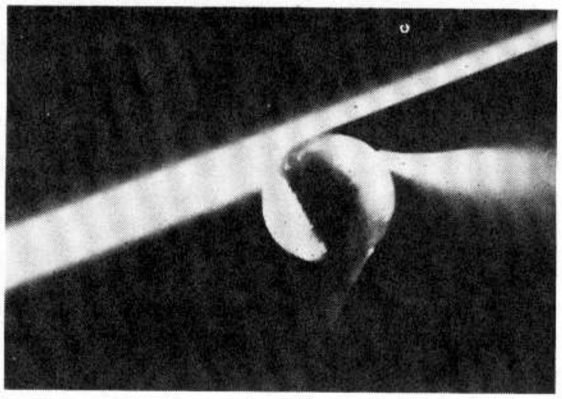

Fig. 1. Rainbow.

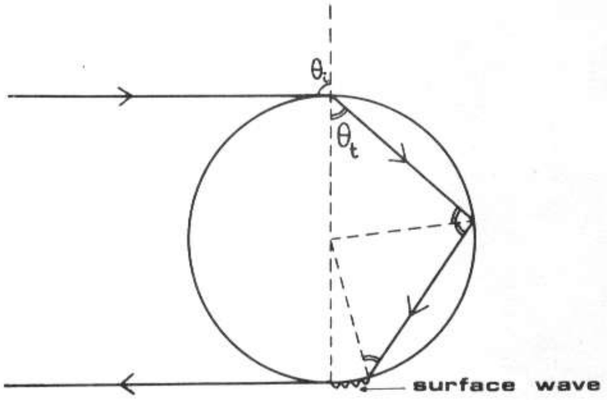

Fig. 2. Path of a light ray in a water droplet.

the formal analogy—at a given energy—between the equation for light propagation

$$\nabla^2\phi(\mathbf{r},t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi(\mathbf{r},t),$$

which for a monochromatic wave of angular frequency ω, with $\phi(\mathbf{r},t) = \varphi(\mathbf{r})e^{-i\omega t}$ becomes

$$\nabla^2\varphi(\mathbf{r}) + (\omega^2/c^2)\varphi(\mathbf{r}) = 0, \quad (2)$$

and the eigenvalues equation for the Hamiltonian of a free particle of mass m,

$$-(\hbar^2/2m)\nabla^2\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$$

or

$$\nabla^2\Psi(r) + (2mE/\hbar^2)\Psi(\mathbf{r}) = 0. \quad (3)$$

We are only interested in the optical properties of the scattering sphere: the only change to bring to Eq. (2) when light spreads into the sphere of radius a concerns the light velocity c, which becomes $c' = c/n$ if n is the refractive index of the sphere.

Equation (2) becomes, for $r \leqslant a$,

$$\nabla^2\varphi(\mathbf{r}) + (n^2\omega^2/c^2)\varphi(\mathbf{r}) = 0. \quad (4)$$

To extend the analogy with the motion of a particle of mass m the coefficient of the second term in Eq. (3) must be changed in the domain $r \leqslant a$, i.e., the particle has to be subjected to a constant potential V_0, so that Eq. (3) becomes

$$\nabla^2\Psi(\mathbf{r}) + [2m(E - V_0)/\hbar^2]\Psi(\mathbf{r}) = 0. \quad (5)$$

The set of Eqs. (2)–(4) and (3)–(5) formally correspond to each other if

$$2mE/\hbar^2 = \omega^2/c^2$$

and if

$$2m(E - V_0)/\hbar^2 = n^2\omega^2/c^2,$$

i.e., if

$$n^2 = 1 - V_0/E. \quad (6)$$

A potential well ($V_0 < 0$) corresponds to a sphere with a refractive index $n > 1$. We can see that the phase velocity of the light $c' = c/n$ in the sphere is smaller than the light velocity in vacuum.

Also the velocity of a particle of mass m,

$$v = \left(\frac{2E}{m}\right)^{1/2},$$

in vacuum becomes

$$v' = \left(2\frac{(E - V_0)}{m}\right)^{1/2} > v$$

in the sphere (if $V_0 < 0$). Therefore a diminuation of the light phase velocity is associated with an increase of the velocity of the particle of mass m.

Nussenzveig's theory is based on this analogy. Studying light scattering by a transparent sphere is equivalent to studying the scattering of a particle of mass m by a potential well that extends to the same sphere. The techniques of the scattering theory can thus be used here.

In the case of a square potential well the scattering amplitude may easily be written with the help of a partial wave expansion. We get for an incident beam of wave number k,

$$f(k,\theta) = \frac{i}{k}\sum_{l=0}^{\infty}(l + \tfrac{1}{2})[1 - S_l(k)]P_l(\cos\theta), \quad (7)$$

where $P_l(\cos\theta)$ is the Legendre polynomial of order l, and where $S_l(k)$ is determined by the continuity conditions at $r = a$ for the wave function and for its first derivative.

Nussenzveig applies to this expansion a modified form of the Watson transformation introduced to solve the problem of light scattering by an opaque sphere.[8] With this transformation one obtains the scattering amplitude by calculating the contributions of the singularities of the S function in the complex plane of angular momentum rather than from the series (7).

We thus have to calculate

$$f(\beta,\theta) = \frac{i}{2\beta}\int_C [1 - S(\lambda,\beta)]P_{\lambda-1/2}(\cos\theta)e^{-i\pi\lambda}\frac{\lambda\, d\lambda}{\cos\pi\lambda}$$

or

$$f(\beta,\theta) = \frac{i}{\beta}\sum_{m=-\infty}^{\infty}(-1)^m\int_0^{\infty}[1 - S(\lambda,\beta)] \times P_{\lambda-1/2}(\cos\theta)e^{2im\pi\lambda}\lambda\, d\lambda, \quad (8)$$

where $\beta = ka$, and where $S(\lambda,\beta)$ and $P_\lambda(\cos\theta)$ are the analytical continuation in the complex plane of $S_l(k)$ and $P_l(\cos\theta)$, and where the contour C is shown in Fig. 3.

The integration contour C may be deformed so that the integral (8) is determined by the contributions of the singularities (the poles) of $S(\lambda,\beta)$ in the complex plane λ. Numerous works have been devoted to this problem.[9]

For the case we are interested in, Nussenzveig showed that these poles belong to two different classes, respectively, along the curves γ_1 and γ_2 of Fig. 3. At the limit $\beta \gg 1$, the poles situated on γ_2 (corresponding to surface effects) are separated from each other by an interval of the order of $\beta^{1/3} \gg 1$, and their series converges rapidly since formula (8) shows that the residue obtained from a pole in $\lambda = \lambda' + i\lambda''$ is proportional to $e^{-2\pi m\lambda''}$.

The poles of class γ_1 situated near the real axis and associated to the interior of the potential give contributions whose series is only slowly converging (the imaginary part of pole λ remains about constant), and the direct application of the modified Watson transformation is faced with this difficulty. One can get around it by introducing an other expansion used by Debye. The construction of this new expansion relies on geometrical optics: the path of a ray is treated in terms of surface interactions between two unbounded media. Transmission and reflection coefficients for every

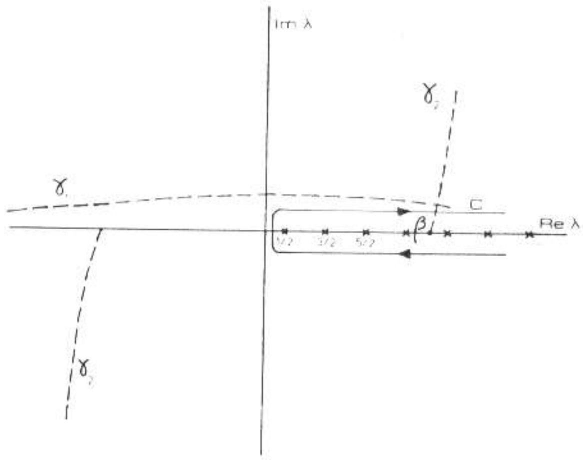

Fig. 3. Poles of $S(\lambda,\beta)$ in the λ plane (along the curves γ_1 and γ_2), and the integration contour.

spherical wave of order l are *a priori* introduced in the wave function, and their expression is determined thanks to the continuity conditions on the surface of the same wave function and of its first derivative. These conditions being the same that were used to determine $S_l(k)$ in formula (7), both forms are identifiable; and $S_l(k)$ can be deduced with respect to the transmission and reflection coefficients. The Debye expansion is thus obtained, and its interpretation is easy; a term of order j is associated to the path of a ray that has been reflected $(j-1)$ times into the sphere, after it has entered and before it comes out. The Watson transformation may then be applied to each term of the Debye expansion, their only singularities being poles of class γ_2.

With this procedure one obtains the behavior at high frequency of the scattering amplitude in every direction, and in particular a quantitative analysis of the glory.

The results essentially confirm the predictions of van de Hulst: the contributions associated to the "Regge poles" can be interpreted in terms of surface waves, which dominate if we study the backward scattering (for a sphere of radius a with an incident beam of light of wave number k with $\beta = ka$ of the order of 130, the ratio of the "geometrical" contribution to the "surface waves" contribution is 0.07; when β is increasing, the relative importance of the surface waves is decreasing). However, for a complete description of the glory it is necessary to take into account terms of order greater than 2 in the Debye expansion: although their contribution is important, rays having been once reflected into the sphere are not enough to explain all the characteristics of the glory, and in particular the periodicity of the backward intensity when β varies.

Mathematically the increase of the backward intensity comes from the asymptotic form of the Legendre polynomials when θ goes to π. The matter is thus a phenomenon of geometrical nature, analogous to "Poisson's bright spot."[10]

This may intuitively be explained as follows: the exponential decrease of the surface waves amplitude is due to the fact that at every point on its way on a main circle of the sphere a constant proportion of the energy escapes tangentially; light rays are thus sent in all directions of a plane $\varphi =$ constant, if we adopt a spherical coordinates system where z is the direction of the incident beam, θ is the inclination angle, and φ the azimuthal angle.

Looking schematically at the light as surface waves along main circles, an incident ray situated in the plane $\varphi = \overline{\varphi}$ scatters into rays of direction $(\theta, \overline{\varphi})$ where θ takes all possible values.

Inversely an observer who is in a direction $(\theta' \varphi')$ will receive two rays originating from the decomposition of only the two incident rays situated into the plane $\varphi = \varphi'$ and touching the drop at the points $r = a, \theta = \pi/2, \varphi = \varphi'$, and $\varphi = \varphi' + \pi$. But if $\theta' = 0$ (this direction being the common intersection of all the planes $\varphi =$ const) the beam leaving the drop will not come from the decomposition of two rays only, but it will come from all rays knocking tangentially the drop whatever may be the angle φ of the contact point. It is thus logical that the ratio between backward intensity and the intensity in any direction be proportional to the length of the impact circle, i.e., $2\pi a$. This fact is seen by the factor that appears in the backward-scattering amplitude, β being the only dimensionless factor proportional to a. This factor is obtained by Nussenzveig's theory.

The above representation explains also why the intensity backscattered by greater drops is no longer dominated by

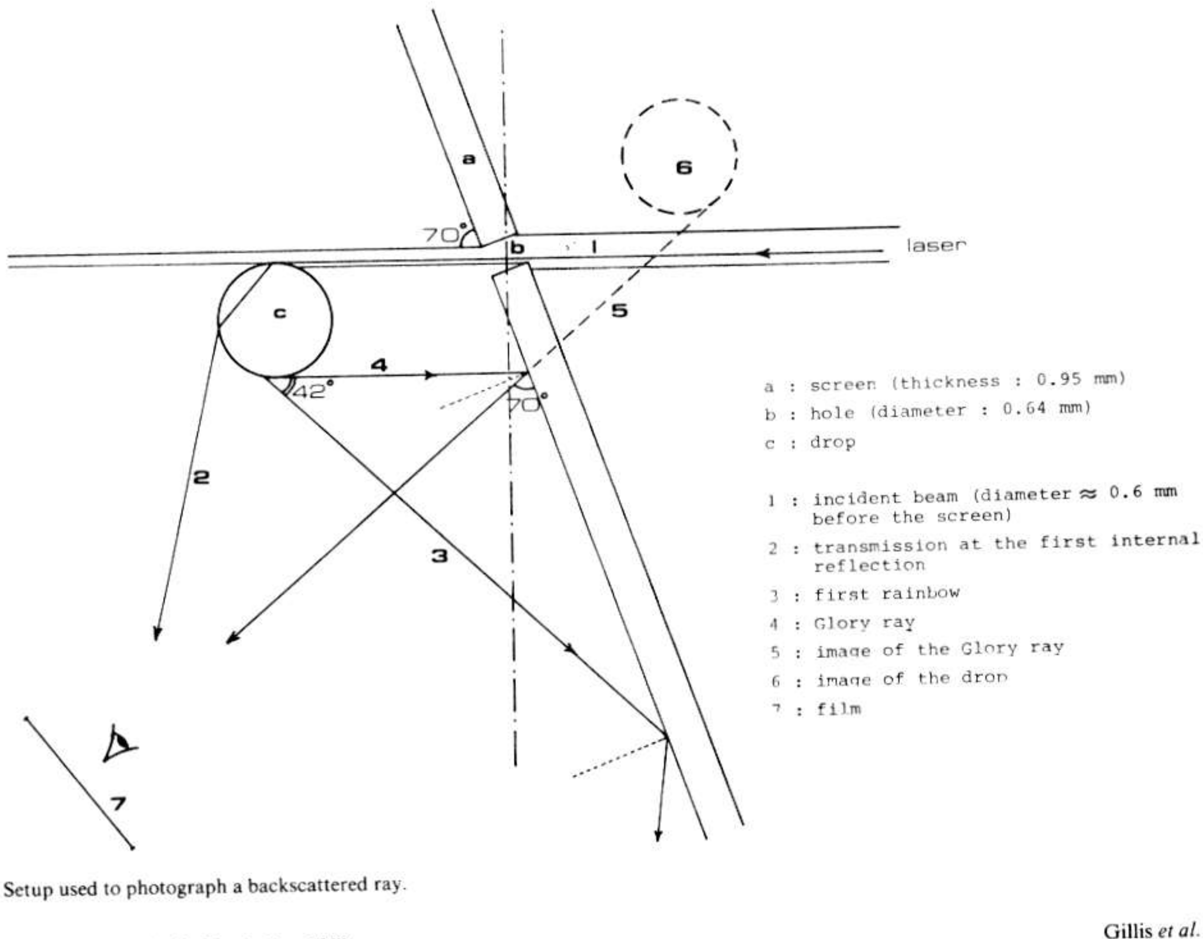

Fig. 4. Setup used to photograph a backscattered ray.

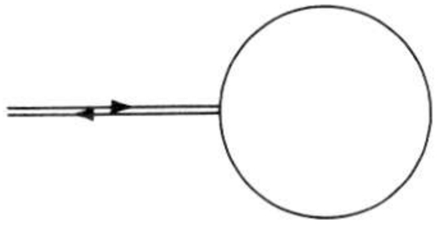

Fig. 5. Geometrical contribution of order 0.

surface waves but by geometrical contributions. In this case the path on the surface (i.e., an arc of 15°) is too long and the exponential decay cuts off the surface effects.

Several facts mentioned above may be experimentally tested. As will be described below.

We first checked by a simulation that the diffraction figure obtained, when it is assumed that the light source is composed of many circumferences (representing the boundaries of a droplet), consists of rings with predicted intensities proportional to $J_0^2(u)$ where $u = \beta\theta$. We then build a setup that allows to see the glory rays backwards and that shows that these rays leave the droplet tangentially.

V. SIMULATION OF THE GLORY

The theories described above imply in the hypothesis of surface waves that the backscattered light beam leaves the drop tangentially: it thus originates from the boundaries of this drop. This situation may be experimentally simulated, by replacing the drops by the ensemble of the emergence points of the surface waves. To do this one "draws" transparent rings on an opaque screen that is lighted from the back by a laser beam.

These rings behave like a cross section of the spherical surface of the drops. This type of simulation has been described previously.[4] We observe in the direction of the incident beam, a diffraction pattern illustrating the addition of the contributions of all drops to the coronae of the glory figure, i.e., a figure made of, respectively, bright and dark concentric rings.

The angular distribution of the intensity of the rings observed on a film follows a law in $J_0^2(ka\epsilon)$ as for the real phenomenon where J_0 is the Bessel function of order 0 and ϵ the angle where the rings are observed.

VI. EXPERIMENTAL OBSERVATIONS

It is possible by means of a simple setup to single out the light backscattered by the water droplets, i.e., the rays that participate to the glory pattern when the drop is no longer isolated for experimental convenience, but integrated in a

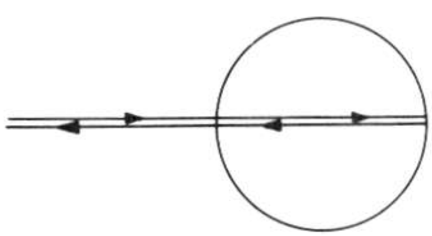

Fig. 6. Geometrical contribution of order 1.

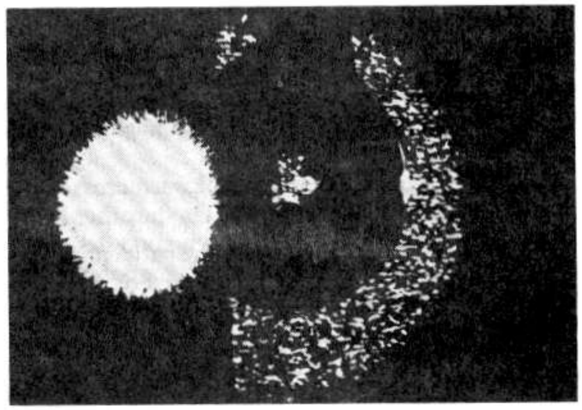

Fig. 7. Photograph of the backscattered ray.

cloud where all the contributions are added.

The setup used is shown on Fig. 4.

A droplet of pure water hanging on a thin hypodermic needle receives on its boundary the beam of a He–Ne laser (we used a 0.5-mW Spectra Physics laser). The monochromatic light passes through a hole perforated in a screen placed between the drop and the light source to receive the picture of the back of the drop.

This screen is thus the "observer," and a small mirror on it sends the rays back to a film in a camera [we used a Canon FT_b with a Macro lens FD 50-mm $f/3.5$].

Let us remark that this setup, as shown on Fig. 5, does not allow the observation of the geometrical contributions of order 0 and 1 (see Figs. 5 and 6) since the beam impinges on the drop's boundary.

We obtain a picture of the back of the droplet (see Fig. 7); its shape stands out against a halo coming from several parasitic reflections.

On the left of this shape a very bright spot corresponds to the diffusion in all direction of the incident beam on the boundary of the drop.

Part of this beam goes into the drop, is once reflected and goes out at a point situated behind the plane of the picture, and is thus invisible in our observations. From this exit point surface waves are emitted in all directions.

Although this setup does not allow one to decide which are the parts due to the "geometrical" and to the "surface" contributions in the backscattered light, it nevertheless allows one to conclude that a glory pattern is formed from the light rays backscattered by each of the water drops. Furthermore, the illumination of a band on the right boundary of the drop may be interpreted as the backwards emergence of surface waves, in the direction where the contribution of these surface waves is the more intense.

VII. CONCLUSIONS

The interest in light phenomena such as the glory or the rainbow widely extends the field of optics: the wave–particle duality, whose role in the rigorous analysis of the glory has been illustrated above, allows the transposing of a few characteristics of the light phenomenon—particularly the ideas of surface wave, or of surface interaction—to the study of particle scattering. Surface wave is useful for the description of the behavior of high-energy particles hitting a nucleus.

For example, experiences of α–^{40}Ca scattering[11] show a strong increase of backscattered intensity, and the shape of the differential cross section at backward angles is similar to the one of the glory.

The attempts at explaining this backward cross section in terms of resonances lead to failure. On the other hand, a semiclassical procedure[12] showed that a good description of the phenomenon can be given by using a potential theory, and that the internal wave ("reflected" wave) is responsible for the backscattered intensity.

The analytic expression for the differential cross section is in this case analogous to the expression for the glory: the scattering amplitude is proportional to a Bessel function of order 0. It is thus reasonable to think that the mechanisms governing heavy-ion scattering, on one hand, and the glory, on the other, are of the same nature.

We feel that whenever possible, as, for example, in a course on optics, it is useful to point out areas of physics where the same technics may be applied for the "explanation" of seemingly unrelated experimental phenomena. The case of the glory and of heavy-ion scattering may be such an example.

[1]"There is one thing I must not leave out—perhaps the greatest that ever happened to any man—and I write this to testify to the divinity and mysteries of God, which He designed to make me worthy of. From the time I had my vision till now, a light—a brillant splendor—has rested above my head, and has been clearly seen by those very few men I have wanted to show it to. It can be seen above my shadow, in the morning, for two hours after the sun has risen; it can be seen much better when the grass is wet with that soft dew; and it can also be seen in the evening, at sunset. I became aware of it in France, in Paris, since in that region the air is so much freer from mists that it can often be seen, far more clearly than in Italy where mists are much more frequent. But this is not to say that I cannot see it on all occasions and can point it out to others, but not so well as in that part of the world." From the autobiography of Benvenuto Cellini (1500–1571) translated by George Bull (Penguin, Toronto, 1956).

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[8]H. M. Nussenzveig, Ann. Phys. **34**, 23 (1965).

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[10]In the Fresnel theory of diffraction it can be shown that the flux density at the center of the diffraction pattern from a circular disk equals the flux density when there is no disk. This effect is known as "Poisson's bright spot" and is valid only for perfectly circular disks. The width of this spot may be calculated: it goes to zero as the inverse of the radius of the disk. See, for example, M. V. Klein, *Optics* (Wiley, New York, 1970).

[11]G. Gaul, H. Lüdecke, R. Santo, H. Schmeing, and R. Stock, Nucl. Phys. A **137**, 177 (1969).

[12]D. M. Brink and N. Takigawa, Nucl. Phys. A **279**, 159 (1977).